Unit 7
Algebraic Models: Making Connections

Grade 9 Applied

Lesson Outline

BIG PICTURE

Students will:

- describe relationships between variables graphically;
- use first differences to determine that relationships are linear or non-linear;
- describe linearly related data graphically, in words, and algebraically;
- make connections among number, graphic, and algebraic models;
- look for patterns and use variables to generalize/summarize/simplify.

Day	Lesson Title	Math Learning Goals	Expectations
1 2	Linear and Non-Linear Investigations	• Investigate linear and non-linear relationships. • Examine first differences and the shape of the graph. • Explore the effects of changing the conditions. • Write equations for linear relationships and describe non-linear relationships.	NA2.07, LR3.02, LR3.04, LR4.03, LR4.05 CGE 5a, 7i
3	Building Models *Presentation file:* **Patterns**	• Use multiple representations (physical, numerical, algebraic). • Develop an understanding that simplification is necessary to determine if two algebraic expressions are equivalent.	NA2.01, NA2.05, LR4.03, LR4.04, LR4.05 CGE 5a, 7i
4	Simplifying Algebraic Models *Presentation files:* **Collecting Terms using Algebra Tiles**, **Expanding and Simplifying Algebraic Equations**	• Use multiple representations (physical, numerical, algebraic). • Simplify algebraic expressions.	NA2.01, NA2.05, NA2.06, LR4.03, LR4.04, LR4.05 CGE 2a, 2b
5	Multiple Representations	• Use multiple representations (concrete, pictorial, numerical, algebraic, graphical). • Simplify algebraic expressions.	NA2.01, NA2.05, NA2.06, LR4.03 CGE 2b, 5a
6	Use of Variables as Bases of Power	• Use variables as a base for powers up to degree 3.	NA2.01, NA2.03, NA2.04, NA2.05 CGE 2a, 2c
7	Algebraic Models in Measurement and Geometry	• Use variables to make connections between symbolic and concrete models from measurement and geometry application problems. • Evaluate expressions after substitution of a value for a variable. • Substitute into algebraic equations and solve for one variable in the first degree.	NA2.01, NA2.03, NA2.04, NA2.08 CGE 5c
8		Instructional Jazz	
9		Instructional Jazz	
10		Assessment	

Unit 7: Day 1: Linear and Non-Linear Investigations (Part 1)

Grade 9 Applied

75 min

Math Learning Goals

- Investigate linear and non-linear relationships.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials

- BLM 7.1.1, 7.1.2
- BLM 7.1.3 (Teacher)
- see BLM 7.1.3 for additional materials

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Explain what the students will be doing at each station.

Review terminology: linear and non-linear; rate of change and initial value (refer to Word Wall).

Action!

Small Groups → Carousel of Activities

Learning Skill (Teamwork)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' contributions to the group as they complete the activities.

Arrange the four stations by placing the appropriate materials and one colour-coded copy of the experiment (BLM 7.1.2) at each station.

Students complete each of the four experiments and record their answers on BLM 7.1.1 (You will need four copies per group).

Consolidate Debrief

Whole Class → Connecting

After students have completed all four of the experiments, help them make the connection between the first differences and the type of relationship (linear and non-linear). If students have not finished all four of the experiments, allocate more time the next day and make connections then. (See Day 2 for guiding questions.)

See Answers to Experiments (BLM 7.1.3).

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Complete the following journal entry:

Sally was not in class today. She doesn't know how to use differences to determine if a relationship is linear or non-linear. Use words, pictures, and symbols to explain it to her.

7.1.1: Record Sheet

Group:

Investigation #:

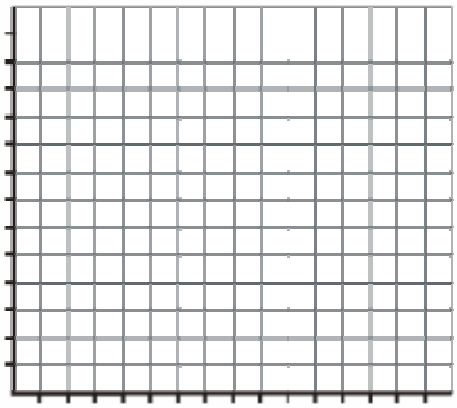
Hypothesis:

Mathematical Models:

Numerical: Complete the table of values and calculate the differences.

		First Differences

Graphical: Make a scatter plot and draw the line of best fit.



Algebraic Model: (or a description of the relationship in words)

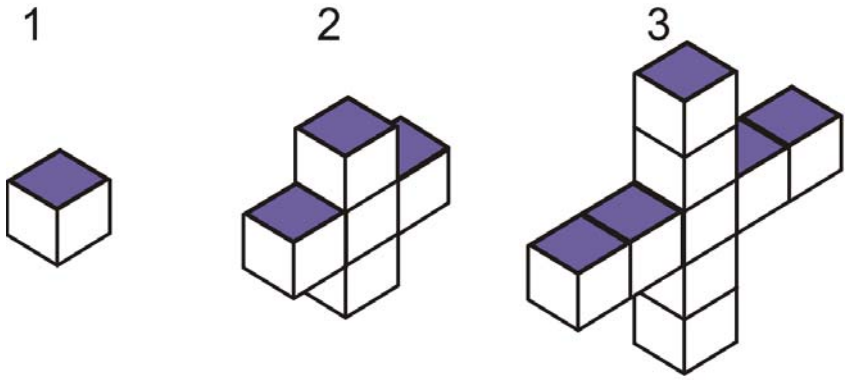
Conclusion:

7.1.2: Linear and Non-Linear Investigations

Investigation 1 – Building Crosses

Purpose
Find the relationship between the figure number and the total number of cubes.

Procedure
Using linking cubes, make **two** more figures by adding a cube to each end of the cross.



Hypothesis
Write your **hypothesis** on the Record Sheet.

- We think that as the figure number increases, the total number of cubes will

  _____ because _____ .
  increase or decrease

- We think that the relationship will be _____ .
  linear or non-linear

- The data is _____ .
  continuous or discrete

Mathematical Models
- Record your observations in the table provided and calculate the first differences.
- Make a scatter plot and draw the line (or curve) of best fit.
- Determine the algebraic model or describe the relationship in words.

Conclusion
Make a conclusion. Refer to your hypothesis.

Answer the following questions on the back of the Record Sheet.

1. How many cubes are required to make model number 10? Show your work.

2. What figure number will have 25 cubes?

3. How would adding two blocks to each end of the cross rather than one affect the graph and the equation?

Investigation 2 – Pass the Chocolate Bar

Purpose
Find the relationship between the number of pieces of "chocolate bar" remaining and the total number of times the chocolate bar was passed around.

Procedure
Every time the chocolate bar is passed, you "eat" half ($\frac{1}{2}$) of what remains.

Hypothesis
Write your hypothesis on the Record Sheet.

- We think that the more times the chocolate bar is passed, the number of pieces remaining will _____ because _____.
  increase or decrease
- We think that the relationship will be _____ .
  linear or non-linear
- The data is _____ .
  continuous or discrete

Mathematical Models
- Record the number of pieces of the chocolate bar that remain after 0 passes, 1 pass, 2 passes (up to 4 passes).
- Calculate the first differences.
- Make a scatter plot and draw the line (or curve) of best fit.
- Determine the algebraic model or describe the relationship in words.

