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The Ontario Curriculum

Mathematics

Mathematics Transfer Course, Grade 9, Applied to Academic

2006

Contents

Une publication équivalente est disponible en français sous le titre suivant : *Le curriculum de l'Ontario : Mathématiques – Mathématiques, 9^e année, transition du cours appliqué au cours théorique, 2006.*

This publication is available on the Ministry of Education's website, at http://www.edu.gov.on.ca.

Introduction

The Ontario Curriculum: Mathematics – Mathematics Transfer Course, Grade 9, Applied to Academic, 2006 will be implemented in Ontario secondary schools starting in 2006 for students who have completed Foundations of Mathematics, Grade 9, Applied (MFM1P) and wish to take Principles of Mathematics, Grade 10, Academic (MPM2D). This document is available only on the Ministry of Education's website, at http://www.edu.gov.on.ca.

This document is designed for use in conjunction with the curriculum policy document *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005 (Revised)*, particularly the sections entitled The Program in Mathematics, The Mathematical Processes, Assessment and Evaluation of Student Achievement, and Some Considerations for Program Planning in Mathematics.

The Place of This Transfer Course in the Curriculum

In Grade 9 and 10 mathematics, two types of courses are offered, an academic course and an applied course. (For definitions of the two course types, see *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005 [Revised]*, p.6.) "Students who successfully complete the Grade 9 academic course may proceed to either the Grade 10 academic or the Grade 10 applied course. Those who successfully complete the Grade 9 applied course may proceed to the Grade 10 applied course, but must successfully complete a transfer course if they wish to proceed to the Grade 10 academic course" *(Ibid.*, p. 6). This transfer course is made available to offer students a means of transferring from the Grade 9 applied to the Grade 10 academic mathematics course. It has been designed as a half-credit course.

The student who enrols in this transfer course has decided to change from the applied course type in Grade 9 to the academic course type in Grade 10. In many cases, the student will have decided to pursue a program pathway in Grades 11 and 12 that requires the Grade 10 academic mathematics course.

The development and application of the mathematical processes, as well as of appropriate learning skills, are critical to students' success in the transfer course and in their further study of mathematics in the Grade 10 academic course. Teachers will therefore support students in developing and applying the mathematical processes and appropriate learning skills as they study the content specified in the transfer-course expectations.

Mathematics Transfer Course, Grade 9, Applied to Academic (MPM1H)

This transfer course will provide students who have successfully completed Foundations of Mathematics, Grade 9, Applied with an opportunity to achieve the expectations not covered in that course but included in Principles of Mathematics, Grade 9, Academic.[1] On successful completion of this transfer course, students may proceed to Principles of Mathematics, Grade 10, Academic (MPM2D).

This transfer course focuses on developing number sense and algebra, linear relations, analytic geometry, and measurement and geometry through investigation, the effective use of technology, and abstract reasoning. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Prerequisite: Foundations of Mathematics, Grade 9, Applied (MFM1P)

Credit value: 0.5

1. It is important to note that some of the expectations that are common to the Grade 9 academic and applied courses (and that are therefore *not* included in this transfer course) may need to be reviewed during the transfer course. Because of differences in instructional approaches in the two course types, students who completed the applied course may not have had as much practice with algebraic representations in connection with these shared expectations as did students taking the academic course.

Mathematical process expectations. The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

PROBLEM SOLVING
- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Number Sense and Algebra

In the Number Sense and Algebra strand, students explore and apply the three basic exponent rules. Students build on their experiences in the Grade 9 applied course by manipulating polynomials with up to two variables; expanding and simplifying polynomial expressions involving one variable; solving equations, including equations with fractional coefficients; and rearranging formulas with and without substitution. Thus, students expand their understanding of the abstract realm of algebra, and become better equipped to select and apply the most appropriate strategy needed to solve a problem. Problem solving in this transfer course encourages further development of algebraic models and methods, and of the mathematical processes of communicating and representing. While conducting investigations, students consolidate numerical skills and develop the mathematical processes of reasoning and proving, communicating, and reflecting.

Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions;
- manipulate numerical and polynomial expressions, and solve first-degree equations.

Specific Expectations

Operating with Exponents

By the end of this course, students will:

– derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

– extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

Manipulating Expressions and Solving Equations

By the end of this course, students will:

– add and subtract polynomials with up to two variables [e.g., $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., computer algebra systems, paper and pencil);

– multiply a polynomial by a monomial involving the same variable [e.g., $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., computer algebra systems, paper and pencil);

– expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

– solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies);

– rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement) (***Sample problem:*** A circular garden has a circumference of 30 m. Write an expression for the length of a straight path that goes through the centre of this garden.);

– solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods (***Sample problem:*** Solve the following problem in more than one way: Jonah is involved in a walkathon. His goal is to walk 25 km. He begins at 9:00 a.m. and walks at a steady rate of 4 km/h. How many kilometres does he still have left to walk at 1:15 p.m. if he is to achieve his goal?).

Linear Relations

Most of the expectations in the Linear Relations strand of the Grade 9 applied and academic courses are common to the two courses. Some of these expectations may require review since the concepts and skills in this strand are critical for student success in the Analytic Geometry strand of this transfer course.

