

Exploring Relationships: Lines and Curves of Best Fit

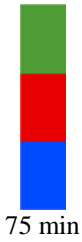
Lesson Outline

BIG PICTURE

Students will:

- describe relationships between variables using graphical models;
- connect graphical features to the characteristics of the relationship (no relationship, strong/weak correlation, positive/negative correlation, linear and non-linear, discrete/continuous);
- use a line of best fit to predict values and solve problems;
- use first differences to determine whether relationships are linear or non-linear.

| Day | Lesson Title | Math Learning Goals | Expectations |
|-----|------------------------------------|--|--|
| 1 | Looking for Relationships (Part 1) | <ul style="list-style-type: none"> • Identify a trend in a scatter plot. • Use a trend to describe the relationship between the variables. • Make a prediction about a relationship in preparation for an investigation on Day 2. | LR1.01, LR1.02, LR1.04, LR4.01 CGE 2c, 5e |
| 2 | Looking for Relationships (Part 2) | <ul style="list-style-type: none"> • Investigate a relationship between measures by constructing a scatter plot. • Describe the trend seen in the plotted points. | LR1.01, LR1.02, LR1.04, LR4.01 CGE 2c, 5e |
| 3 | Is There a Relationship? (Part 1) | <ul style="list-style-type: none"> • Identify the trend and correlation of a scatter plot. • Describe the relationship between the variables. • Make a prediction about a linear relationship using a line of best fit. • Use technology to graph scatter plots. | LR1.01, LR1.02, LR1.04, LR4.01 CGE 2b, 2c, 5a |
| 4 | Is There a Relationship? (Part 2) | <ul style="list-style-type: none"> • Identify the trend and correlation of a scatter plot. • Describe the relationship between the variables. • Make a prediction about a relationship in preparation for using a line of best fit. • Use technology to graph scatter plots. | LR1.01, LR1.02, LR1.04, LR4.01 CGE 5b, 5e |
| 5 | Carousel of Relationships (Part 1) | <ul style="list-style-type: none"> • Identify the trend and correlation of data using a scatter plot. • Describe the relationship between the variables. • Collect data from experiments. • Make a prediction about a relationship in preparation for using a line or curve of best fit. | LR1.01, LR1.02, LR1.04, LR2.02, LR2.03, LR4.01 CGE 5a |
| 6 | Carousel of Relationships (Part 2) | <ul style="list-style-type: none"> • Identify the trend and correlation of data using a scatter plot. • Describe the relationship between the variables. • Collect data from experiments. • Make predictions about a relationship using a line or curve of best fit. | LR1.01, LR1.02, LR1.04, LR2.02, LR2.03, LR4.01 CGE 5a |
| 7 | First Differences | <ul style="list-style-type: none"> • Investigate the pattern in the first differences to determine if a relationship is linear or non-linear. • Construct tables of values and scatter plots. | LR2.02, LR2.03, LR4.01 CGE 2b, 3c |
| 8 | | Instructional Jazz | |
| 9 | | Instructional Jazz | |
| 10 | Sunflower Performance Task | Assessment | |



Math Learning Goals

- Identify a trend in a scatter plot.
- Use a trend to describe the relationship between the variables.
- Make a prediction about a relationship in preparation for an investigation on Day 2.

Materials

- BLM 3.1.1, 3.1.2, 3.1.3, 3.1.4, 3.1.5
- overhead projector
- measuring tapes/metre sticks

Assessment Opportunities

Minds On ...

Whole Class → Activating Prior Knowledge

Students respond to statements about a scatter plot (BLM 3.1.1), focusing on the pattern in the scatter plot, and providing a rationale for their choice of statement. They respond according to the following: Stand up facing forward means Agree, Stand up facing backward means Disagree, Sit down means Pass. Discuss responses with the whole class.

Individually and then in pairs, students predict relationships in response to statements (BLM 3.1.2). Pairs form groups of four to share their responses and rationale, and repeat by forming new groups of four.

Whole Class → Sharing

Share student responses and rationales as a whole class. (Consensus is not required.)

Station materials may be placed in bags or bins ahead of time.

Action!

Pairs or Small Groups → Investigation

Students gather data (BLM 3.1.3).

Record class data on overhead of BLM 3.1.4.

Learning Skills/Observation/Anecdotal: As students work together to gather data, observe related learning skills.

Students choose a relationship to investigate on Day 2 (BLM 3.1.2). Ensure that at least one group investigates the relationship between armspan and height in preparation for Day 3.



Consider setting up measurement stations complete with measuring tapes or metre sticks attached to the wall or floor.

Consolidate Debrief

Individual → Practice

Review scatter plots.

Students complete the worksheet (BLM 3.1.5).

Provide copies of the class data sheet to the pairs.

Concept Practice

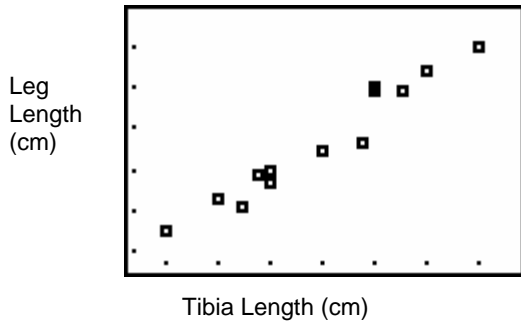
Home Activity or Further Classroom Consolidation

Complete the worksheet 3.1.5, Relationships Summary.

Complete practice problems about relationships.

Provide appropriate practice problems.

3.1.1: Plotted Points

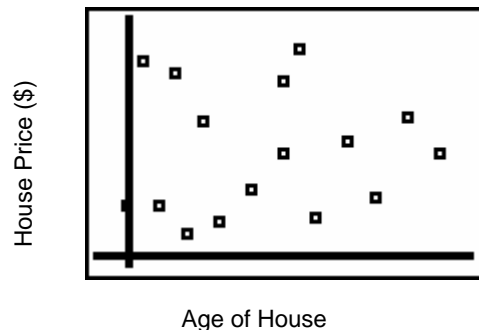


1. The graph shows the plotted points rising upwards to the right.
 - Agree
 - Disagree
 - Pass
2. As the length of the tibia increases the length of the leg increases.
 - Agree
 - Disagree
 - Pass
3. The graph can be used to determine the length of a person's leg if you know the length of the tibia bone.
 - Agree
 - Disagree
 - Pass



1. The graph shows the plotted points falling to the right.
 - Agree
 - Disagree
 - Pass
2. As the distance from the net increases the number of baskets made decreases.
 - Agree
 - Disagree
 - Pass
3. The graph can be used to determine the number of baskets you will make if you know the distance from the basket.
 - Agree
 - Disagree
 - Pass