Conclusion
Make a conclusion. Refer back to your hypothesis.

Answer the following questions on the back of the Record Sheet.

1. How many pieces of chocolate bar will remain after 6 passes? Show your work.

2. Using this method of eating the chocolate bar, when will it be fully "eaten"? Explain.

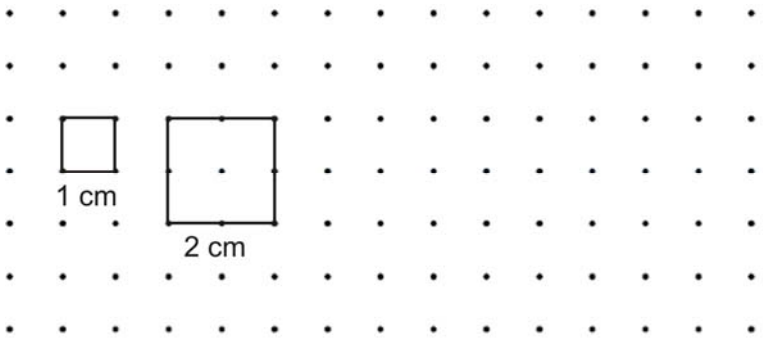
3. If the chocolate bar began with 32 pieces instead of 16, how would the graph be different? Include a sketch of the original graph and the new graph on the same set of axes. Give reasons for your answer.

7.1.2: Linear and Non-Linear Investigations (continued)

Investigation 3 – Area vs. Length of a Square

Purpose
Find the relationship between the area and the length of a side of a square.

Procedure
- On grid paper, draw squares with side lengths of 1 cm, 2 cm, 3 cm, and 4 cm.
- Draw and calculate the area of squares with sides measuring 1 cm, 2 cm, 3 cm, and 4 cm.



Hypothesis
Write your hypothesis on the Record Sheet.
- We think that as the side length increases, the area will _____ (*increase or decrease*)

  because _____ .
- We think that the relationship will be _____ (*linear or non-linear*) .
- The data is _____ (*continuous or discrete*) .

Mathematical Models
- Record your observations in the table provided and calculate the first differences.
- Make a scatter plot and draw the line (or curve) of best fit.
- Determine the algebraic model or describe the relationship in words.

Conclusion
Make a conclusion. Refer back to your hypothesis.

Answer the following questions on the back of the Record Sheet.
1. What is the area of a square with a side length of 9 cm?
2. What side length does a square with an area of 100 cm^2 have?
3. Describe the pattern in the first differences.

7.1.2: Linear and Non-Linear Investigations (continued)

Investigation 4 – Burning the Candle at Both Ends

Purpose
Find the relationship between the number of blocks and the figure number.

Procedure
- Using cube links, build a long chain with 20 blocks.
- To create the next figure, remove 1 block from each end.
  Record the number of blocks remaining.
- Repeat this process four more times.

Hypothesis
Write your hypothesis on the Record Sheet.
- We think that as the figure number increases, the total number of blocks will

  _____ because _____.
  increase or decrease

- We think that the relationship will be _____ .
  linear or non-linear
- The data is _____ .
  continuous or discrete

Mathematical Models
- Record your observations in the table provided and calculate the first differences.
- Make a scatter plot and draw the line (or curve) of best fit.
- Determine the algebraic model or describe the relationship in words.

Conclusion
Form a conclusion. Refer back to your hypothesis.

Answer the following questions on the back of the Record Sheet.
1. How many cubes are required to make figure number 7? Show your work.

2. What figure number will have 4 cubes?

3. How would removing 2 blocks from each end of the "candle" rather than 1 affect the graph and the equation?

4. If 5 more blocks were added to the original model, how would that affect the graph and the equation?

7.1.3: Answers to Investigations (Teacher)

Investigation	Materials Required
1 – Building Crosses	cube links (49)
2 – Pass the Chocolate Bar	square colour tiles to represent the chocolate bar (16)
3 – Area vs. Length of a Square	1 cm grid paper
4 – Burning the Candle at Both Ends	20 cube links

Investigation 1

Figure No.	No. of Cubes	First Difference
1	1	
		4
2	5	
		4
3	9	
		4
4	13	
		4
5	17	

T = No. of cubes
n = Figure no.
$T = 4n - 3$
Linear
Discrete

Investigation 2

Number of Passes	No. of Pieces	First Difference
0	16	
		-8
1	8	
		-4
2	4	
		-2
3	2	
		-1
4	1	

Non-Linear
Discrete
No. of pieces = $2^{(4-p)}$
p = No. of passes

Investigation 3

Side Length	Area	First Difference
1	1	
		3
2	4	
		5
3	9	
		7
4	16	
		9
5	25	

Non-Linear
Continuous
Area = $(\text{Length})^2$

Investigation 4

Figure No.	No. of Cubes	First Difference
1	20	
		-2
2	18	
		-2
3	16	
		-2
4	14	
		-2
5	12	

$T = 22 - 2n$
Linear
Discrete
No. of cubes = 22 – 2 (Figure no.)

7.1.3: Answers to Investigations (Teacher)
(continued)

Investigation 1 – Building Crosses

1. Figure number 10 requires 37 cubes to construct it.

2. Figure number 7 requires 25 cubes to construct it.

3. The graph would be steeper. The rate of change would be 8 instead of 4.
   (**Note:** new equation is T = 8*n* – 7)

Investigation 2 – Pass the Chocolate Bar

1. After 6 passes, there will be $\frac{1}{4}$ of a piece of the chocolate bar remaining.

2. Using this method the chocolate bar will never be fully eaten, because you always eat $\frac{1}{2}$ of what is remaining.

3. The graph will have a higher initial value. It will start at 32 instead of 16.

Investigation 3 – Area vs. Length of a Square

1. The area of a square with a side length of 9 cm is 81 cm^2.

2. A square with an area of 100 cm^2 will have a side length of 10 cm.

3. The pattern in the first differences is, 3, 5, 7, 9, in other words, odd numbers starting at 3.

Investigation 4 – Burning the Candle at Both Ends

1. Figure number 7 requires 8 cubes to construct it.

2. Figure number 8 requires 6 cubes to construct it.

3. The rate of change of the graph would be steeper and the rate of change would be 4 instead of 2. (**Note:** The new equation is No. cubes = 24 – 4*n*)

4. If 5 more blocks are added to the original model, the initial value will be higher.
   (**Note:** The new equation is No. of cubes = 27 – 2*n*.)

Unit 7: Day 2: Linear and Non-Linear Investigations (Part 2)

Grade 9 Applied

75 min

Math Learning Goals

- Investigate linear and non-linear relationships through investigation.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials

- graph paper
- BLM 7.2.1
- BLM 7.2.2 (Teacher)

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Summarize how to identify whether a relationship is linear or non-linear using first differences. (BLM 7.2.1)

Action!

Small Groups → Carousel of Activities

Students continue to complete the experiments if not completed from Day 1.

Learning Skill (Initiative)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' initiative as they work in their groups.