Three of the specific expectations in the Linear Relations strand of the Grade 9 academic course are not covered in the Grade 9 applied course. Because these expectations are closely linked to analytic geometry, they are incorporated into the Analytic Geometry strand of the transfer course, and relate to the first overall expectation. These expectations extend students' experience of investigating relationships between two variables to include designing and carrying out an investigation or experiment, thereby enhancing students' ability to apply the mathematical processes of reflecting, and reasoning and proving. Students develop the process skill of representing as they construct equations to represent linear relations and connect the equations to tables of values and to graphs, and as they determine the equation of the line of best fit for a scatter plot, using an informal method.

Analytic Geometry

In the Analytic Geometry strand, students extend their understanding of linear relations as they investigate and determine the relationship between the form of an equation and the shape of its graph, and examine different forms of the equation of a line through investigation and problem solving. Students progress from their previous experiences of analysing the constant rate of change and initial values of linear relations in context towards an abstract understanding of the significance of slope and *y*-intercept. Students extend their understanding of rate of change as $\frac{rise}{run}$ to include various formulas for the slope of a line or line segment and the properties of slopes. When applying the properties of linear relations, including the slope and *y*-intercept, students solve some problems that have a context and some problems that do not have a context and hence require a greater degree of abstract thinking. The Analytic Geometry strand demonstrates that mathematical ideas can be represented in a variety of ways, with an emphasis on the algebraic representation of linear relations and its connection to the graphical and numeric representations.

Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the characteristics of a linear relation;
- determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity;
- determine, through investigation, the properties of the slope and *y*-intercept of a linear relation;
- solve problems involving linear relations.

Specific Expectations

Understanding Characteristics of Linear Relations

By the end of this course, students will:

– design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs) (***Sample problem:*** Design and perform an experiment to measure and record the temperature of ice water in a plastic cup and ice water in a thermal mug over a 30 min period, for the purpose of comparison. What factors might affect the outcome of this experiment? How could you design the experiment to account for them?);

– construct equations to represent linear relations derived from descriptions of realistic situations, and connect the equations to tables of values and graphs, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil) (***Sample problem:*** Construct a table of values, a graph, and an equation to represent a monthly cellphone plan that costs \$25, plus \$0.10 per minute of airtime.);

– determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

By the end of this course, students will:

– determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation);

– identify, through investigation, the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;

– express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.

Investigating the Properties of Slope

By the end of this course, students will:

– determine, through investigation, various formulas for the slope of a line segment or a line (e.g., $m = \frac{rise}{run}$, $m = \frac{\text{the change in } y}{\text{the change in } x}$ or $m = \frac{\Delta y}{\Delta x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$), and use the formulas to determine the slope of a line segment or a line;

– identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

– determine, through investigation, connections between slope and other representations of a constant rate of change of a linear relation (e.g., if the cost of producing a book of photographs is \$50, plus \$5 per book, then the slope of the line that represents the cost versus the number of books produced has a value of 5, which is also the rate of change; the value of the slope is the value of the coefficient of the independent variable in the equation of the line, $C = 50 + 5p$, and the value of the first difference in a table of values);

– identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Using the Properties of Linear Relations to Solve Problems

By the end of this course, students will:

– graph lines by hand, using a variety of techniques (e.g., graph $y = \frac{2}{3}x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

– determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points) (***Sample problem:*** Compare the equations of the lines parallel to and perpendicular to $y = 2x - 4$, and with the same x-intercept as $3x - 4y = 12$. Verify using dynamic geometry software.);

– describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the y-intercept, 40, represents the \$40 cost of renting the gym; the value of the slope, 2, represents the \$2 cost per person);

– identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550).

Measurement and Geometry

In the Measurement and Geometry strand, students extend their understanding of volume as they solve problems involving the volumes of composite figures. Students also extend their knowledge of the area of composite shapes to explore the surface areas of pyramids and solve problems involving surface areas of three-dimensional figures, including composite figures. Students build on their knowledge of optimal values by investigating the effect of varying dimensions on surface area and volume for square-based prisms and cylinders. In geometry, students extend their mathematical process skill of reasoning and proving by investigating and verifying geometric properties and relationships, including properties of polygons. As students investigate and problem solve in the Measurement and Geometry strand, they strengthen their skills in applying the mathematical processes of reflecting and communicating.

Overall Expectations

By the end of this course, students will:

- solve problems involving the surface areas and volumes of three-dimensional figures;
- verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Solving Problems Involving Surface Area and Volume

By the end of this course, students will:

– solve problems involving the volumes of composite figures composed of prisms, pyramids, cylinders, cones, and spheres;

– determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square-based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);

– solve problems involving the surface areas of prisms, pyramids, cylinders, cones, and spheres, including composite figures (***Sample problem:*** Break-bit Cereal is sold in a single-serving size, in a box in the shape of a rectangular prism of dimensions 5 cm by 4 cm by 10 cm. The manufacturer also sells the cereal in a larger size, in a box with dimensions double those of the smaller box. Compare the surface areas and the volumes of the two boxes, and explain the implications of your answers.);

– identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

– explain the significance of optimal surface area or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

– pose and solve problems involving maximization and minimization of measurements of geometric figures (i.e., square-based prisms or cylinders) (***Sample problem:*** Determine the dimensions of a square-based, open-topped prism with a volume of 24 cm^3 and with the minimum surface area.).

Investigating and Applying Geometric Relationships

By the end of this course, students will:

– determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

– pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables) (***Sample problem:*** How many diagonals can be drawn from one vertex of a 20-sided polygon? How can I find out without counting them?);

– illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software (***Sample problem:*** Confirm or deny the following statement: If a quadrilateral has perpendicular diagonals, then it is a square.).

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