1. The graph shows the plotted points scattered.
 - Agree
 - Disagree
 - Pass
2. As the age of the house increases the price of the house is either large or small.
 - Agree
 - Disagree
 - Pass
3. The graph can't be used to determine the price of the house if you know how old it is.
 - Agree
 - Disagree
 - Pass



3.1.2: Relationships

Complete the following statements by yourself, then share your answers with your partner. Explain the reasons for your choice. Indicate if you and your partner agree or disagree.



| Is There a Relationship? | My Partner and I: |
|--|-------------------------------|
| As a person gets taller their armspan _____. <i>(gets wider, gets smaller, stays the same)</i> | ___ agree ___ disagree |
| The longer a person's legs are _____ they run. <i>(the faster, the slower, will make no difference to how fast)</i> | ___ agree ___ disagree |
| As a person's foot size increases, their walking stride _____. <i>(gets longer, gets shorter, stays the same)</i> | ___ agree ___ disagree |
| As a person's forearm gets longer, their armspan _____. <i>(gets longer, gets shorter, stays the same length)</i> | ___ agree ___ disagree |
| The longer a person's thumb is _____ their index finger. <i>(the longer, the shorter, will make no difference to the length of)</i> | ___ agree ___ disagree |
| As a person gets taller, their foot size _____. <i>(gets longer, gets shorter, is not affected)</i> | ___ agree ___ disagree |

3.1.3: Data Collection – Is There a Relationship Here?

With a partner, measure and record each measurement to the nearest centimetre. Enter your data into the class data collection chart.

a) total height _____ cm

b) forearm _____ cm

c) armspan from fingertips to fingertips _____ cm

d) foot length _____ cm

e) walking stride length _____ cm



With a partner, choose two sets of data that you think might show some type of correlation. Create a scatter plot of the data. On the scatter plot discuss your findings.

3.1.5: Relationships Summary

A **scatter plot** is a graph that shows the _____ between **two** variables.

The points in a scatter plot often show a pattern, or _____.

From the pattern or trend you can describe the _____.

Example:

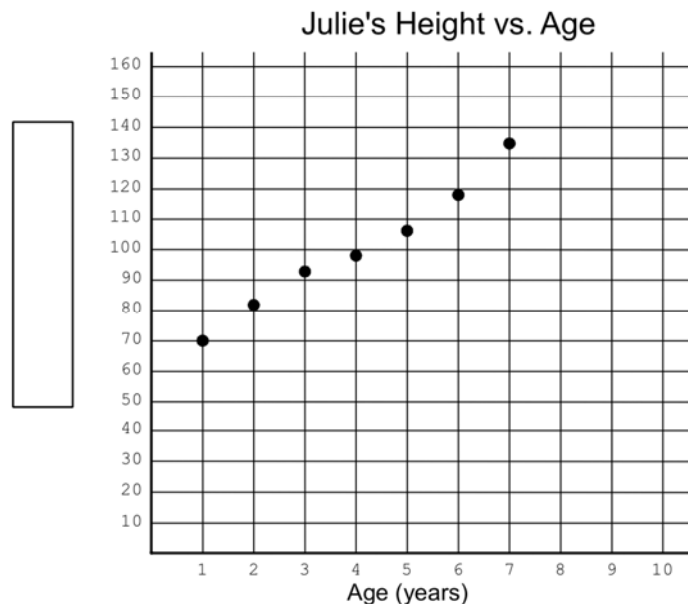
Julie gathered information about her age and height from the markings on the wall in her house.

| | | | | | | | | |
|--------------------|----|----|----|----|-----|-----|-----|-----|
| Age (years) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Height (cm) | 70 | 82 | 93 | 98 | 106 | 118 | 127 | 135 |

a) Label the vertical axis.

b) Describe the trend in the data.

c) Describe the relationship.



Variables

The *independent variable* is located on the _____ axis.

This variable does not depend on the other variable.

The *dependent variable* is located on the _____ axis.

This variable depends on the other variable.

Independent variable: _____

Dependent variable: _____

Note:
The independent variable comes *first* in the table of values.



75 min

Math Learning Goals

- Investigate a relationship between measures by constructing a scatter plot.
- Describe the trend seen in the plotted points.

Materials

- BLM 3.2.1, 3.2.2, 3.2.3
- grid paper

Assessment Opportunities

Minds On ...

Whole Class → Demonstration

Take up Julie’s height graph from Day 1 Home Activity (BLM 3.1.5).
 Ask: Based on the data, what would Julie’s height be at age 10? age 12?
 How do you know?
 Discuss the need for a line of best fit to make predictions [interpolation, extrapolation].
 Discuss the limitations of extrapolation too far away from the collected data, e.g., when Julie is age 30.
 Students complete BLM 3.2.1.

Action!

Individual → Graphing Relationships

Using class data from Day 1, students make a scatter plot of their chosen relationship (e.g., armspan vs. height) on grid paper.

Have students write a response to the following prompts:

- Which phrase describes the direction of the plotted points in the graph?
 - The plotted points rise upward to the right.
 - The plotted points fall downward to the right.
 - The plotted points are scattered across the graph.
- Describe the relationship.
- How could you use this graph to predict additional measurements? Explain.

Communicating/Observation/Anecdotal: Observe students as they respond to the prompts.

Students work with their original partner from Day 1 and share their findings. They peer edit the written component and submit it with the scatter plot.

Opportunity to assess communication skills.

Consolidate Debrief

Whole Class → Discussion

For practice in recognizing relationships in data, students complete BLM 3.2.2 by identifying the scatter plot data according to the strengths of the relationship.

To prepare for Day 3, introduce correlation by presenting the information on BLM 3.2.3.

Word Wall
correlation

No axes are included in the scatter plots to allow students to focus on the relationship of the data points. Axes are introduced on Day 3.

Home Activity or Further Classroom Consolidation

Concept Practice Exploration

Using the class data from Day 1, choose two different variables and investigate the relationship between them. Record your findings and conclusions.

3.2.1: Relationships Summary (continuation of 3.1.5)

Line of Best Fit

To be able to make predictions, we need to model the data with a line or a curve of best fit.

Rules for drawing a line of best fit:

1. The line must follow the _____.
2. The line should _____ through as many points as possible.
3. There should be _____ of points above and below the line.
4. The line should pass through points all along the line, not just at the ends.

Making Predictions

Use your line of best fit to estimate the following:

| Question | Answer | Method of Prediction |
|---|--------|----------------------|
| How tall was Julie when she was 5 years old? | | |
| How tall will Julie be when she is 9 years old? | | |
| How old was Julie at 100 cm tall? | | |
| How tall was Julie when she was born? | | |

Interpolate

When you interpolate, you are making a prediction _____ the data.