Consolidate Debrief

Whole Class → Connecting

Use the following guiding questions:

- Which experiments had a linear relationship? (Take up equations using the graph, and identify the rate of change and the initial value.)
- Identify the rate of change and initial value for each linear relation. Write the equation for each relation.
- How can you use the table of values to predict if a relationship will be linear or non-linear? (Emphasize that the x values are increasing by 1, and that the differences are all the same.)

Discuss how changing the conditions of the experiments affects the graph (linear only).

Discuss with the students whether or not it makes sense to join the points on the graph based on whether the relationship is discrete or continuous.

As an alternate approach to taking up the activities, have students present their answers to the activities.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Graph the relationships from worksheet 7.2.1 and identify the rate of change and the initial value for the linear relationships. Write the equation for each relation.

Solutions to BLM 7.2.1 are provided on BLM 7.2.2.

7.2.1: Linear or Non-Linear

Complete the tables of values and determine if the relationship is linear or non-linear.
Give reasons for your answers.

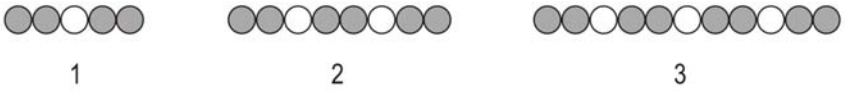


Figure Number	Number of Shaded Circles	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

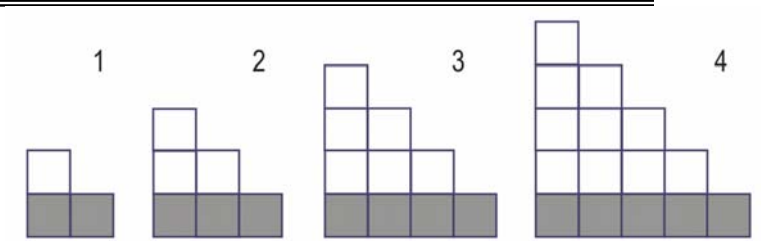
Figure Number	Number of Unshaded Circles	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

Figure Number	Total Number of Circles	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

7.2.1: Linear or Non-Linear
(continued)



Complete the tables of values and determine if the relationship is linear or non-linear. Give reasons for your answers.

Figure Number	Number of Shaded Squares	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

Figure Number	Number of Unshaded Squares	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

Figure Number	Total Number of Squares	First Differences
1		
2		
3		
4		
5		

This relationship is _____ because _____
linear or non-linear

7.2.2: Linear or Non-Linear Answers (Teacher)

Figure Number	Number of Shaded Circles	First Differences
1	4	
2	6	6 – 4 = 2
3	8	8 – 6 = 2
4	10	2
5	12	2

This relationship is linear because the model number increases by 1 each time and the first differences are all the same value (2).

Figure Number	Number of Unshaded Circles	First Differences
1	1	
2	2	2 – 1 = 1
3	3	3 – 2 = 1
4	4	1
5	5	1

This relationship is linear because the model number increases by 1 each time and the first differences are all the same value (1).

Figure Number	Total Number of Circles	First Differences
1	5	
2	8	8 – 5 =3
3	11	11 – 8 = 3
4	14	3
5	17	3

This relationship is linear because the model number increases by 1 each time and the first differences are all the same value (3).

7.2.2: Linear or Non-Linear Answers (Teacher)
(continued)

Figure Number	Number of Shaded Squares	First Differences
1	2	
		3 – 2 = 1
2	3	
		4 – 3 = 1
3	4	
		1
4	5	
		1
5	6	

This relationship is linear because the model number increases by 1 each time and the first differences are all the same value (1).

Figure Number	Number of Unshaded Squares	First Differences
1	1	
		3 – 1 = 2
2	3	
		6 – 3 = 3
3	6	
		10 – 6 = 4
4	10	
		5
5	15	

This relationship is non-linear because the model number increases by 1 each time and the first differences are NOT all the same value.

Figure Number	Total Number of Squares	First Differences
1	3	
		6 – 3 = 3
2	6	
		10 – 6 = 4
3	10	
		5
4	15	
		6
5	21	

This relationship is non-linear because the model number increases by 1 each time and the first differences are NOT all the same value.

75 min

Math Learning Goals

- Use multiple representations (physical, numerical, algebraic).
- Develop an understanding that simplification is necessary to determine if two algebraic expressions are equivalent.

Materials

- computer/data projector
- BLM 7.3.1, 7.3.2
- algebra tiles
- pattern blocks

Assessment Opportunities

Minds On ...

Whole Class → Presentation

Use the electronic presentation Patterns to introduce number patterns and terminology.

Patterns.ppt

Pattern recognition is a valuable skill. Discuss how it is used in different careers/disciplines, e.g., music, sports, art, history.

Pairs → Discussion

Students complete BLM 7.3.1 in pairs, comparing and refining responses.

Students work with another pair to compare/refine their responses to the problems.

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior knowledge.

Pair more capable students with a partner who needs support.

Action!

Whole Class → Setting Context

Introduce the task (BLM 7.3.2) and establish a purpose for finding a pattern (e.g., Frieda may want to know how many chairs she needs for 32 tables or how many tables she needs for 108 people).

Small Groups → Guided Exploration

Students complete BLM 7.3.2.

Consolidate Debrief

Whole Class → Discussion

Debrief the Feeding Frenzy activity to determine that students can build an algebraic model from a number pattern and that students recognize that there may be more than one correct algebraic model. Compare the equivalent models and simplify them to demonstrate that they are the same. Discuss and compare the patterns in both parts.

Home Activity or Further Classroom Consolidation

Concept Practice
Making Connections

- Use algebra tiles to show that the given three expressions are equivalent:
  (i) $2 + 4n$ (ii) $1 + 2n + 2n + 1$ (iii) $6 + (n - 1)(4)$
- Journal entry: Jason thinks that both of the Feeding Frenzy examples show a linear relationship between the number of tables and number of chairs. What evidence can you offer to support his claim?
- Practise your skills with algebraic expressions.

Provide appropriate practice questions.

7.3.1: Number Patterns

Determine an algebraic expression for the *n*th term.

Hints:

a) What's the same in each term?
b) What's different in each term?
c) When determining an expression for *n*, write the "same" parts first, then think about the "different" parts.
d) Is there a relationship between the term number and the "different part"?

1.

Term Number	Term	Expression
1	1	1(1)
2	2	2(1)
3	3	3(1)
4	4	4(1)
n		

2.

Term Number	Term	Expression
1	0	1 – 1
2	1	2 – 1
3	2	3 – 1
4	3	4 – 1
n		

3.

Term Number	Term	Expression
1	10	7 + 3(1)
2	13	7 + 3(2)
3	16	7 + 3(3)
4	19	7 + 3(4)
n		

4.

Term Number	Term	Expression
1	7	7 + 5(0)
2	12	7 + 5(1)
3	17	7 + 5(2)
4	22	7 + 5(3)
n		

5.

Term Number	Term	Expression
1	14	2 + 4(3)
2	18	2 + 4(4)
3	22	2 + 4(5)
4	26	2 + 4(6)
n		

6.

