Lesson Outline

*Note: This unit could stand alone and be placed anywhere in the course.

**BIG PICTURE**

Students will:

- investigate properties of geometric objects using dynamic geometry software and manipulatives;
- illustrate and explain the relationship between angles formed by parallel lines cut by a transversal and interior and exterior angles of triangles and quadrilaterals;
- determine some properties of sides and diagonals of quadrilaterals.

**Note:** Students may have a very broad range of experience with using The Geometer’s Sketchpad® 4.

Skills can be taught as they are needed for each lesson, or alternatively, Introduction to The Geometer’s Sketchpad® 4 file (included in Day 2) could be used at the beginning of the unit.

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title</th>
<th>Math Learning Goals</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>It’s a Parallel World</td>
<td>• Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.</td>
<td>MG3.01, MG3.02 CGE 5a, 5c</td>
</tr>
<tr>
<td>2</td>
<td>Plane Geometry – Introduction (Part 1) GSP®4 files: Plane Geometry, Introduction to Geometer’s Sketchpad</td>
<td>• Review angles, triangles, and parallel lines through exploration. • Build skills required for future use of The Geometer’s Sketchpad®4 (GSP®4).</td>
<td>MG3.01, MG3.02 CGE 5a, 5e</td>
</tr>
<tr>
<td>3</td>
<td>Plane Geometry – Introduction (Part 2)</td>
<td>• Explore geometrical concepts (angles, triangles, parallel lines). • Build skills required for future use of GSP®4.</td>
<td>MG3.01, MG3.02 CGE 4b, 5b</td>
</tr>
<tr>
<td>4</td>
<td>What’s So Special? (Part 1) GSP®4 file: What’s So Special?</td>
<td>• Build investigation skills by exploring geometric concepts, using GSP®4. • Develop communication skills and geometric vocabulary.</td>
<td>MG3.01, MG3.02 CGE 2a, 5a</td>
</tr>
<tr>
<td>5</td>
<td>What’s So Special? (Part 2)</td>
<td>• Build investigation skills by exploring geometric concepts, using GSP®4. • Develop communication skills and geometric vocabulary.</td>
<td>MG3.01, MG3.02 CGE 5a, 5b</td>
</tr>
<tr>
<td>6</td>
<td>Interior and Exterior Angles of Triangles and Quadrilaterals GSP®4 files: Tutorial GSP, Sum of the Exterior Angles of a Triangle, Sum of the Interior Angles of a Polygon</td>
<td>• Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using GSP®4 and demonstration. • Develop skills with GSP®4 in preparation for summative assessments.</td>
<td>MG3.01, LR1.03, LR4.04 CGE 2a, 3c</td>
</tr>
<tr>
<td>7</td>
<td>Using the Properties: Connecting Algebra to Geometry</td>
<td>• Practise solving problems using the geometry explored in previous lessons. • Make connections to solving equations.</td>
<td>MG3.01, MG3.02, NA2.07, LR2.02, LR4.04 CGE 4b, 5a</td>
</tr>
<tr>
<td>8</td>
<td>Freaky Folds</td>
<td>• Use paper folding to illustrate geometric properties.</td>
<td>MG3.03 CGE 3c, 4b, 5g</td>
</tr>
<tr>
<td>9</td>
<td>Instructional Jazz</td>
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<tr>
<td>10</td>
<td>Instructional Jazz</td>
<td></td>
<td></td>
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<tr>
<td>11</td>
<td>Assessment presentation and/or test</td>
<td></td>
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</tr>
</tbody>
</table>
Unit 8: Day 1: It's a Parallel World

Math Learning Goals
- Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.

Materials
- BLM 8.1.1, 8.1.2
- washable markers
- stickers
- butterfly fasteners
- protractors

Assessment Opportunities
- Have pictures, overheads, or an electronic presentation for followup to reinforce the possibilities.
- Teachers can differentiate for students by helping them select an appropriate tool.
- Students must understand that equal corresponding or alternate angles only occur with parallel lines.
- Provide appropriate practice questions for students.

Minds On ...
Small Groups ➔ Brainstorm/Pass the Paper
In groups, students pass one piece of paper and write examples of everyday objects or situations that can be modelled using parallel lines. Examples could include fencing, ironing board top with the floor, any rectangular shape, railway tracks, etc. Record verbal responses from the groups, asking clarifying questions with respect to where the parallelism is. As a class, define parallel lines.

Action!
Pairs ➔ Guided Exploration
Provide each pair with an acetate copy of BLM 8.1.1, washable markers, a protractor, and some stickers. Cut the acetate in half, and poke a hole through each piece on the dot. Fasten the pieces of acetate with butterfly fasteners through the dots. Explain that the images on the acetates should be superimposed and rotated to create parallel lines.

Students explore, record, and justify any angle relationships they observe on BLM 8.1.2. Students can use any of the tools (e.g., stickers, markers, protractors) to determine angle relationships.

Selecting Tools and Computation Strategies/Observation/Checklist:
Circulate and observe the students as they choose their tools and strategies and as they investigate.

Consolidate Debrief
Whole Class ➔ Summarizing
Record and consolidate the relationships established by student groups. Establish that the transversal is an essential condition for angles with parallel lines. Ensure that students recognize all the possible variations of each relationship, i.e., alternate (Z), corresponding (F), co-interior (C). Students make summarizing notes.

Home Activity or Further Classroom Consolidation
Identify the types of angles found in the examples used in Minds On. Justify your answers.
Complete the practice questions.
8.1.1: Parallel Lines Exploration
# 8.1.2: Parallel Lines Exploration

<table>
<thead>
<tr>
<th>Explore and Reflect</th>
<th>Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How did you know when Line 1 and Line 2 were parallel?</td>
<td></td>
</tr>
</tbody>
</table>

## Angle Relationships

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Find one pair of equal angles. Explain how you know they are equal.</td>
<td></td>
</tr>
<tr>
<td>3. Find another pair of equal angles. Explain how you know they are equal.</td>
<td></td>
</tr>
<tr>
<td>4. Find as many pairs of angles that are supplementary (add to 180°) as you can. Explain how you know.</td>
<td></td>
</tr>
</tbody>
</table>

## Summary (to be completed as a whole class)
### Math Learning Goals
- Review angles, triangles, and parallel lines through exploration.
- Build skills required for future use of The Geometer’s Sketchpad®4 (GSP).

### Materials
- BLM 8.2.1, 8.2.2, 8.2.3
- BLM 8.2.4 (Teacher)
- chart paper

### Assessment Opportunities
- Intro GSP.qsp
- Plane Geometry.qsp
- Word Wall

### Minds On ...
Small Groups ➔ Brainstorm

**Curriculum Expectations/Observation/Mental Note:** Circulate while students are working to assess prior learning diagnostically.