These predictions are usually _____.

Hint:
You are interpolating when the value you are finding is somewhere between the first point and the last point.

Extrapolate

When you extrapolate, you are making a prediction _____ the data.

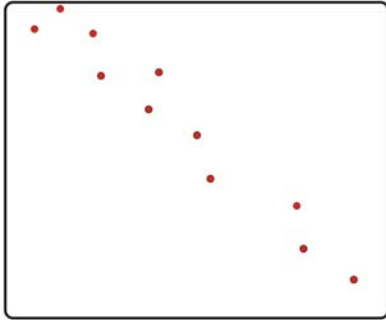
It often requires you to _____ the line.

These predictions are less reliable.

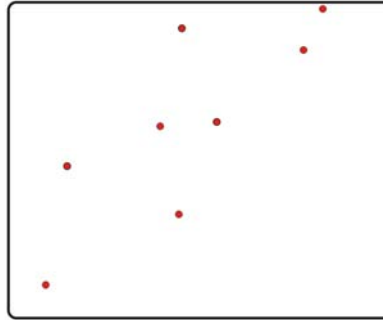
You are extrapolating when the value you are finding is before the first point or after the last point. This means you may need to extend the line.

3.2.2: Describing Scatter Plots and Lines of Best Fit

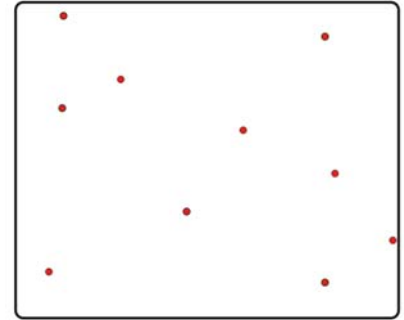
Draw a line of best fit for each of the scatter plots that show a linear relationship below. Write two or three key words to describe each relation on the line below the scatter plot. (*rises upward to the right, falls downward to the right, no relationship, strong, weak, linear, non-linear*)



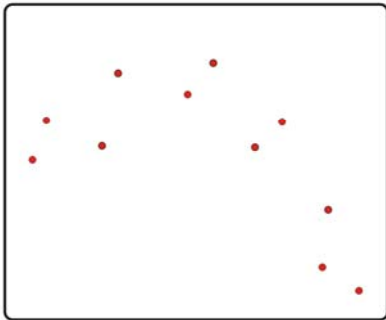
a) _____



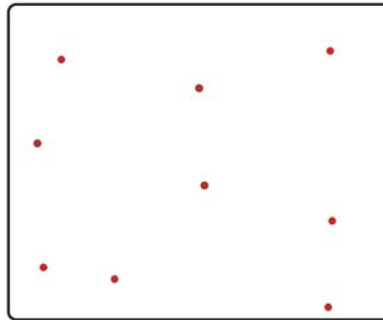
b) _____



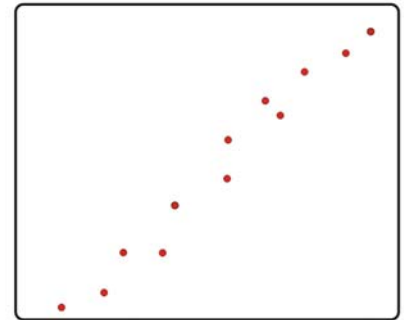
c) _____



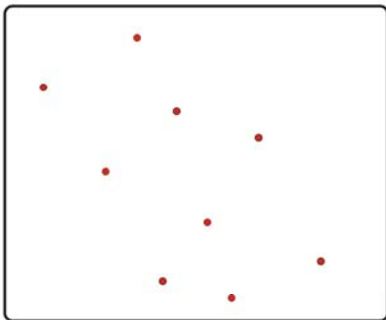
d) _____



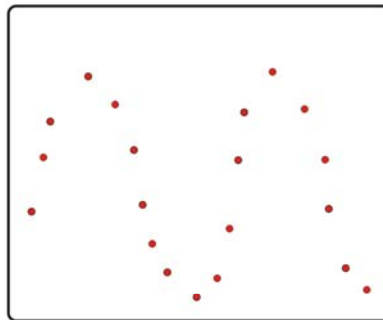
e) _____



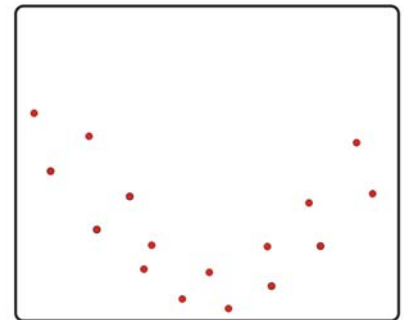
f) _____



g) _____



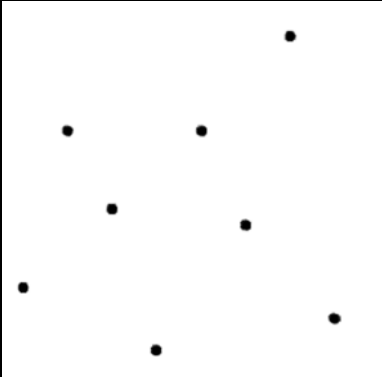


h) _____



i) _____

3.2.3: Correlation

| | |
|--|---|
|  | <p>A scatter plot shows a _____ correlation when the pattern rises up to the right.</p> <p><i>This means that the two quantities increase together.</i></p> |
|  | <p>A scatter plot shows a _____ correlation when the pattern falls down to the right.</p> <p><i>This means that as one quantity increases the other decreases.</i></p> |
|  | <p>A scatter plot shows _____ correlation when no pattern appears.</p> <p><i>Hint:</i> <i>If the points are roughly enclosed by a circle, then there is no correlation.</i></p> |

Strong or Weak?

If the points nearly form a line, then the correlation is

_____.

If the points are dispersed more widely, but still form a rough line, then the correlation is _____.

Hint:

To visualize this, enclose the plotted points in an oval.

If the oval is *thin*, then the correlation is *strong*.

If the oval is *fat*, then the correlation is *weak*.



Math Learning Goals

- Identify the trend and correlation of a scatter plot.
- Describe the relationship between the variables.
- Make a prediction about a linear relationship using a line of best fit.
- Use technology to graph scatter plots.

Materials

- BLM 3.3.1, 3.3.2, 3.3.3
- graphing calculators or Fathom software

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Clarify the problem and discuss the purpose (BLM 3.3.1).

Students write their hypothesis on the first page. Summarize what students are to do and answer any questions.