Term Number	Term	Expression
1	26	5(6) – 4
2	31	5(7) – 4
3	36	5(8) – 4
4	41	5(9) – 4
n		

7.

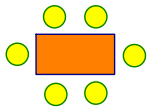
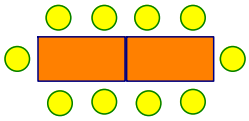
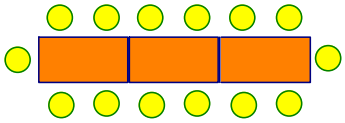
Term Number	Term	Expression
1	1	2(2) – 3
2	3	2(3) – 3
3	5	2(4) – 3
4	7	2(5) – 3
n		

7.3.2: Feeding Frenzy – Patterning to Algebraic Modelling

Part A

Frieda runs a catering business. She often has to set up table arrangements like the ones shown below.
Help her determine the number of chairs and/or tables that she needs.

1. Start by completing the Number of Chairs column.
   (You may wish to use the algebra tiles to create physical models for terms 4, 5, and 6.)

Term Number	Picture	Number of Chairs	Expression
1		6	
2			
3			
4			
5			
6			

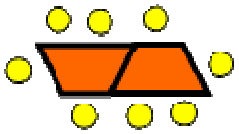
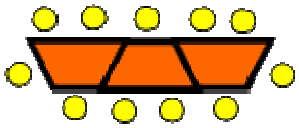
2. a) Build a number pattern in the last column.
   b) Use the number pattern to find an expression for the Number of Chairs if the term number is *n*.

7.3.2: Feeding Frenzy (continued)

Part B

Frieda runs a catering business. She often has to set up table arrangements like the ones shown below. Frieda sometimes uses trapezoidal tables.
Help her determine the number of chairs and/or tables that she needs.

1. Start by completing the Number of Chairs column. (You may wish to use pieces from the pattern blocks set to create physical models for terms 4 and 5.)

Term Number	Picture	Number of Chairs	Expression
1		5	
2			
3			
4			
5			

2. a) Build a number pattern in the last column.

   b) Use the number pattern to find an expression for the Number of Chairs for *n* tables.

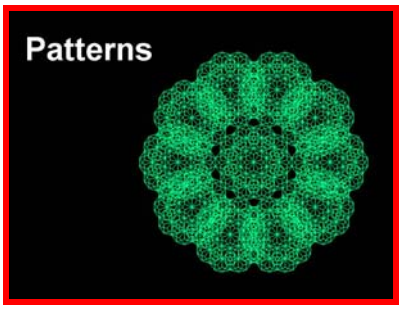
   c) Explain how your answer to part (b) relates to the picture model.

3. Find a different but equivalent algebraic model. Explain how it relates to the picture model.

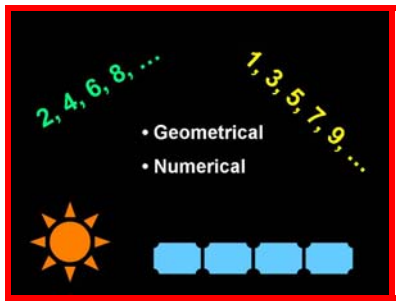
Patterns (Presentation Software File)

Patterns.ppt

1

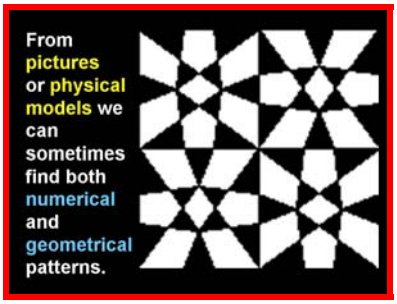


2



3

4



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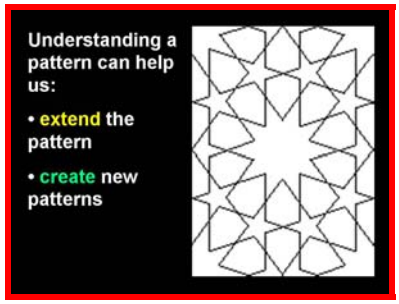

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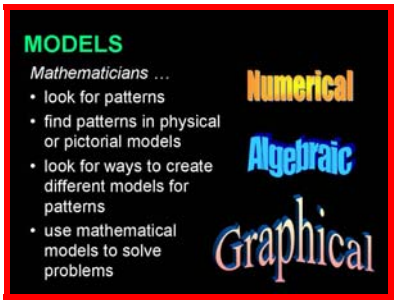

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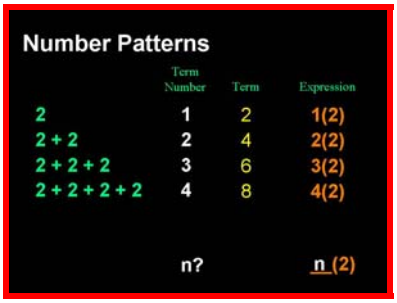

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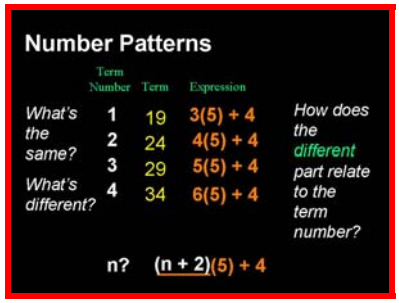

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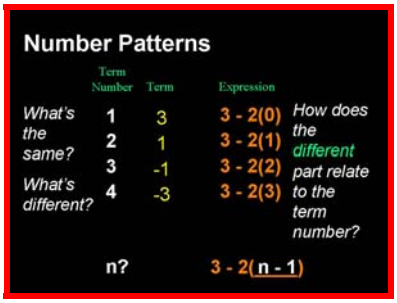

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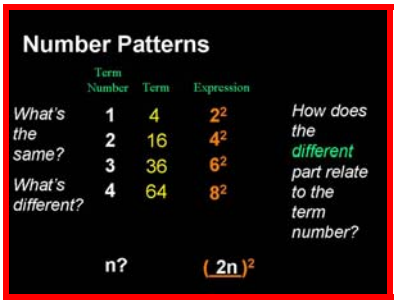

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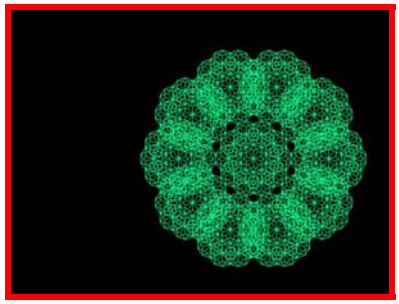


Unit 7: Day 4: Simplifying Algebraic Models

Grade 9 Applied

75 min

Math Learning Goals

- Use multiple representations (physical, numerical, algebraic).
- Simplify algebraic expressions.

Materials

- overhead projector
- algebra tiles
- computer/data projector
- BLM 7.4.1, 7.4.3 (Teacher)
- BLM 7.4.2, 7.4.4

Assessment Opportunities

Minds On ...

Small Groups → Practice

Students found a relationship between the term number and the expression for the term (Day 3). Provide each group with a set of answer cards. Hold up a cue card with an expression and students match it with one of their set.

Students explain their choices.