For the Graffiti activity, post sheets of chart paper with the following topic titles: Triangles, Quadrilaterals, Polygons, Angles, Lines/Line Segments/Rays. Instruct students to record prior learning on one of the concept sheets. Students might draw sketches, write definitions, state properties, etc. Groups rotate so that each group visits each topic sheet and adds information.

**Whole Group ➔ Demonstration**
Demonstrate how to navigate through the GSP®4 file Plane Geometry using the How Do I...? feature and the GSP®4 Notes Booklet (BLM 8.2.2). Students may refer to these notes throughout the unit.

### Action!
Pairs ➔ Guided Exploration

Students use the GSP®4 file Plane Geometry and BLM 8.2.1 to review geometric concepts from Grades 7/8 and to build skills required for future activities with The Geometer’s Sketchpad®4. Student roles: Driver (runs the mouse), Recorder (takes notes for the pair).

Students exchange roles part way through the activity. Provide feedback to student responses.

**Learning Skills/Work Habits/Observation/Rating Scale:** Circulate to assess how individual students stay on task and in role to complete the investigations.

### Consolidate Debrief
Whole Class ➔ Discussion
Discuss what has been accomplished to date.
Ask students to bring an optical illusion for Day 3.

### Home Activity or Further Classroom Consolidation
- Make a title page for this unit.
- Complete practice questions using the theorems.
- Complete worksheet 8.2.3.
8.2.1: Plane Geometry Record Sheet

Use this page to record your observations and conclusions from the Plane Geometry GSP®4 file.
Determine the unknown angle in the right column. Give reasons for your answer.

1) Supplementary Angles
Given: \( \triangle ABC \) is a straight angle.
Explore: Measure \( \angle ABD \) and \( \angle DBC \).
Calculate the sum of these two angles.
Drag point D.
Conclude: What is the sum of the two angles?

2) Complementary Angles
Given: \( \angle AOC \) is a right angle.
Explore: Measure \( \angle AOB \) and \( \angle COB \).
Calculate the sum of these two angles.
Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
Conclude: What is the sum of the two angles?

3) Opposite Angle Theorem
Given: \( AB \) intersects \( CD \) at \( O \).
Explore: Measure \( \angle AOC \) and \( \angle DOB \).
Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
Conclude: What do you notice about \( \angle AOC \) and \( \angle DOB \)?
4) The Sum of the Interior Angles of a Triangle

Given: \( \triangle ABC \) with the interior angles identified.

Explore: Click on the action button below.

- Show the Sum of the Angles
- Reset

Drag each vertex of the triangle.
(Make sure the coloured sections marking interior angles are always inside the triangle.)

Conclude: What is the sum of the interior angles of a triangle?

5) Isosceles Triangle Theorem

Given: Isosceles \( \triangle ABC \), where \( AB = AC \)

Explore: Measure the angles opposite the equal sides
(\( \angle ABC \) and \( \angle ACB \))

Drag each of the vertices.

Conclude: What do you notice about the angles opposite the equal sides?

6) Equilateral Triangles

Given: \( \triangle ABC \), where \( AB = AC = BC \)

Explore: Measure each of the interior angles then drag point C.

Conclude: What do you notice about each of the interior angles?

7) Exterior Angle Theorem

Given: \( \triangle ABC \) and exterior angle \( \angle ABD \)

Explore: Measure the non-adjacent angles \( \angle ACB \) & \( \angle CAB \)

Calculate the sum of these two angles. Measure the exterior angle \( \angle ABC \)

Drag point A.

Conclude: What do you notice about the sum of the two non-adjacent angles, \( \angle ACB \) & \( \angle CAB \), inside the triangle and the exterior angle, \( \angle ABC \)?
8.2.1: Plane Geometry Record Sheet (continued)

8) Parallel Line Theorem - Corresponding Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the corresponding angles \( \angle BWX \) and \( \angle DXQ \).

\[ \text{Drag points } B \text{ and } X. \]

Conclude: What do you notice about the corresponding angles \( \angle BWX \) and \( \angle DXQ \)?

More... Explain why this relationship is called the F-pattern.

There are four "F-patterns" in this diagram. Can you find them?

9) Parallel Line Theorem - Alternate Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the alternate angles \( \angle BWX \) and \( \angle DXC \).

\[ \text{Drag points } B \text{ and } X. \]

Conclude: What do you notice about the alternate angles \( \angle BWX \) and \( \angle DXC \)?

More... Explain why this relationship is called the Z-pattern.

There are two "Z-patterns" in this diagram. Can you find them?

10) Parallel Line Theorem - Co-Interior Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the co-interior angles \( \angle BWX \) and \( \angle DXC \).

\[ \text{Calculate the sum of these two angles.} \]

\[ \text{Drag points } B \text{ and } X. \]

Conclude: What do you notice about the sum of the co-interior angles \( \angle BWX \) and \( \angle DXC \)?

More... Explain why this relationship is called the C-pattern.

There are two "C-patterns" in this diagram. Can you find them?
How do I …?

**Geometer’s Sketchpad Version 4**

Instruction Booklet
Created by:
8.2.2: GSP® Notes Booklet (continued)
Define each principle and determine the unknown angles.

1. Supplementary Angles
   \[ x \quad 85^\circ \]
   \[ x^\circ = = \]

2. Complementary Angles
   \[ m \quad 30^\circ \]
   \[ m^\circ = = \]

3. Opposite Angle Theorem
   \[ r \quad 70^\circ \]
   \[ r^\circ = = \]

4. The Interior Angles of a Triangle
   \[ q \]
   \[ 50^\circ \quad 55^\circ \]
   \[ q^\circ = = \]
5. Isosceles Triangle Theorem

\[ a^\circ = \quad = \]

6. Equilateral Triangles

\[ x^\circ = \quad = \]

7. Exterior Angle Theorem

\[ b^\circ = \quad = \]
8.2.3: Theorems Practice Sheet (continued)

8. Parallel Lines
   a) Corresponding Angles

   \[ m = \]

   \[ \text{\textdegree} 68 \]

   \[ m \]

   b) Alternate Angles

   \[ w = \]

   \[ \text{\textdegree} 83 \]

   \[ w \]

   c) Co-interior Angles

   \[ x = \]

   \[ \text{\textdegree} 72 \]

   \[ x \]
# 8.2.4: Word Wall List (Teacher)