Whole Class → Demonstration

Demonstrate how to use the technology, i.e., a graphing calculator or Fathom software, to make a scatter plot (BLM 3.3.2).

Action!

Pairs → Investigation

Students enter data using technology and graph scatter plots to investigate height vs. each of the other measurements (four of them) outlined in the problem (BLM 3.3.1) to see which one provides the strongest correlation. The strongest correlation will be the most reliable prediction tool.

Students complete the worksheet (BLM 3.3.3).

Learning Skill (Teamwork)/Observation/Checklist: Observe and record students' collaboration skills.



Examine the student measurement data from Day 1 using Fathom before giving it to students. If students measure incorrectly it will throw off the entire investigation. Modify any data that seems unreasonable. Make a printout of the data to give to students.

Consolidate Debrief

Whole Class → Discussion

Use the following prompts, depending on how far the students are in their investigation:

- Describe the relationships in the graphs.
- Which relationship shows the strongest correlation?
- Do any of the graphs show *no* relationship?

Home Activity or Further Classroom Consolidation

Think of two variables that will show each of the following types of correlation: positive, negative, and none.

Concept Practice

3.3.1: Could I Be a Forensic Scientist?

Name: _____

Date: _____

Exploring the Problem

Remnants of a human skeleton were found at an archaeological dig that is thought to be the ruins of an ancient civilization. From the bones discovered, the scientists have determined the following:

- length of the forearm is 23 cm
- armspan is 185 cm
- handspan is 23 cm
- foot length is 24 cm

The scientists call you in as an expert in anthropology who is currently researching relationships between body measurements to help them determine an estimated height of the skeleton in question.

As the expert, your job will be to:

- estimate the height of the skeleton;
- explain the procedure you used to determine the height of the skeleton;
- include evidence (tables, graphs, and other models) to support your conclusion;
- explain the limitations of your method or discuss a different way to conduct your investigation.

Clarifying the Problem

Review the problem and highlight any important information.

- What are the variables?
- What exactly are you being asked to find?
- Are there certain variables that would be more useful than others?

Formulating an Hypothesis

- Decide which pairs of variables you think could show a relationship that would aid the scientists in their predictions.
- Explain your reasoning.

3.3.1: Could I Be a Forensic Scientist? (continued)

Method

Write the steps you used to determine the height of the skeleton.

Hint:
Your method should be written so that the scientists can repeat exactly what you did!

Inferring and Concluding

Guiding Questions

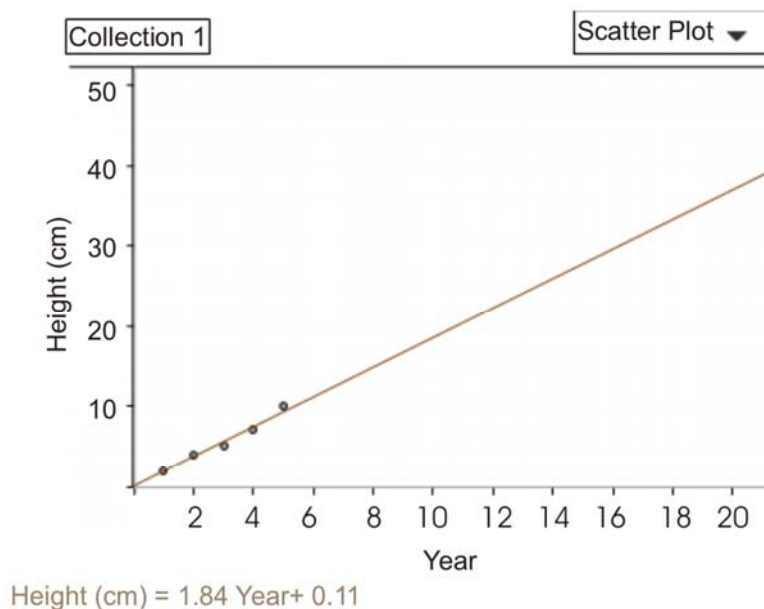
- What is the height of the skeleton? Give evidence to support your answer.
- How does the relationship support your hypothesis?
- Identify any relationships that show a stronger correlation than others.
- What are the limitations of using this method?

3.3.2: Introduction to FATHOM → Creating Scatter Plots

- Step 1) Pull down a case table. Enter the heading YEAR in the <new> column and enter Height in the next column. Enter the data as shown.
- Step 2) Pull down an empty scatter plot. Grab and drag the YEAR heading to the horizontal axis, and the HEIGHT heading to the vertical axis. Your scatter plot is done!
- Step 3) Pull down the GRAPH menu and choose MOVABLE LINE. This will be your line of best fit when you move it to its best position.
- Step 4) Pull down the GRAPH menu and choose SHOW SQUARES. Try to position the line such that the sum of the squares is a MINIMUM. Watch the SUM change as you reposition the line.
- Step 5) Change the horizontal and vertical scales by grabbing and dragging them towards zero. This will change the scale of the scatter plot and allow you to make predictions beyond the data collected.
- Step 6) Predict the Height for Year 10 →
Predict the Height for Year 20 →
Predict the Year when the Height will be 30 →
- Step 7) Add a text box to record your description of the scatter plot and the predictions by pulling down the INSERT menu and choosing TEXT.
Type "Scatter Plot" by "your name."
Describe your scatter plot with three sentences.
 - * One sentence will describe the correlation.
 - * The next sentence will describe the relationship and how strong it is.
 - * The third sentence will use examples to support your conclusion.

Use the line of best fit to make your predictions.

| Collection 1 | | |
|--------------|------|-------------|
| | Year | Height (cm) |
| 1 | 1 | 2 |
| 2 | 2 | 4 |
| 3 | 3 | 5 |
| 4 | 4 | 7 |
| 5 | 5 | 10 |
| | | |



3.3.3: Choosing the Best Model

Use Fathom to examine each pair of variables from your data set. Print your scatter plots. Glue your graphs in the boxes below and describe the correlation.

Height vs. Length of Forearm

Height vs. Armspan

Height vs. Handspan

Height vs. Foot Length

Which model is the best predictor of the height? Give reasons for your answers.



Math Learning Goals

- Identify the trend and correlation of a scatter plot.
- Describe the relationship between the variables.
- Make a prediction about a relationship in preparation for using a line of best fit.
- Use technology to graph scatter plots.

Materials

- BLM 3.4.1, 3.4.3
- BLM 3.4.2 (Teacher)
- graphing calculator or Fathom software

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Review students’ work from Day 3. Take up some of the Home Activity responses.

Action!

Individual → Investigation

Students complete the investigation, including a graph of the strongest relationship.

Curriculum Expectations/Performance Task/Rubric: Use BLM 3.4.2 to assess the student’s graph.