Clear up any misunderstandings.

Whole Class → Practice

Do several examples like the following:

- What is an algebraic expression for "3 more than double a number"? (Answer: $3 + 2x$)
- Use algebra tiles to create a model for the expression. (Answer: Use the overhead to show 2 x-tiles and 3 one-tiles.)

Collecting Terms.ppt,

Algebraic Equations.ppt

The activity Cue Cards is designed to give students practice in creating algebraic expressions from words.

Set a date with students for a proficiency test on simplifying algebraic expressions.

Action!

Whole Class → Presentation

Use electronic presentations Collecting Terms Using Algebra Tiles, and Expanding and Simplifying Algebraic Expressions.

Pairs → Practice

Students complete BLM 7.4.2.

Learning Skill (Work Habits)/Observation/Rating Scale: Observe and record how students work to complete the assignment.

Consolidate Debrief

Whole Class → Connecting

Use Debrief Notes to consolidate learning and make connections between numerical, algebraic, and graphical models (BLM 7.4.3).

Application

Home Activity or Further Classroom Consolidation

Complete worksheet 7.4.4 We're All Correct!

How can you determine if two expressions are equivalent?

Students may need additional practice questions.

7.4.1: Reading in Mathematics – Cue Cards (Teacher)

Purpose

- To practise symbol vocabulary

Method

- Give each individual (or pair of students or small group) a set of cards.
- Read a cue card. Ask students to select and hold up the matching response(s). Visually check for correct responses.

Student Cards (enlarge before making sets)

$n + 7$	$7 - n$	$n \div 7$	$7 \div n$
$n - 7$	$7n$	$7n + 1$	$7(n+1)$

Cue Cards

A number n less than 7	7 more than a number n	A number n decreased by 7	The sum of a number n and 7
7 less than a number n	The difference between a number n and 7	7 divided by a number n	The quotient of a number n and 7
7 subtracted from a number n	7 times a number n increased by 1	The product of a number n and 7	A number n increased by 7
7 times a number n	The product of number n increased by 1 and 7		

7.4.2: Algebraic Expressions

Simplify each algebraic expression.
Create a word statement for your answer.
The first question is completed as an example.

	Algebraic Expression	Word Statement
1.	$4x + 20 - 3x + 6$ $= x + 26$	twenty-six more than a number
2.	$3(2x - 4)$	
3.	$2(x + 4)$	
4.	$5x - 3 + 2(x + 1)$	
5.	$3(2x + 3) - 2(2x + 3)$	
6.	$(3x^2 + 4x - 3) + (2x^2 - 2x + 1)$	

7.4.3: Debrief Notes (Teacher)

Using algebra tiles is one way to explore expanding and simplifying algebraic expressions. Use the notes below to consolidate student understanding by discussing numerical, algebraic and graphical models.

The following example uses the algebraic equation $y = 3(x - 1)$. Ensure that students understand the difference between this equation and just the right side $3(x - 1)$, which is an expression.

Complete the tables of values below. Students will see that the tables are identical even though different expressions were used to calculate y.

a) $y = 3(x - 1)$

x	***y***
-2	
-1	
0	
1	
2	

b) $y = 3x - 3$

x	***y***
-2	
-1	
0	
1	
2	

Use a graphing calculator or graphing software to show that the graphs of $y = 3(x - 1)$ and $y = 3x - 3$ are identical.

Ask students to describe the process that would transform $3(x - 1)$ into $3x - 3$.

If time permits, students work in small groups to practise simplifying and expanding these expressions:

a) $4(2x + 3)$

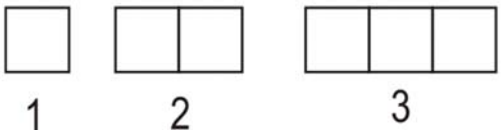
b) $2(2x + 4) + 3(x - 5)$

c) $3(2x) - 4 + 2(5x)$

d) $(x - 4) - 2(3x + 1)$

7.4.4: We're All Correct!

Reconciling Equivalent Algebraic Expressions



How many toothpicks are needed for *n* squares?

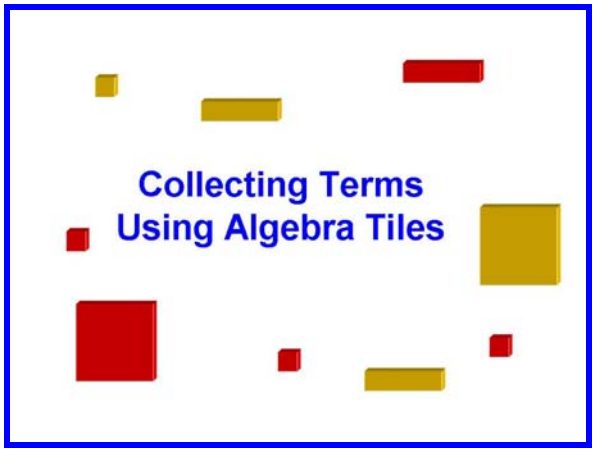
Show a picture of each student's thinking. Explain why each solution is correct.

Anju's Solution  "If *T* is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 1 plus three times the number of squares." My equation is $T = 1 + 3n$.	**Erin's Solution**  "If *T* is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 4 plus three times one less than the number of squares." My equation is $T = 4 + 3(n - 1)$.
Silva's Solution  "If *T* is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 2 times the number of squares plus one more than the number of squares." My equation is $T = 2n + (n + 1)$.	**Bijuan's Solution**  "If *T* is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 4 times the number of squares minus one less than the number of squares." My equation is $T = 4n - (n - 1)$.

Collecting Terms Using Algebra Tiles (Presentation software file)

Collecting Terms.ppt

1



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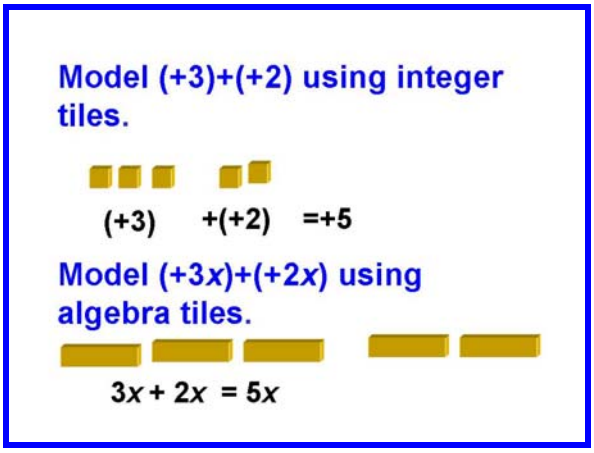

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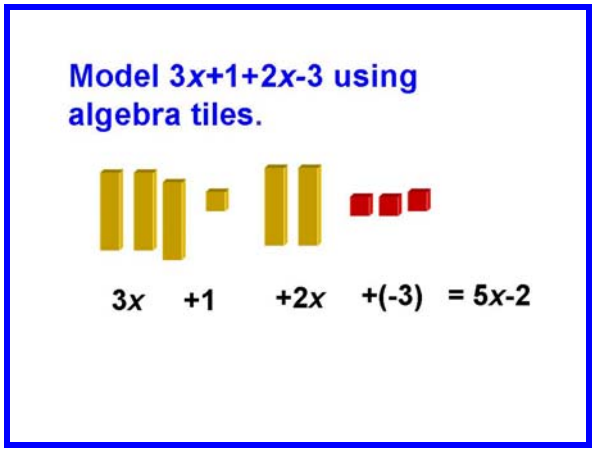