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute angle</td>
<td>Obtuse triangle</td>
</tr>
<tr>
<td>acute triangle</td>
<td>Octagon</td>
</tr>
<tr>
<td>adjacent angles</td>
<td>Opposite angle</td>
</tr>
<tr>
<td>alternate angles</td>
<td>Parallelogram</td>
</tr>
<tr>
<td>bisector</td>
<td>Pentagon</td>
</tr>
<tr>
<td>circle</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>co-interior angles</td>
<td>Perpendicular bisector</td>
</tr>
<tr>
<td>complementary angles</td>
<td>Point</td>
</tr>
<tr>
<td>congruent</td>
<td>Polygon</td>
</tr>
<tr>
<td>corresponding angles</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>diagonal</td>
<td>Radius</td>
</tr>
<tr>
<td>diameter</td>
<td>Ray</td>
</tr>
<tr>
<td>equilateral triangle</td>
<td>Rectangle</td>
</tr>
<tr>
<td>exterior angle</td>
<td>Reflex angle</td>
</tr>
<tr>
<td>hexagon</td>
<td>Rhombus</td>
</tr>
<tr>
<td>interior angle</td>
<td>Right angle</td>
</tr>
<tr>
<td>isosceles triangle</td>
<td>Right angle triangle</td>
</tr>
<tr>
<td>kite</td>
<td>Scalene triangle</td>
</tr>
<tr>
<td>line</td>
<td>Side</td>
</tr>
<tr>
<td>line segment</td>
<td>Similar figures</td>
</tr>
<tr>
<td>midpoint</td>
<td>Supplementary angles</td>
</tr>
<tr>
<td>obtuse angle</td>
<td>Transversal</td>
</tr>
</tbody>
</table>
Introduction to the Geometer’s Sketchpad (GSP®4 file)
Intro GSP.gsp

Introduction to Geometer’s Sketchpad.

Geometer’s Sketchpad allows us to construct, measure, and animate and move geometric objects easily.

The four tools for constructing objects are at the left of the screen.
1) Point Tool (dot)
2) Compass Tool (circle)
3) Segment Tool (line segment)
4) Text Tool (letter A)

Try experimenting with these tools by constructing various objects on the right of the screen.

When you are finished, click on the Link button to go to Page 2.

Labeling Objects.

A point and a line segment are drawn at right.

To label them click on the Text Tool (letter A) then click on the point or the line segment.

To hide a label click on the object, choose the Display tab then choose Hide Label.

Another way to hide or display a label is to highlight the object, then from the Display tab, choose Show/Hide Label.

By highlighting an object you can delete the object from the Display tab. This does not delete it, but hides it from view.

Click on the link button to go to Page 3.

Constructions.

To construct the midpoint of CD, click on the line segment then, from the Construct menu, choose Midpoint.

To construct a line parallel to FG highlight point E and segment FG then, from the Construct menu, choose Parallel Line.

To construct a line perpendicular to FG, highlight point E and segment FG then, from the Construct menu, choose Perpendicular Line.

Click on the link button to go to Page 4.

Constructions 2

To construct the interior of ABC, click on the points ABC (order is important) then, from the Construct Menu, choose Angle Bisector.

To construct the interior of ADEF, click on the points ADEF. Then, from the Construct Menu, choose Triangle Interior.

You can change the color of the interior by highlighting it then, from the Display Menu, choose Color.

Click on the link button to go to Page 5.

Measuring Objects (segments and angles).

A line segment is drawn at right. To measure the length of the line segment, click on it then go to the Measure menu above and choose Length.

Be sure that only the line segment is highlighted. To de-highlight an object click anywhere on the screen.

The angle $\angle$CDE is drawn at right. To measure the angle click on points CDE (order is important) then go to the Measure menu and choose Angle.

To change the precision of the measurements, go to the Edit tab and choose Preferences.

Click on the link button to go to Page 6.

Measuring Objects (perimeter and area).

Construct the interior of ABC (click on points ABC then choose Triangle Interior from the Construct Menu).

Highlight the interior of the triangle then, from the Measure Menu, choose Area.

Highlight the interior of the triangle then, from the Measure Menu, choose Perimeter.

Practice: Determine the area and perimeter of parallelogram DEFG.

Click on the link button to go to Page 7.
Introduction to the Geometer’s Sketchpad (GSP®4 file) (continued)

Measuring Angles (tabulating)

\( \Delta DEF \) is constructed at right.
All of the angles have been measured as well as the sum of the angles.
Move point D to change the measures of the angles, then double-click on the tabulation table.

What do you notice? ______

Move point F then double-click on the tabulation table again.

On the next page you will learn how to make a tabulation table.

Measuring (sum of angles)

Determine the measures of the three angles of the triangle (see page 5).
When you have determined the three measures, click on the Measure menu and choose Calculate.

When you see the calculator, click on the first measure followed by the plus sign (+) then click on the second measure followed by the plus sign (+) then click on the third measure followed by OK.

Highlight all three measures as well as the sum.
Then, from the Graph menu, choose Tabulate.

Change the size of the triangle then click the tabulate box.
What do you notice? Do this a few times.

Click on the link button to go to Page 9

Practice

1- Construct any triangle and determine the midpoints of the three sides.
2- Join the midpoints of the sides.
3- Determine the area of the small triangle in the middle formed by joining the midpoints.
4- Determine the area of the large triangle.
5- Use the Calculator to divide the area of the large triangle by the area of the smaller triangle.
6- What do you notice? ______

Click on the link button to go to Page 10

Practice

Construct line segments and triangles and practice constructing the following:
1- parallel lines
2- perpendicular lines
3- angle bisectors
4- lengths of line segments
5- area and perimeters
8.2.1 Plane Geometry

Explore each of the ten activities.  
(Start by clicking on number 1.)

Each activity has a button which will bring you back to this page.

If you need help on how to use Geometer’s Sketchpad go to the ‘How do I ...?’ page by pressing the button:

Angles
- Supplementary Angles
- Complementary Angles
- Opposite Angles

Triangles
- Sum of the Interior Angles in a Triangle
- Isosceles Triangles
- Equilateral Triangles
- Exterior Angle of a Triangle

Parallel Lines
- Corresponding Angles (F-Pattern)
- Alternate Angles (Z-Pattern)
- Co-interior Angles (C-Pattern)

Explore each of the ten activities.
(Start by clicking on number 1.)

Each activity has a button which will bring you back to this page.

If you need help on how to use Geometer’s Sketchpad go to the ‘How do I ...?’ page by pressing the button:

1) Supplementary Angles

Given: \( \angle ABC \) is a straight angle.

Explore: Measure \( \angle ABD \) and \( \angle DBC \).