Students complete:

- Creating Scatterplots and Lines of Best Fit (BLM 3.4.1).
- Forensic Analysis (BLM 3.4.3).

Consolidate Debrief

Whole Class → Discussion

Use the following prompts to debrief the activity, Could I Be a Forensic Scientist?

- Describe the relationships in the graphs.
- Which graph showed the strongest correlation?
- Do any of the graphs show *no* relationship?
- How did you determine the height of the skeleton?

Reflection

Home Activity or Further Classroom Consolidation

In your journal, write a response: What have you learned about how scientists use data in their jobs?

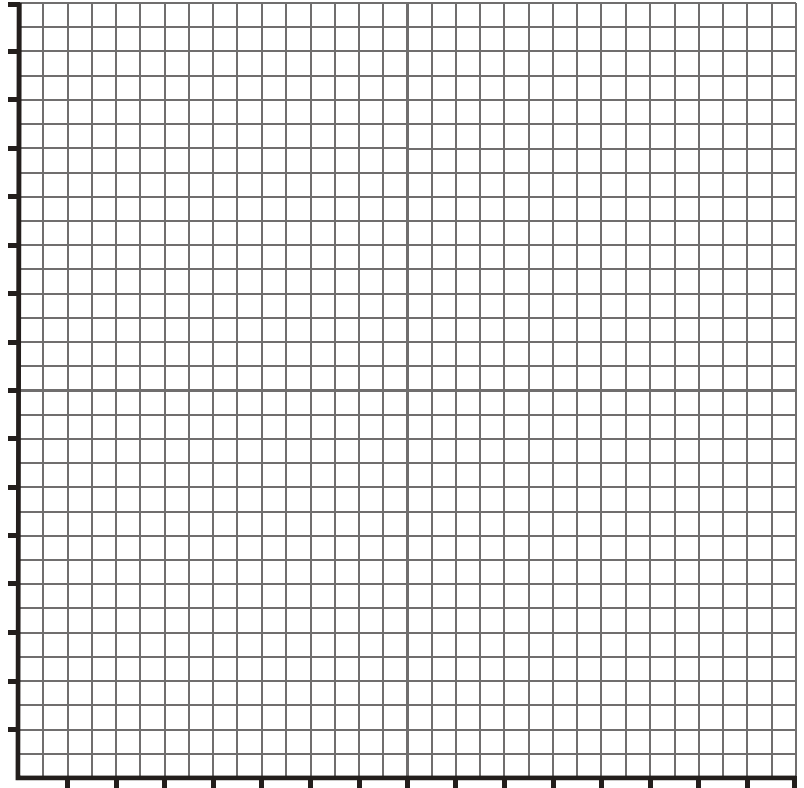


3.4.1: Creating Scatter Plots and Lines of Best Fit

Test the hypothesis: The older you are, the more money you earn.

Plot the data on the scatter plot below, choosing appropriate scales and labels.

| Age | Earnings (\$) |
|-----|---------------|
| 25 | 22000 |
| 30 | 26500 |
| 35 | 29500 |
| 37 | 29000 |
| 38 | 30000 |
| 40 | 32000 |
| 41 | 35000 |
| 45 | 36000 |
| 55 | 41000 |
| 60 | 41000 |
| 62 | 42500 |
| 65 | 43000 |
| 70 | 37000 |
| 75 | 37500 |



Note: The symbol _____ is used to signal a “break” in the axis when the scale does not start at zero to avoid a large empty space in one corner of the graph.

- 1) Draw a line of best fit. Describe the trend in the data.
- 2) Does the data support the hypothesis? Give reasons to support your answer. (Refer to the scatter plot.)
- 3) Explain why the data for ages over 65 do not correspond with the hypothesis.
- 4) Explain what the point (41, 35000) represents.

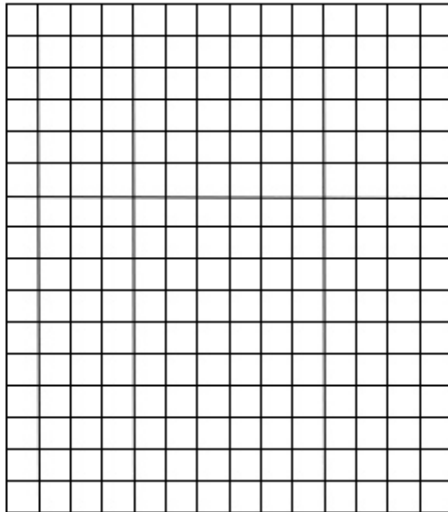
3.4.2: Investigation Rubric – Could I Be a Forensic Scientist?

| Criteria | Below Level 1 | Level 1 | Level 2 | Level 3 | Level 4 |
|--|---|---|--|--|--|
| Representing Constructs table of values, scatter plot, and line of best fit | - the representations do not match the data or each other | - constructs table of values, scatter plot, and line of best fit with limited understanding | - constructs table of values, scatter plot, and line of best fit with some understanding | - constructs table of values, scatter plot, and line of best fit with considerable understanding | - constructs table of values, scatter plot, and line of best fit with thorough understanding |
| Connecting Describes the trend, relationship, and correlation of the data | - describes the trend, relationship, and correlation of the data inaccurately or not at all | - describes the trend, relationship, and correlation of the data with limited effectiveness | - describes the trend, relationship, and correlation of the data with some effectiveness | - describes the trend, relationship, and correlation of the data with considerable effectiveness | - describes the trend, relationship, and correlation of the data with a high degree of effectiveness |
| Reasoning and Proving Forms conclusions using convincing arguments | - forms conclusions without presenting an argument or arguments are inaccurate | - forms conclusions using arguments with limited effectiveness | - forms conclusions using arguments with some effectiveness | - forms conclusions using arguments with considerable effectiveness | - forms conclusions using arguments with a high degree of effectiveness |
| Communicating Expression and organization of ideas and mathematical thinking | - no evidence of ability to express and organize ideas and mathematical thinking | - expresses and organizes ideas and mathematical thinking with limited effectiveness | - expresses and organizes ideas and mathematical thinking with some effectiveness | - expresses and organizes ideas and mathematical thinking with considerable effectiveness | - expresses and organizes ideas and mathematical thinking with a high degree of effectiveness |
| Use of conventions, vocabulary, and terminology | - uses conventions, vocabulary, and terminology inaccurately or not at all | - uses conventions, vocabulary, and terminology with limited accuracy | - uses conventions, vocabulary, and terminology with some accuracy | - uses conventions, vocabulary, and terminology with considerable accuracy | - uses conventions, vocabulary, and terminology accurately and precisely |

3.4.3: Forensic Analysis

Anthropologists and forensic scientists use data to determine information about people. Scientists can make predictions about the height, age, and sex of the person they are examining by looking for relationships in large amounts of data.