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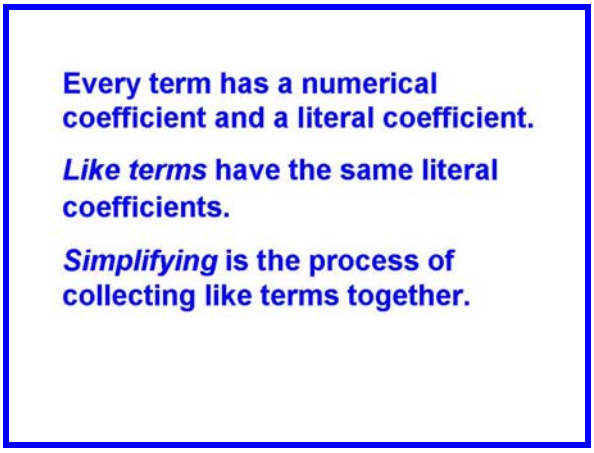

Expanding and Simplifying Algebraic Expressions

(Presentation software file)

Algebraic Expressions.ppt

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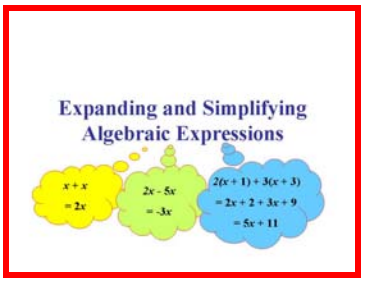

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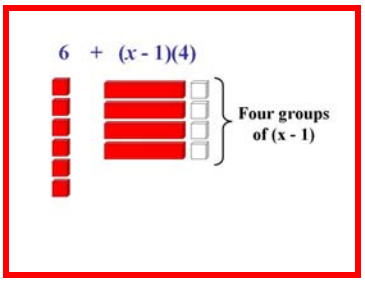

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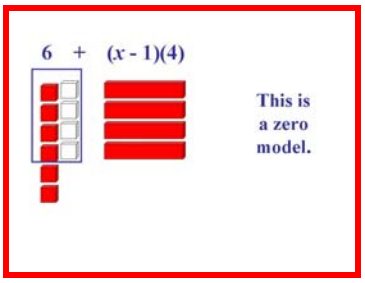

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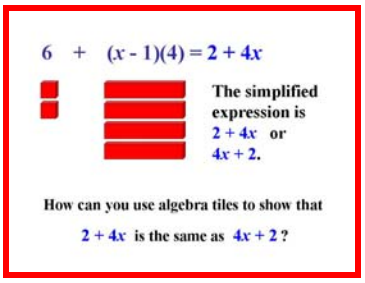

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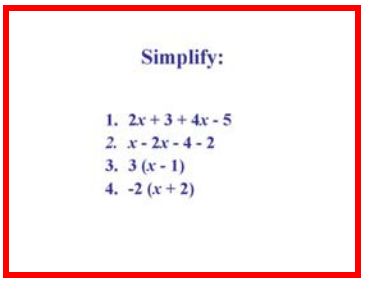

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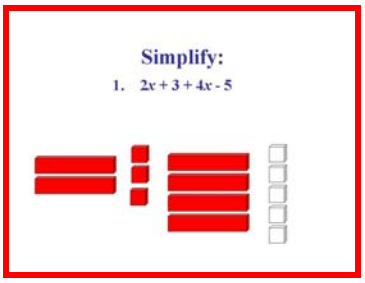

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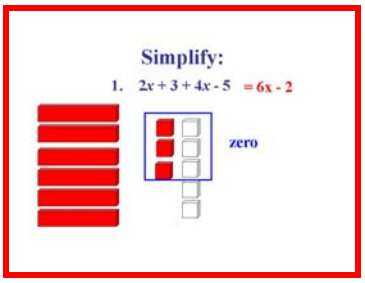

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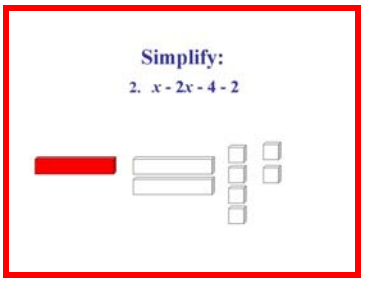

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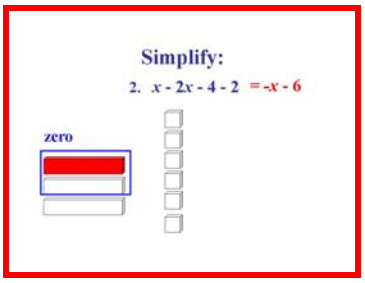

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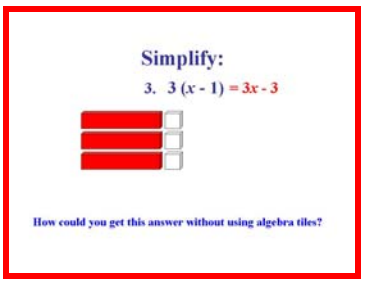

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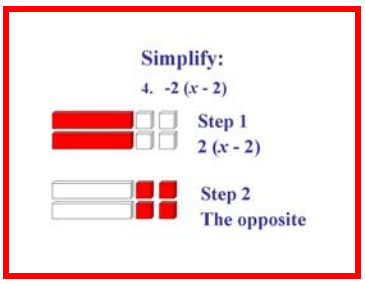

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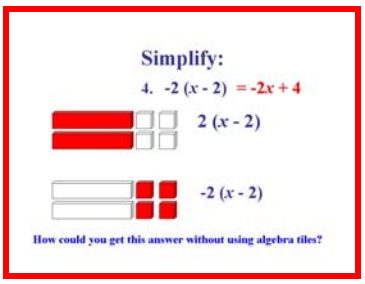

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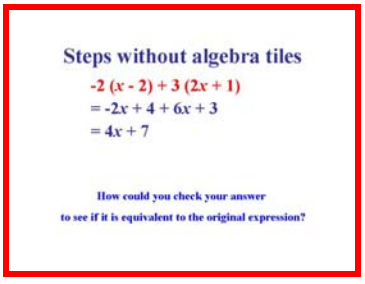


75 min

Math Learning Goals

- Use multiple representations, (concrete, pictorial, numerical, algebraic, graphical).
- Simplify algebraic expressions.

Materials

- BLM 7.5.1, 7.5.2
- pattern blocks
- colour tiles
- toothpicks

Assessment Opportunities

Minds On ...

Whole Group → Discussion

Using toothpicks, students demonstrate how to build the thinking models from We're All Correct! (Day 4).

Action!

Groups → Problem Solving

Students work in groups of three or four.

Each group completes one activity, BLM 7.5.1 or 7.5.2, using manipulatives.

Curriculum Expectations/Observation/Mental Note: Assess communication as students discuss the solution to their group's problem.