Conclude: What is the sum of the two angles?

2) Complementary Angles

Given: \( \angle AOC \) is a right angle.

Explore: Measure \( \angle AOB \) and \( \angle COB \).

Conclude: What is the sum of the two angles?

3) Opposite Angle Theorem

Given: \( AB \) intersects \( CD \) at \( O \).

Explore: Measure \( \angle AOC \) and \( \angle DOB \).

Conclusion: What do you notice about \( \angle AOC \) and \( \angle DOB \)?

4) The Sum of the Interior Angles of a Triangle

Given: \( \triangle ABC \) with the interior angles identified.

Explore: Click on the action button below:

Conclude: What is the sum of the interior angles of a triangle?

5) Isosceles Triangle Theorem

Given: Isosceles \( \triangle ABC \), where \( AB = AC \)

Explore: Measure the angles opposite the equal sides

Conclude: What do you notice about the angles opposite the equal sides?
Given: $\triangle ABC$, where $AB = AC = BC$

Explore: Measure each of the interior angles then drag point C.

Conclude: What do you notice about each of the interior angles?

6) Equilateral Triangles

7) Exterior Angle Theorem

Given: $\triangle ABC$ and exterior angle $\angle ABD$.

Explore: Measure the non-adjacent angles $\angle ACB$ & $\angle CAB$.

Calculate the sum of these two angles. Measure the exterior angle $\angle ABD$.

Drag point A.

Conclude: What do you notice about the sum of the two non-adjacent angles, $\angle ACB$ & $\angle CAB$, inside the triangle and the exterior angle, $\angle ABD$?

8) Parallel Line Theorem - Corresponding Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the corresponding angles $\angle BXW$ and $\angle DXP$.

Drag points B and X.

Conclude: What do you notice about the corresponding angles $\angle BXW$ and $\angle DXP$?

Note: Explain why this relationship is called the F-pattern.

There are two "F-patterns" in this diagram. Can you find them?

9) Parallel Line Theorem - Alternate Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the alternate angles $\angle BXW$ and $\angle WXC$.

Drag points B and X.

Conclude: What do you notice about the alternate angles $\angle BXW$ and $\angle WXC$?

More... Explain why this relationship is called the Z-pattern.

There are two "Z-patterns" in this diagram. Can you find them?

10) Parallel Line Theorem - Co-interior Angles

Given: AB is parallel to CD with transversal PQ.

Explore: Measure the co-interior angles $\angle BXW$ and $\angle WXD$.

Calculate the sum of these two angles.

Drag point B and X.

Conclude: What do you notice about the sum of the co-interior angles $\angle BXW$ and $\angle WXD$?

Note: Explain why this relationship is called the C-pattern.

There are two "C-patterns" in this diagram. Can you find them?

More... Explain why this relationship is called the X-pattern.

There are two "X-patterns" in this diagram. Can you find them?
Unit 8: Day 3: Plane Geometry – Introduction (Part 2)

Math Learning Goals
- Explore geometrical concepts (angles, triangles, parallel lines).
- Build skills required for future use of The Geometer’s Sketchpad®4 (GSP).

75 min

Materials
- computer/pair
- BLM 8.2.1

Assessment Opportunities
- Acronyms are good time savers when students are asked to give reasons for their answers. Students should use full terminology in work handed in. They could write acronyms on their practice sheet (BLM 8.2.3).
- Examining the theorem with GSP®4 provides convincing evidence that the theorem always works.
- Encourage students to record GSP®4 instructions on BLM 8.2.2 GSP®4 Notes Booklet for future reference.
- Consider using the Word Wall list for a charade word game as a vocabulary review.
- Provide appropriate practice questions.

Minds On ...

Pairs ➔ Discussion
Students post their optical illusions and find concrete examples in the classroom of the geometric concepts that they explored yesterday. (For example, swing the door and ask what type of angles does the door create with the wall?)

Whole Group ➔ Discussion
Review some of the concepts from the last lesson and ask students to share their concrete examples. Discuss some of the problems you observed with the students’ use of GSP®4 and BLM 8.2.2 GSP®4 Notes Booklet (see Day 2).

As a class make up acronyms for some of the theorems e.g., students may suggest OAT for the Opposite Angles Theorem. In later lessons students may use the acronyms when asked to give reasons for their answers. Discuss the idea of a theorem and the need to confirm your hypothesis with measurement.

Action!

Pairs ➔ Guided Exploration
Learning Skills/Work Habits/Checklist/Rating Scale: Circulate to check homework and see unit title page while students are working.

Students use the GSP®4 file Plane Geometry and BLM 8.2.1 to complete the review of geometric concepts from Grades 7/8 and to build skills required for future activities with Geometer’s Sketchpad. As well, they should work on completing BLM 8.2.2 GSP®4 Notes Booklet.

Student roles: Driver, Recorder. Students exchange roles part way through the activity. Provide feedback to student responses during circulation. (Students can check their answers by using the GSP®4 sketch.)

Whole Class ➔ Discussion
Using the Graffiti sheets from Day 2, students determine if anything needs to be added/changed/deleted on the topic sheets, based on what they learned during the computer exploration.

Students provide more concrete illustrations of the concepts, using objects in the classroom, e.g., pages in a book, ceiling/floor tiles.

As a class, continue to make up acronyms for the theorems, e.g., students may suggest OAT for the Opposite Angles Theorem. Students record them on the Word Wall and/or in their notes. In later lessons students may use the acronyms when asked to give reasons for their answers.

Home Activity or Further Classroom Consolidation
Ask students to bring an optical illusion to the next day’s class if they haven’t already.

Continue working on the title page for this unit, if necessary.

Complete practise questions using the theorems.
**Math Learning Goals**
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

**Assessment Opportunities**

**Special.gsp**
When students identify a property they should consider: Is it always true? Have I provided evidence to support my answers?

Emphasize that students should look for special properties and measure to show the special feature.

**Materials**
- BLM 8.4.1, 8.4.2
- data projector

**Minds On ... Whole Class ➔ Demonstration**
Students post their optical illusions. Display the GSP®4 file What's So Special? using a data projector. After each demonstration, invite students to share their observations.
- Discuss the two optical illusions on the first sketch. Students will see different things when they look at geometric diagrams.
- Click the first two demonstration buttons. Emphasize the importance of discussing and recording observations so students can learn from each other. Use the posted optical illusions to reinforce messages.
- Click the Demonstration 3 button. Discuss the hidden geometry in the constructed equilateral triangle. (Students may respond that some sketches are special because some things always remain true in any drag test.)
- Click the Demonstration 4 button and drag the points to demonstrate how to collect evidence that shows that two quantities are proportional. Show students how to measure the areas of both circles and how to determine the ratio of the areas by using the Calculate command under the Measure menu.
- Use the first activity (Special Triangles) to demonstrate how to look for something special in a sketch. Explore the first two triangles. Demonstrate how to use “tabulate” to collect evidence that supports an hypothesis, e.g., for the second triangle collect at least three table entries that show that angle LKM has a measure of 90 degrees.
- Review the process by using BLM 8.4.1 and 8.4.2, and suggest that students use them as guides for GSP®4 explorations.