1. Construct a graph of the length of the humerus bone vs. the length of the radius.



| Length of Radius (cm) | Length of Humerus (cm) |
|-----------------------|------------------------|
| 25 | 29.7 |
| 22 | 26.5 |
| 23.5 | 27.1 |
| 22.5 | 26 |
| 23 | 28 |
| 22.6 | 25.2 |
| 21.4 | 24 |
| 21.9 | 23.8 |
| 23.5 | 26.7 |
| 24.3 | 29 |
| 24 | 27 |

2. Circle the point on the graph that represents the data for a radius that is 21.9 cm long. How long is the humerus? _____.
3. Put a box around the point on the graph that represents the data for a humerus that is 27.1 cm long. How long is the radius? _____.
4. Describe the trend.
5. Describe the relationship: As the length of the radius gets longer, the humerus _____.
6. a) Draw a line of best fit.
b) Use the line of best fit to predict the length of the humerus, if the radius is 24.5 cm long. Did you interpolate or extrapolate?
c) Use the line of best fit to predict the length of the radius, if the humerus is 25 cm long. Did you interpolate or extrapolate?



Math Learning Goals

- Identify the trend and correlation of data using a scatter plot.
- Describe the relationship between the variables.
- Collect data from experiments.
- Make a prediction about a relationship in preparation for using a line or curve of best fit.

Materials

- BLM 3.5.1, 3.5.2, 3.5.3
- BLM 3.5.4 (Teacher)
- see BLM 3.5.4 for additional materials

Minds On ...

Whole Class → Demonstration

Explain what students are to do at each station and how to complete the Investigations Record Sheet (BLM 3.5.1).

Assessment Opportunities

As an alternative, have students do one activity and present it to the group. Data is then shared with the entire group, so all students can analyse the graphs.

Action!

Small Group → Carousel of Investigations

Learning Skill (Teamwork)/Observation/Checklist: Observe and record students' collaboration skills.

Students complete each of the five investigations (BLM 3.5.2) and record their answers.

Note: Continue the next day until each group has visited each station.



Alternate investigation: Height of Object vs. Length of Shadow (Linear with positive correlation)

Use different-coloured handouts for each investigation.

Consolidate Debrief

Pairs → Timed Retell

Pair students with someone from a different group.

Students describe what they did and observed, using a Timed Retell.

(See *Think Literacy: Cross-Curricular Approaches, Grades 7–12 Mathematics*, pp. 102–103).

Application Exploration

Home Activity or Further Classroom Consolidation

Complete the questions on worksheet, Going Further for the investigations that you finished in class.

See BLM 3.5.4 for answers to BLM 3.5.3.

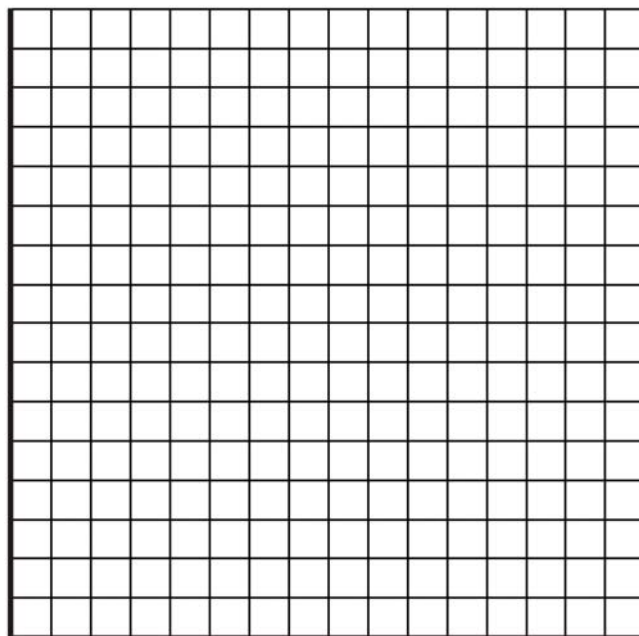
3.5.1: Investigations Record Sheet

Group Investigation # ____:

Write a **hypothesis** about the relationship.

Make the following **Mathematical Models**:

Complete the table of values and scatter plot. Draw the line (or curve) of best fit.

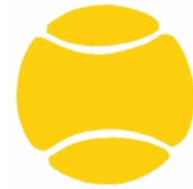


Write a **conclusion**.

Answer the questions for your investigation.

3.5.2: Investigations

Investigation 1 – Ball Bounce



Purpose

To determine if there is a relationship between the drop height of a ball and its rebound height.

Hypothesis

I think that as the drop height increases, the rebound height _____ because...

Materials

- golf, tennis, or ping-pong ball
- masking tape
- metre stick
- measuring tape

Procedure

- Attach measuring tape to the wall so you can measure the heights.
- Drop the ball from various heights and record the rebound height.
- Always drop the ball so that the bottom of the ball is just over the drop height.

Models

Record your observations on the Investigations Record Sheet.

Conclusion

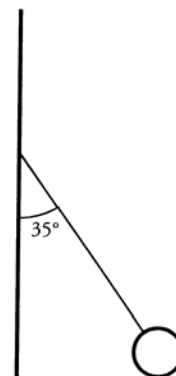
- Describe the relationship in your own words.
- Was your hypothesis correct?
- Describe any factors that may have affected your results.

3.5.2: Investigations (continued)

Investigation 2 – Pendulum Swing

Purpose

To determine if there is a relationship between the mass of the swinging object and the time it takes for it to make five complete swings.



Hypothesis

I think that as the mass increases, the time to complete five swings will _____ because...

Materials

- retort stand
- string
- heavy-duty paper clips
- timer
- protractor

Procedure

- Attach one weight (paper clip) to end of the pendulum string.
- Release the pendulum from a 35° angle and start the timer.
- Measure and record the length of time for five complete swings.
- Repeat, after increasing the mass at the end of the pendulum by one paper clip each time.

Models

Record your observations on the Investigations Record Sheet.

Conclusion

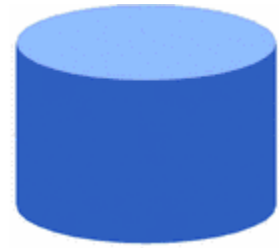
- Describe the relationship in your own words.
- Was your hypothesis correct?
- Describe any factors that may have affected your results.

3.5.2: Investigations (continued)

Investigation 3 – Cylinder Size

Purpose

To determine if there is a relationship between the height of various cylindrical containers and their diameter.