Individual→ Problem Solving

Each student completes the activity that their group did not do.

Learning Skills/Observation/Checklist: Assess how students work independently.

Consolidate Debrief

Whole Group → Connecting (Guided)

In a whole group discussion, guide students as they respond to the question:
How would you determine if any of the relationships are linear?

Refer to the tables and first differences, and the equation (compare different methods for finding the equation e.g., using rate of change and initial value or using patterns), to check the appearance of the graphs.

Help students see connections between the different types of models.

Concept Practice

Home Activity or Further Classroom Consolidation

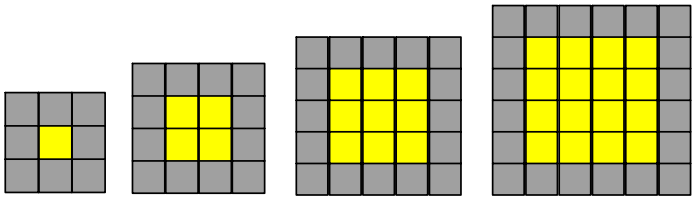
Complete the practice questions.

Provide appropriate practice questions.

7.5.1: The Frame Problem

Problem

The Capture-It Company makes picture frames.
Tiles are used for the border of the frames.
The light area represents the square space for the picture.



Procedure

Marla and Tim work together to find an algebraic model to represent this problem.
They build models with colour tiles and count the number of dark tiles needed on pictures of different sizes.

Frame Number (n)	Number of Dark Tiles (d)
1	
2	
3	
4	
5	
6	

Marla and Tim determined different equations to represent the relationship between the frame number (n) and the number of dark tiles (d).

Marla's equation: $d = 2(n + 2) + 2n$

Tim's equation: $d = 4(n + 1)$

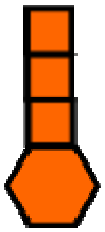
1) Determine your own equation to represent the relationship between the frame (n) and the number of dark tiles (d).

2) Compare your equation with Marla's and Tim's to determine that they represent equivalent algebraic models.

7.5.2: The Walkway Problem

Problem

The Larry's Landscaping Company makes walkways. One walkway starts with a hexagonal piece of concrete. To make the walkway longer, square pieces are added.



This walkway begins with a hexagon and has three square pieces added. The length of each side is the same. The perimeter of this walkway is 12.

Procedure

Cara and Cal work together to find an algebraic model to represent this problem. They build a model with pattern blocks and determine the perimeter of the walkway. The perimeter only includes sides on the outer edge of the walkway.

Number of Squares (n)	Perimeter (P)
1	
2	
3	
4	
5	
6	

Cara and Cal determined different equations to represent the relationship between the number of squares (n) and the perimeter (P).

Cara's equation: $P = 5 + 2n + 1$

Cal's equation: $P = 6 + 4n - 2n$

1) Determine your own equation to represent the relationship between the number of squares (n) and the perimeter (P)?

2) Compare your equation with Cara's and Cal's to determine that they represent equivalent algebraic models.

75 min

Math Learning Goals

- Use variables as the bases of powers up to degree 3.

Materials

- graphing calculators
- BLM 7.6.1 (Teacher)
- BLM 7.6.2

Assessment Opportunities

Minds On ...

Whole Class → Activating Prior Knowledge

Lead a demonstration of building models to show the powers of x using geometric representations. Refer to BLM 7.6.1.

Action!

Pairs → Guided Exploration

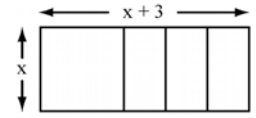
Students extend their understanding of expansions involving the multiplication of powers with variable bases up to degree 3 (BLM 7.6.2).

In the last question students collect like terms to complete a solution.

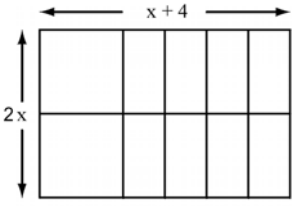
Consolidate Debrief

Whole Class → Connecting (Guided)

Revisit the expressions (BLM 7.6.2), showing visual representations such as:

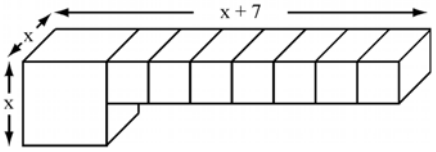


Shows $x\,(x+3) = x^2 + 3x$



Shows $2x\,(x+4) = 2x^2 + 8x$

Length, Width, Area



Shows $x\,(x^2+7) = x^3 + 7x$

Depth, Surface Area, Volume

Model only positive terms to connect these concepts to algebra tiles.

It may be appropriate to spend another day helping students connect algebraic manipulations (expansions and collection of like terms) with using manipulatives.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete the practice questions using variable as bases of powers.

Provide appropriate practice questions.

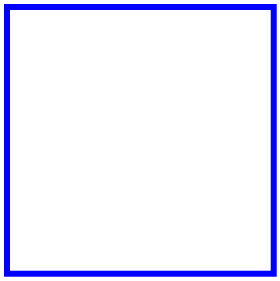
7.6.1: Modelling Powers of x (Teacher)

Build models using concrete materials such as

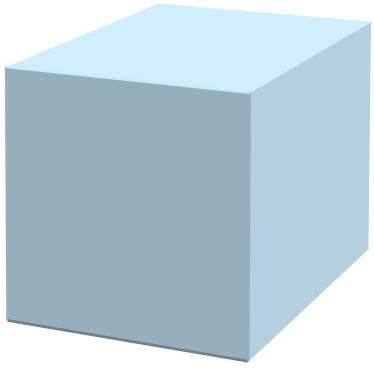
- toothpick and marshmallow
- chopsticks and playdough
- science molecule building kits
- skewers and polystyrene balls

1. Use an object of a fixed length, e.g., a toothpick, chopstick. Assign the length the value of x units.

2. Build a square of side length x to represent the value of x^2. (Reinforce the idea that x^2 is the area.)



3. Build a cube of side length x to represent the value of x^3. (Reinforce the idea that x^3 is the volume.)



7.6.2: Powers with Variable Bases (Numeric, Graphical, and Algebraic Models)

a) Using a graphing calculator or spreadsheet, create a table of values for:

$y = x(x + 3)$

x	y
-2	
-1	
0	
1	
2	

$y = x^2 + 3x$

x	y
-2	
-1	
0	
1	
2	

How do the tables compare?

b) Using a graphing calculator or graphing software, set the window at ZOOM 4.
- Graph $y = x(x + 3)$. Record the graph, in blue, on the grid paper.
- Graph $y = x^2 + 3x$ and record the graph, in red, on the same paper.
- How do the graphs compare?

The tables of values in a) and the graphs in b) should be identical.
- What must this mean about the expressions $x(x + 3)$ and $x^2 + 3x$?
- What process would transform $x(x + 3)$ into $x^2 + 3x$?

c) On the back of this paper, create tables of values and compare them for:
$y = x(x^2 + 2)$ $y = x^3 + 2x$
- What process would transform $x(x^2 + 2)$ into $x^3 + 2x$?

d) Graph $y = 2x(x – 2)$ and $y = 2x^2 – 4x$ on the same axes and compare the graphs.
What process would transform $2x(x – 2)$ into $2x^2 – 4x$?

e) Explain why $y = x(x)(x)$ and $y = x(x^2)$ and $y = x^3$ have identical graphs.

f) The process of distributing through the brackets is called “expansion” or “distribution.”