**Action! Pairs ➔ Investigation**
Students work on triangle ABC and KLM of Investigation 1 in the GSP®4 file What’s So Special? Circulate and assist groups who are experiencing difficulties.

**Consolidate Debrief**
Whole Class ➔ Discussion
Using the demonstration, discuss what is so special about the triangle. Discuss what must be measured to show the special feature.

**Curriculum Expectations/Quiz/Marking Key:** Create a quiz that is based on material from Days 1–3.

**Home Activity or Further Classroom Consolidation**
Find or create a logo that has a geometric design. Describe the design so someone else has a clear picture of it.
8.4.1: “What’s So Special?” Guide Sheet

Explore!
Drag each vertex in the figure.

As you drag vertices, look for some of the following:
- measurements that always seem to be equal to each other
- measurements that never seem to change
- measurements that might have a constant ratio (proportional)
- lines that always seem to be parallel or perpendicular
- line segments that always seem to be bisected
- figures that always seem to be congruent
- objects that don’t seem to be connected, yet they move together when something is dragged

Make an Hypothesis
Decide which measurements you need to test your hypothesis.
Drag each vertex again while you pay close attention to the way the object moves and to the way the measurements change.

Test Your Hypothesis
Collect and record evidence to test your hypothesis.

<table>
<thead>
<tr>
<th>What can you measure?</th>
<th>What can you calculate?</th>
</tr>
</thead>
<tbody>
<tr>
<td>angles</td>
<td>sums</td>
</tr>
<tr>
<td>lengths</td>
<td>ratios</td>
</tr>
<tr>
<td>areas</td>
<td>formulas</td>
</tr>
<tr>
<td>perimeters</td>
<td></td>
</tr>
<tr>
<td>slopes</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8.4.2: Guiding Questions

Examine the angles...

• Are any of the angles equal?
• Do any of the angle measures always add to give the same total?
• Does the measure of any angle always stay the same?
• Are any of the angles cut in half (bisected)?

Examine the line segments...

• Are any of the lengths equal?
• Is any length proportional with any other length?
• Are any of the line segments cut in half (bisected)?

Examine the lines...

• Are any of the lines parallel?
• Are any of the lines perpendicular?
• Are any of the slopes of the lines equal?

Examine areas and perimeters...

• Are any of the areas equal?
• Are any of the perimeters equal?
• Are any of the shapes congruent?
• Are any of the shapes similar?
• Are any of the areas proportional?
• Are any of the perimeters proportional?
What's So Special? (GSP®4 file)

Special.gsp

Main Menu

Investigation 1: Special Triangles?

Exploring:
- Drag each vertex of every triangle.

Hypothesizing:
- Make a hypothesis about each triangle. Are any of the triangles special?

Collecting Evidence:
- What evidence can you find to support your hypothesis?
- What are you going to measure? Calculate? Tabulate?

Making a Conclusion:
- What kinds of triangles are shown? Were your hypotheses correct?
- Justify your answers using the evidence you collected.

Investigation 2: Parallel or Perpendicular?

Exploring:
- Drag the endpoints of each line segment.

Hypothesizing:
- Make a hypothesis based on your exploration.

Collecting Evidence:
- What evidence can you find to support your hypothesis?
- What are you going to measure? Calculate? Tabulate?

Making a Conclusion:
- What is special about the intersecting line segments? Were your hypotheses correct?
- Justify your answer using the evidence you collected.

Investigation 3: Bisected?

Exploring:
- Explore all three figures. Do you notice anything special? Record your observations.

Hypothesizing:
- Make a hypothesis.

Collecting Evidence:
- What evidence can you find to support your hypothesis?
- What are you going to measure? Calculate? Tabulate?

Making a Conclusion:
- What is special about the intersecting line segments? Were your hypotheses correct?
- Justify your answer using the evidence you collected.
Exploring:
Drag each vertex. Do you notice anything special? Measure the area and the perimeter of figures AEFG and ABCD.

Hypothesizing:
Is there anything special in this diagram?

Collecting Evidence:
What evidence can you find to support your hypothesis? What are you going to measure? calculate? tabulate?

Making a Conclusion:
Was your hypothesis correct? Is anything in the diagram proportional? Justify your answers using the evidence you collected.

Investigation 5: Special Quadrilaterals?

How do I ...?

Angle Bisector: A line that divides an angle into two equal parts.
Bisector of a line segment: A line that is perpendicular to line segment and divides the line segment into two equal parts.
Definitions:

- Acute $\triangle$: A triangle in which each of the three interior angles is acute.
- Obtuse $\triangle$: A triangle containing one obtuse angle.
- Right $\triangle$: A triangle containing a 90° angle.
- Equilateral $\triangle$: A triangle with all sides equal.
- Isosceles $\triangle$: A triangle with two equal sides.
- Scalene $\triangle$: A triangle with no equal sides.
- Perpendicular: Intersecting at 90°
- Parallel Lines: Lines in the same plane that do not intersect. They are always the same distance apart.
- Bisect: Means to cut in half. Angles or lengths can be bisected.
Math Learning Goals
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

Materials
- BLM 8.5.1

Assessment Opportunities
- Have available some examples of logos with geometric designs.

Minds On ...

**Small Groups ➔ Brainstorm**
Students form small groups and challenge each other to sketch the logo that they describe from the Home Activity assignment. Students brainstorm how to create an effective description.

**Whole Class ➔ Discussion**
Demonstrate the two triangles from Day 4.
Emphasize that there is very little that is always special about triangle ABC. (It will always have three sides and three angles).
Emphasize that triangle KLM always has a right angle at K.
Explore the other triangles in the sketch to determine what is special.

Action!

**Pairs ➔ Investigation**
Students investigate geometric relationships using sketches in the GSP®4 file, What’s So Special? (See Day 4.)
Assign one investigation to each pair for reporting purposes during Consolidate/Debrief.
Pairs may have time to do more than one of the investigations.

**Learning Skills (Teamwork)/Observation/Rating Scale:** Circulate to assess how students contribute to the group to complete the activity.