Hypothesis

I think that as the height increases, the diameter _____ because...

Materials

- 8 cylindrical containers of different heights
- ruler

Procedure

Measure and record the height and the diameter of the cylinders.

Models

Record your observations on the Investigations Record Sheet.

Conclusion

- Describe the relationship in your own words.
- Was your hypothesis correct?
- Describe any factors that may have affected your results.

3.5.2: Investigations (continued)

Investigation 4 – Bag Stretch

Purpose

To determine if there is a relationship between the height of a bag suspended by elastics over the floor and the number of books in the bag.



Hypothesis

I think that as the number of books increases, the distance of the bag from the floor

_____ because...

Materials

- 8 identical books
- shopping bag
- strong, long rubber bands
- metre stick

Procedure

- Hang the shopping bag from elastics so that the bottom is about 1 m above the floor.
- Measure and record the distance from the bottom of the bag to the floor.
- Add one book to the bag. Measure and record the distance from the bottom of the bag to the floor.
- Repeat, adding one book at a time until all the books are in the bag.

Models

Record your observations on the Investigations Record Sheet.

Conclusion

- Describe the relationship in your own words.
- Was your hypothesis correct?
- Describe any factors that may have affected your results.

3.5.2: Investigations (continued)

Investigation 5 – Water Drains

Purpose

To determine if there is a relationship between the height of water in a container and the time it takes to drain the container.

Hypothesis

I think that as time increases, the height of the water in the container _____ because...

Materials

- small plastic bottle, e.g., water bottle with the cap removed
- sharp object (compass) to poke hole in the bottle
- ruler
- timer (clock)
- collection container

Procedure

- Fill the bottle with water up to the point where the container slopes towards the top.
- Record the height of the water in the bottle.
- When your timer is ready, poke a hole in the bottom and top of the bottle.
- Record the height of water every 20 seconds until the water is past the cylindrical part of the bottle.

Models

Record your observations on the Investigations Record Sheet.

Conclusion

- Describe the relationship in your own words.
- Was your hypothesis correct?
- Describe any factors that may have affected your results.

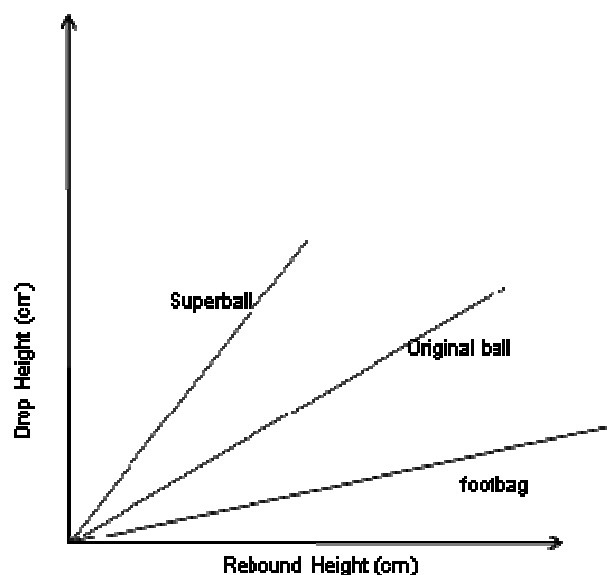
3.5.4: Teacher's Notes

Overview of Investigations

| Investigation | Materials Required | Experiment Information |
|--------------------|--|------------------------------|
| 1 – Ball Bounce | <ul style="list-style-type: none"> • golf, tennis or ping-pong ball • masking tape • metre stick • measuring tape | linear, positive correlation |
| 2 – Pendulum Swing | <ul style="list-style-type: none"> • retort stand • string • heavy duty paper clips (or washers) • timer • protractor | linear, constant |
| 3 – Cylinder Size | <ul style="list-style-type: none"> • 8 cylindrical containers with different heights and diameters | no relationship |
| 4 – Bag Stretch | <ul style="list-style-type: none"> • cloth shopping bag • 8 identical books • strong, long rubber bands • metre stick | linear, negative correlation |
| 5 – Water Drains | <ul style="list-style-type: none"> • plastic bottle with cap removed (1 for each group – they will be destroyed each round) • sharp object (compass) • ruler • timer (clock) • collection container | non-linear, negative trend |

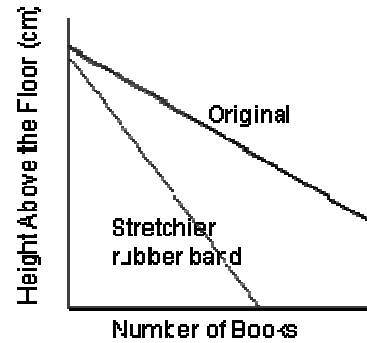
Going Further Answers

- Answers will vary.
(It will be less than 120 cm.)
 - Answers will vary.
(It will be greater than 40 cm.)
- The superball is more bouncy than the original ball, so the line would be steeper.
 - A footbag barely bounces at all (if at all) when dropped, so the line will be gradual.
- The graph forms a horizontal line. This means that number of paper clips does not affect the swing time.

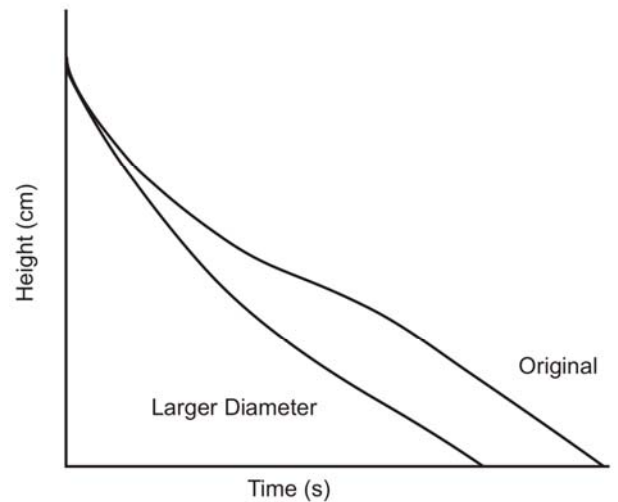


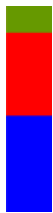
3.5.4: Teacher's Notes (continued)

- Answers will vary.
- Yes, the circumference is always about 3.14 times the diameter ($C = \pi d$).
- Answers will vary. It will be the x-intercept of the graph.
- The line would end sooner since the bag would hit the floor sooner.



- Answers will vary.
- Answers will vary.
- The water would take longer to empty because there would be more water.





75 min

Math Learning Goals

- Identify the trend and correlation of data using a scatter plot.
- Describe the relationship between the variables.
- Collect data from experiments.
- Make a prediction about a relationship in preparation using a line or curve of best fit.