Expand the following on the back of this worksheet:
1. $2x(x + 4)$
2. $3x(x^2 + 2x)$
3. $4x(3x^2 + 2x – 5)$
4. $-3a(a^2 – 4a)$
5. $5x^2(3x – 4)$

Check your understanding
Three students were asked to expand this expression: $x(x^2 – 2x + 3x)$

Kevin’s answer	Sal’s answer	Ari’s answer
$x(x^2 – 2x + 3x)$ $= x^3 – 2x^2 + 3x^2$	$x(x^2 – 2x + 3x)$ $= x(x^2 + x)$ $= x^3 + x^2$	$x(x^2 – 2x + 3x)$ $= x^3 – 2x^2 + 3x^2$ $= x^3 + x^2$

Which solution is the most efficient?
Explain your choice.

75 min

Math Learning Goals

- Use variables to make connections between symbolic and concrete models from measurement and geometry application problems.
- Evaluate expressions after substitution of a value for a variable.
- Substitute into algebraic equations and solve for one variable in the first degree.

Materials

- graphing calculators
- BLM 7.7.1

Assessment Opportunities

Minds On ...

Whole Class → Introduction

Check understanding with questions taken from the Home Activity from Day 6. Discuss these problems to clarify any misunderstandings.

Learning Skill (Work Habits)/Rating Scale: Check Home Activity work.

Demonstrate the solution of a two-step algebraic equation verbally. [Two more than 3 times a number is 11 (or 3 times a number plus 2 is 11).] Ask students to verbalize their thinking.

Action!

Whole Class → Problem Solving

Students solve measurement problems leading to equations (BLM 7.7.1). Using the equation created, review substituting for a variable and simplifying or solving by systematic trial. Discuss the opportunity to "work at your own pace."

Guide the worksheet activity. Provide help and prompts for students as they work. Allow students to check their answers with you after they have completed each page so you can correct any misunderstandings.

As individual students gain confidence, they proceed to work on the tasks individually. Students who need extra support continue to work with the teacher while all students are encouraged to self-assess.

Consolidate Debrief

Whole Class → Planning

Discuss the power of algebraic models to represent "all cases" and allow a solution for one variable when other variables have known values.

Collect work.

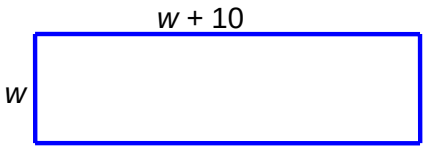
Home Activity or Further Classroom Consolidation

Exploration Reflection

Respond in your journal: What jobs might use algebra to model measurement relationships?

7.7.1: Solving Measurement Problems

A) Sam makes rectangular paving stones that are 10 cm longer than they are wide.

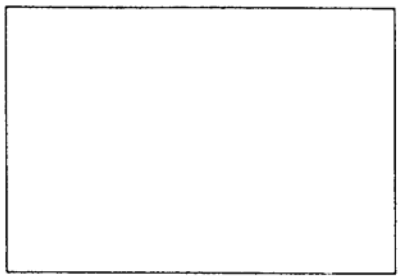


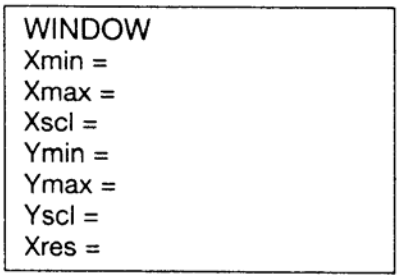
1. Determine a formula for the **perimeter** in terms of *w*.
   (Hint: formula for finding the perimeter of a rectangle is $P = 2(l + w)$)

2. Use this formula to calculate the perimeter when the width is 6.75 cm.

3. Use a graphing calculator to graph the equation describing the perimeter.
   a) Write the equation you entered: $Y =$ _____

   b) Sketch a graph in the space provided below.



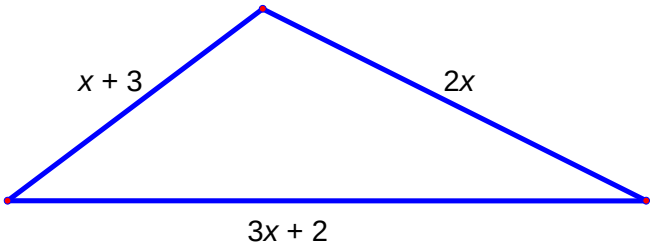


   c) Trace to locate the (width, perimeter) corresponding to the calculation in question 2.
   $X =$ _____ $Y =$ _____

4. Use the formula to calculate the width when the perimeter is 60 cm.

7.7.1: Solving Measurement Problems (continued)

B) This diagram shows the size of the sides in terms of x.



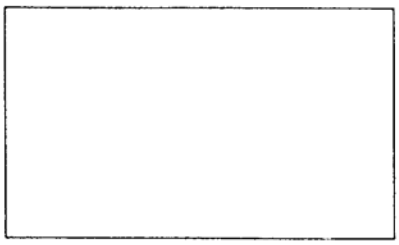
1. Determine a formula for the perimeter in terms of x.
   (Hint: formula for finding the perimeter of a triangle is $P = a + b + c$)

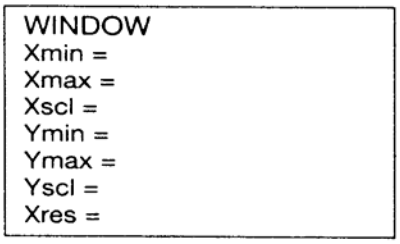
2. Use this formula to calculate
   a) the perimeter when x is 3 cm

   b) the length of each of the sides when $x = 3$.

3. Use the graphing calculator to graph the equation describing the perimeter.
   a) Write the equation you entered: $Y =$ _____

   b) Sketch a graph in the space provided below.



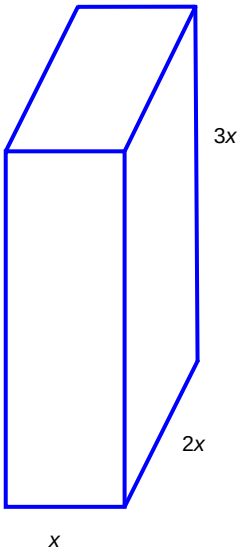


   c) Use Trace to locate the point (x, perimeter) corresponding to the calculation in question 2a). You can also use [2nd] TABLE (over the GRAPH key) to see the table of values.
   $X =$ _____ $Y =$ _____

4. Use the formula to calculate the value of x when the perimeter is 41 cm.

7.7.1: Solving Measurement Problems (continued)

C) Explain how this model shows that the length is 2 times the width and the height is 3 times the width.



1. Determine a formula for the volume.

2. Use this formula to calculate the volume when the width is 225 m.

Challenge

Can you find the width of the shape that has a volume of 162 cm^3?