Consolidate Debrief

**Pairs ➔ Presentation**
For each of the four activities choose pairs of students to present their work to the class.

**Whole Class ➔ Discussion**
Debrief the presentations to help students understand how GSP®4 shows geometric principles.
Ask such questions as:
- How does The Geometer’s Sketchpad®4 help to collect evidence?
- How much evidence is needed to convince you that something is always true?

Application

**Home Activity or Further Classroom Consolidation**
Complete worksheet 8.5.1 Learn the Lingo.
**8.5.1: Learn the Lingo**

1. Part a) shows an example of how to complete a word chart. Complete the remaining word charts.

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
<th>Definition</th>
<th>Association</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral Triangle</td>
<td>![Equilateral Triangle Diagram]</td>
<td><strong>Definition:</strong> An <em>equilateral triangle</em> is a triangle for which all sides have the same length.</td>
<td><strong>Association:</strong> A Yield sign</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>![Triangle Diagram]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior Angle</td>
<td>![Exterior Angle Diagram]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior Angle</td>
<td>![Interior Angle Diagram]</td>
</tr>
</tbody>
</table>
### 8.5.1: Learn the Lingo (continued)

<table>
<thead>
<tr>
<th>Term: Parallel Lines</th>
<th>Visual Representation:</th>
<th>Term: Transversal</th>
<th>Visual Representation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition:</td>
<td>Association:</td>
<td>Definition:</td>
<td>Association:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term: Perpendicular Bisector</th>
<th>Visual Representation:</th>
<th>Term: Diagonal</th>
<th>Visual Representation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition:</td>
<td>Association:</td>
<td>Definition:</td>
<td>Association:</td>
</tr>
</tbody>
</table>
2. Determine the unknown angle. Give reasons for your answer.

a) \(\angle AB = AC = BC\)
\(\angle ACB = \square\)

b) \(\angle DEG = \square\)

c) \(\angle UY = 108^\circ\)
\(\angle TW = 75^\circ\)
\(\angle UVW = \square\)

d) \(\angle COD = 64^\circ\)
\(\angle FOE = \square\)
\(\angle COF = \square\)

e) \(\angle BOC = 43^\circ\)
\(\angle COE = \square\)
\(\angle EOD = \square\)

f) \(\angle BO = 118^\circ\)
\(\angle OR = 115^\circ\)
\(\angle MRQ = \square\)

g) \(\angle RVW = 42^\circ\)
\(\angle RT = RU\)
\(\angle SRT = 19^\circ\)
\(\angle RSZ = \square\)

h) \(\angle WX = WY\)
\(\angle YWX = 118^\circ\)
\(\angle WXZ = \square\)

i) Create your own question!
Unit 8: Day 6: Interior and Exterior Angles of Triangles and Quadrilaterals

**Math Learning Goals**
- Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using The Geometer’s Sketchpad®4 and demonstration.
- Develop skills with The Geometer’s Sketchpad®4 in preparation for summative assessments.

**Materials**
- data projector
- BLM 8.6.1, 8.6.2

**Assessment Opportunities**

**Minds On ...**

**Pairs → Tutorial**
In pairs, students review how to use The Geometer’s Sketchpad®4 tools for today’s activity using the GSP®4 file, Quick Tutorial and BLM 8.2.2 GSP®4 Notes Booklet.

**Whole Class → Demonstration**
Demonstrate how to construct a triangle using rays.
Demonstrate how to measure the exterior angles. Have students make a prediction.
Demonstrate how to calculate the sum of the exterior angles and tabulate the sum. Drag each of the vertices of the triangle and tabulate the results.
Students make a prediction about the exterior angles of a quadrilateral.

**Action!**

**Pairs → Exploration**
Students title a blank sketch “Exterior Angles of a Quadrilateral.” They write their hypothesis about the sum of the exterior angles of the quadrilateral. They construct a quadrilateral, measure the angles, calculate the sum of the angles and tabulate their results, then write a conclusion.
Students create another sketch titled “Exterior Angles of a Polygon.” They may choose a polygon with any number of sides and repeat the same process.
Students can use BLM 8.6.1 to help guide their activity.

**Reasoning and/or Proving/Observation/Mental Note:** Observe students’ facility with the inquiry process to determine if they need differentiated instruction to prepare for the assessment.

**Whole Class → Demonstration**
Consolidate what students should have learned from their investigation using the GSP®4 files, Sum of the Exterior Angles of a Triangle and Sum of the Interior Angles of a Polygon.

**Consolidate Debrief**

**Pairs → Application**
Students complete BLM 8.6.2.

**Reflection**

**Home Activity or Further Classroom Consolidation**
Write a letter to Abe, who missed Math class, explaining how he can determine the sum of the interior and exterior angles in a decagon (10-sided polygon).

Sample response:

\[ S = 10 \times 180^\circ - 360^\circ = 1440^\circ \]
8.6.1: Exterior and Interior Angles of a Polygon

**Part A – Exterior Angles of a Triangle**
Follow the instructions carefully to make the diagram shown.

- Plot point A and draw a ray.
- Drag point B along the ray AB and then draw a ray from B.
- Select points C and A (order is important) then use the construct menu to construct ray CA.
- **Construct** points D, E, and F on rays AB, BC, and CA respectively.
- Drag test your construction.

**Hypothesis**
I think that the sum of the exterior angles of a triangle is ________ because

**Conclusion**
Form a conclusion based on your evidence. Refer to your hypothesis.

**Part B – Exterior Angles of a Quadrilateral**
Use the steps from Part A to construct and explore the sum of the exterior angles of a quadrilateral.

**Part C – Exterior Angles of Any Polygon**
Compare the conclusions you reached in Part A and Part B. Test your hypothesis about the sum of the exterior angles of a polygon by constructing and measuring the angles of another polygon (pentagon, hexagon, etc.). Record your information and write your final conclusion about the sum of the exterior angles of any polygon.
1. Complete the chart.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Number of sides</th>
<th>Sum of interior angles</th>
<th>Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>180°</td>
<td>The sum of the angles in any triangle is 180°.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. a) Determine the sum of the interior angles in a polygon with 15 sides. Show your work.

   b) Determine the number of sides in a polygon if the sum of the interior angles is 5400°. Show your work.
8.6.2: Interior Angle Sums (continued)

3. Derek is building a deck for his summer job in the shape of a regular octagon.
   a) Define: regular octagon

   b) Determine the measure of the interior angles of the deck.
      Show your work.

4. A Canadian $1 coin, known as a loonie, is a regular polygon with 11 sides, called an
   undecagon.
   a) Define a regular polygon with 11 sides.
   
   b) Determine the sum of the interior angles of the loonie.

   c) What is the size of one of the interior angles?
GSP® Quick Tutorial (GSP®4 file)

TutorialGSP.gsp

Quick Tutorial- Geometer's Sketchpad.

Geometer's Sketchpad allows us to construct, measure, and animate and move geometric objects easily.

On the left side of your screen are your tools!

- The Selection Arrow allows you to select and drag objects.
- The Point Tool is used to construct points. It looks like a dot.
- The Compass Tool is used to construct circles. It looks like a circle with cross hairs.
- The Straightedge Tool is used to construct straight objects. If you hold the selection arrow over it it will change to allow you to select a segment, ray or line tool.
- The Text Tool is used to create and edit text. It can be used to label objects. It looks like the letter A.

TRY IT—Try experimenting with these tools by constructing various objects on the right of the screen.

1) Labeling Objects.

A point and a line segment are drawn at right.

To label them click on the Text Tool (letter A) then click on the point or the line segment.

To hide a label, click on the Object, choose the Display tab then choose Hide Label.

Another way to show or hide the label is to highlight the object then from the Display menu, choose Show/Hide Label.

By highlighting an object you can also hide the object from the Display tab. This does not delete it, but hides it from view.

2) Measuring Segments and Angles

Line segment AB is drawn at right.

One way to measure the length of the line segment, click on the line segment then go to the Measure menu above and choose Length.

Another way to measure the length of the line is to measure the distance between points. Select Point A then Point B. Go to the Measure menu above and choose Distance.

Which way do you like better?

\( \angle CDE \) is drawn at right.

To measure the angle, click on points CDE (order is important) then go to the Measure menu and choose Angle.

HINT: To change the precision of the measurements, go to the Edit tab and choose Preferences.

3) Constructing Midpoints and Angle Bisectors

To construct the angle bisector of ABC, click on the points ABC (order is important) then, from the Construct menu, choose Angle Bisector.

Did it really cut the angle in half? Measure it!

To construct the midpoint of DE, click on the line segment then, from the Construct menu, choose Midpoint

Label the Midpoint M.

Is the distance between D and M the same as M and E?

Construct a ray through point P. Hold down the straightedge tool button and choose the middle option - ray tool. Click in the blank space then on point P.

4) Constructing Interiors

To construct the interior of \( \triangle DEF \), click on the points D, E, and F. Then, from the Construct menu, choose Triangle Interior.

You can change the color of the interior by highlighting it then, from the Display menu, choose Color.

TRY IT—Make a quadrilateral and shade the interior blue.

5) Measuring Perimeter and Area of Objects

Construct the interior of \( \triangle ABC \).

Highlight the interior of the triangle then, from the Measure menu, choose Area.

Highlight the interior of the triangle then, from the Measure menu, choose Perimeter.

TRY IT—Determine the area and perimeter of parallelogram DEFG.
GSP® Quick Tutorial (GSP®4 file)  
(continued)

6) Measuring the sum of angles
Determine the measures of the three angles of the triangle \( \triangle ABC \).  
When you have determined the three measures, click on the Measure menu and choose Calculate.  
When you see the calculator, click on the first measure followed by the plus sign (+) then click on the second measure followed by the plus sign (+) then click on the third measure followed by OK.  
Highlight all three measures as well as the sum.  
Change the size of the triangle then double click the tabulate box. What do you notice? Do this a few times.

7) Constructing Parallel and Perpendicular Lines
To construct a line parallel to BC through A. Highlight point A and segment BC  
Then, from the Construct menu, choose Parallel Line.  
To construct a line perpendicular to FG through E. Highlight point E and segment FG then, from the Construct menu, choose Perpendicular Line.

8) Practice
1- Construct any triangle, Label it ABC.  
2- Determine the midpoints of the three sides. Label them P, Q and R.  
3- Join the midpoints of the sides.  
4- Determine the area of the small triangle in the middle formed by joining the midpoints.
5- Determine the area of the large triangle.  
6- Use the Calculator to divide the area of the large triangle by the area of the smaller triangle.  
7- Tabulate your measurements.  
8- What do you notice?  

9) Picture Practice
Make a picture that includes construction of the following:  
1- parallel lines  
2- perpendicular lines  
3- angle bisectors  
4- lengths of the line segments  
5- area and perimeters
Sum of the Exterior Angles of a Triangle (GSP®4 file)

Angles Triangle.gsp

Sum of the exterior angles of a triangle:
Exterior angles of a triangle are shown.
If we decrease the size of the triangle, what do you notice about the sum of the exterior angles?

Show Measurements
Make the triangle smaller
Reset the triangle

A
B
C

Exterior angles of a quadrilateral are shown.
If we decrease the size of the quadrilateral, what do you notice about the sum of the exterior angles?

Reset the quadrilateral
Make the quadrilateral smaller

A
B
C
D

4
3
2
1
5

Angle Measurements
1. 2. 3. 4. 5.

What do you notice about the sum?
Drag each vertex and observe the angle measurements.

Drag each vertex and observe the angle measurements.

Exterior Angle Demonstration

Drag each vertex and observe the angle measurements.

Drag each vertex and observe the angle measurements.

Drag each vertex and observe the angle measurements.

Drag each vertex and observe the angle measurements.

Drag each vertex and observe the angle measurements.
Sum of the Exterior Angles of a Triangle (GSP®4 file)
(continued)

**Sum of the Interior Angles of a Polygon**

- **Tearing Corners**
  - Triangle
  - Quadrilateral

- **Diagonal Divisions**
  - Summary
  - Quadrilateral
  - Pentagon
  - Hexagon vs. Regular Hexagon

- **Tiny Triangles**
  - Quadrilateral

These investigations simulate tearing off the corners of the polygon and joining them.

These investigations use the fact that any polygon can be divided into triangles by drawing diagonals. It can also be used to explore the pattern of the sum of the angles and number of sides.

This investigation uses the fact that any polygon can be divided into triangles by connecting a center point to two of the vertices. It also uses the idea of subtracting one complete rotation.

**QUADRILATERAL - Sum of the Interior Angles**

Hypothesis: What is the sum of the interior angles of a quadrilateral?

If you move the vertices of the quadrilateral does the sum remain the same?

**Interior Angles of Polygons - Looking for Patterns**

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Number of Sides</th>
<th>Sum of Interior Angles</th>
<th>Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>3</td>
<td>180°</td>
<td>FACT: The sum of the angles in any triangle is 180°.</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>4</td>
<td>Show max</td>
<td>The interior of a quadrilateral has two triangles, and sum of the angles in each triangle is 180°, so 2 x 180° = 360°.</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td>5</td>
<td>Show max</td>
<td></td>
</tr>
</tbody>
</table>
Sum of the Interior Angles of a Polygon (GSP®4 file)

Angles Polygon.gsp

Examining the Interior Angles of a Quadrilateral

Drag any point to form a new quadrilateral.

Examining the Interior Angles of a Pentagon

Drag any point to form a new pentagon.

Hexagon
Drag any vertex

Regular Hexagon
Drag point A

Sum of the Interior Angles of a Quadrilateral

Quadrilateral WXYZ consists of 4 triangles. The 4
triangles share vertex O inside the quadrilateral.
The sum of the angles at point O is 360° (since
there are 360° in one complete rotation).
The rest of the angles in the triangles form the interior
angles of the quadrilateral.

Therefore, Sum of the interior angles in WXYZ
= (sum of interior angles in 4 triangles) - 360°
= 4 x 180° - 360°
= 360°

\[ m_\angle WNZ = 92° \]
\[ m_\angle WZY = 158° \]
\[ m_\angle ZYX = 86° \]
\[ m_\angle YZW = 87° \]
Sum of Interior Angles = 360°
Math Learning Goals
• Practise solving problems using the geometry explored in previous lessons.
• Make connections to solving equations.

Materials
• BLM 8.7.1 (Teacher)
• BLM 8.7.2
• envelopes
• Smart Ideas software

Assessment Opportunities
Extension:
Some students are ready to solve equations with variables on both sides. Challenge these students to create new Equation Strips.

Some students will need more instruction on solving simple equations. This lesson provides an opportunity to differentiate instruction based on student readiness.

Application
Create five quiz questions, write solutions, and check them for accuracy.
### 8.7.1: Equation Strips

<table>
<thead>
<tr>
<th>1.</th>
<th>[2a + a = 90]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[3a = 90]</td>
</tr>
<tr>
<td></td>
<td>[\frac{3a}{3} = \frac{90}{3}]</td>
</tr>
<tr>
<td></td>
<td>[a = 30]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.</th>
<th>[2b = 40 + 70]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[2b = 110]</td>
</tr>
<tr>
<td></td>
<td>[\frac{2b}{2} = \frac{110}{2}]</td>
</tr>
<tr>
<td></td>
<td>[b = 55]</td>
</tr>
</tbody>
</table>
8.7.1: Equation Strips (continued)

3. 

\[2b + 3x = 100\]

\[5x = 100\]

\[\frac{5x}{5} = \frac{100}{5}\]

\[x = 20\]

4. 

\[4c + 50 = 170\]

\[4c + 50 - 50 = 170 - 50\]

\[4c = 120\]

\[\frac{4c}{4} = \frac{120}{4}\]

\[c = 30\]
8.7.1: Equation Strips (continued)

5. 

\[ 5g - 32 = 108 \]

\[ 5g - 32 + 32 = 108 + 32 \]

\[ 5g = 140 \]

\[ \frac{5g}{5} = \frac{140}{5} \]

\[ g = 28 \]
8.7.2: Connecting Algebra to Geometry

1. a) The sum of the interior angles in a triangle is:

b) An equation that models the sum of the interior angles in this triangle is:

c) Solve the equation to determine the value of $x$.

d) Use the value of $x$ to calculate the size of:

$\angle W$:  
$\angle Y$:  
$\angle Z$:  

2. a) The sum of the angles in a right angle is:

b) Write 2 equations to model the sums of the 2 sets of angles that add to $90^\circ$:

(i) 

(ii) 

c) Solve these equations to determine the values.

(i) solve for $x^\circ$  
(ii) solve for $y^\circ$ 

d) Use the values of $x$ and $y$ to calculate the size of:

$\angle CBP$:  
$\angle ABQ$:  

TIPS4RM: Grade 9 Applied – Unit 8: Plane Geometry
3. Write an equation and solve for the unknown. State the theorem used to make the equation.

a) \[ 60^\circ \quad 2a^\circ \]

b) \[ 130^\circ \quad b^\circ + 20^\circ \]

c) \[ 120^\circ \quad 2c^\circ + 40^\circ \]

d) \[ 3d^\circ + 10^\circ \quad 100^\circ \]

e) \[ 75^\circ \quad e^\circ - 15^\circ \]

f) \[ f^\circ - 32^\circ \quad 78^\circ \]

g) \[ 3g^\circ + 44^\circ \quad 5g^\circ - 12^\circ \]

h) \[ 4h^\circ - 56^\circ \quad 2h^\circ + 44^\circ \]
8.7.2: Connecting Algebra to Geometry (continued)

i)  

j)  

k)  

l)  

m)  

n)
Unit 8: Day 8: Freaky Folds

Math Learning Goals
- Use paper folding to illustrate geometric properties.

Materials
- origami paper
- scissors
- BLM 8.8.1
  (Teacher)

Assessment Opportunities
This website is designed for young children but provides a good selection of appropriate origami patterns.
For example:
- fish
- iris (a bit tricky)
- cat (easy!)
- dragon fortune teller

Whole Class ➔ Demonstration
Introduce origami to the class as the art of paper folding. Give each student an origami or square piece of paper. Demonstrate how to fold the paper to form a paper drinking cup (use an overhead of BLM 8.8.1). Students perform the same folds.

Students then unfold their paper cup. Discuss the kinds of geometry they see in the folds.
For example:
- triangle EFB, triangle BFG, triangle DHI, triangle DIJ are congruent and isosceles
- triangle HCG and triangle JAE are right isosceles triangles
- EF is parallel to IH
- JI is parallel to FG
- JE || DB || GH

Pairs ➔ Investigation
Students make two of the same item – leave one folded and the second one unfolded. They tape their unfolded one onto another piece of paper and label all the geometry in preparation for their presentation. Ask students to make measurements to provide evidence that they have correctly identified the geometry.

Pairs ➔ Presentation
Students present to the class their origami figure and explain the geometry they found in the unfolded shape.

Communicating/Presentation/Rubric: Assess students on their use of appropriate terminology and the clarity of their justifications as they make their presentations.

Home Activity or Further Classroom Consolidation
Look in newspapers, magazines, or on the Internet for a logo that has some geometric properties. Paste a picture of your logo on a piece of paper and identify and describe the geometry in the logo.
Submit your work for assessment.
8.8.1: Origami Paper Cup (Teacher)

1. Cut out the pattern and fold the square paper in half (lines facing out).
2. Fold right on the dotted line towards you, fold left outwards from you.
3. Tuck the top triangle inside the paper, one on each side paper.