Materials

- see Day 5

Assessment Opportunities



Minds On ...

Small Groups → Reflection

Students compare their home activity responses with other group members.



Action!

Small Groups → Carousel of Investigations

Learning Skill (Teamwork)/Observation/Checklist: Observe and record students' collaboration skills.

Students complete the investigations from Day 5.



Consider sharing this virtual activity where students can plot points and watch the effect on a line of best fit: http://matti.usu.edu/nlvm/nav/topic_t_5.html – choose scatter plots



Consolidate Debrief

Whole Class → Discussion

Use the following prompts:

- Did any of the graphs show a strong positive correlation?
- Were any of the relationships unexpected?
- For any relationship did you draw a curve rather than a line of best fit?
- Did any of the graphs have a negative correlation?

Consider using an integer game at the end of the lesson.

Application Exploration

Home Activity or Further Classroom Consolidation

Complete the worksheet, Going Further for the investigations you did today.



Math Learning Goals

- Investigate the pattern in the first differences to determine if a relationship is linear or non-linear.
- Construct tables of values and scatter plots.

Materials

- BLM 3.7.1, 3.7.2, 3.7.3
- 1 cm cubes

Assessment Opportunities

Minds On ...

Whole Class → Demonstration and Discussion

Create two graphs, one suggesting a linear, the other a non-linear relationship (continuous data, both beginning at the origin).

Students describe the graphs and match a table of values to the appropriate graph.

Discuss their rationale for the matched graphs (accept all answers without judgement).

You may need to review integer subtraction here and provide practice examples.

Action!

Small Group → Guided Discovery

Demonstrate how to calculate the first differences using BLM 3.7.1 First Differences (Problem 1A).

Students complete BLM 3.7.1 to determine the relationship between the shape of the graph and the first differences. They learn how to use first differences to determine linearity. They encounter a piecewise linear relationship in Problem 2.

Take care to line up the first differences halfway between rows in the table of values.

Curriculum Expectations/Self-Assessment/Marking Scheme: Students check their understanding after completing BLM 3.7.2.

Students must know to subtract the upper value from the lower value in the chart when computing first differences.

Consolidate Debrief

Whole Class → Discussion

Review the key learning that linear relations have constant first differences, referring to examples from the task and additional tables of values.

Pairs → Peer Tutoring

Students work in pairs to respond to practice questions. Encourage students to consult one another before asking for help. Circulate and provide assistance as needed.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete the Using What You Have Discovered worksheet 3.7.2.

3.7.1: First Differences

Problem 1

A. Jody works at a factory that produces square tiles for bathrooms and kitchens. She helps determine shipping costs by calculating the perimeter of each tile.

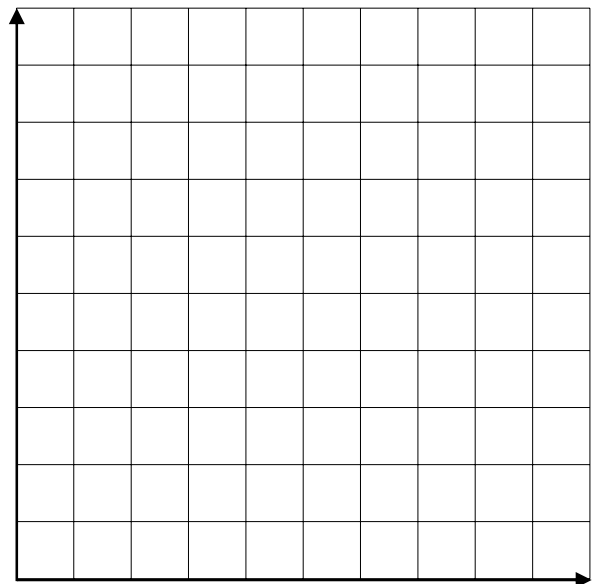
i) **Calculate** the perimeter and record your answers in the Perimeter column of the table.

| Side Length (cm) | Perimeter (cm) | First Differences |
|------------------|----------------|-------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

ii) **Describe** what happens to the perimeter of each tile when the side length increases by one centimetre. _____

iii) **Construct** a graph of the perimeter vs. the side length. Include labels and titles.

- Which variable is the independent variable?
- Which variable is the dependent variable?
- Use the graph to describe the relationship between the perimeter and side length of a tile.
- Describe the shape of the graph.



iv) **Calculate** the first differences in the First Differences column of the table. What do you notice about the first differences?

- Summarize** your observations.
 - When the side length increases by one centimetre, the perimeter increases by _____.
 - The plotted points suggest a...
 - The first differences are...

3.7.1: First Differences (continued)

B. Jody is paid \$8.50/hour to calculate perimeters.

i) **Calculate** her pay and record your answers in the Pay column of the table.

| Number of Hours | Pay (\$) | First Differences |
|-----------------|----------|-------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

ii) **Describe** what happens to her pay when the number of hours she works increases by one hour. _____

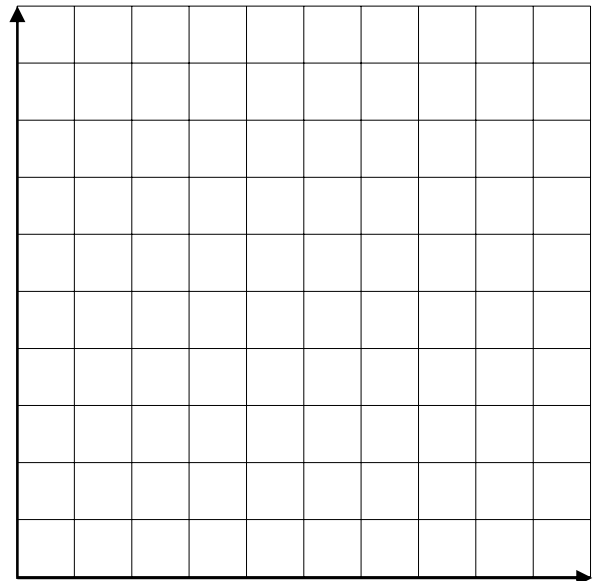
iii) **Construct** a graph of her pay vs. the number of hours she works. Include labels and titles.

a) Which variable is the independent variable?

b) Which variable is the dependent variable?

c) Use the graph to describe the relationship between her pay and the number of hours she works.

d) Describe the shape of the graph.



iv) **Calculate** the first differences in the First Differences column of the table. What do you notice about the first differences?

v) **Summarize** your observations.

a) When the number of hours worked increases by one, the pay increases by _____.

b) The plotted points suggest a...

c) The first differences are...

3.7.1: First Differences (continued)

C. Raj, another employee at the factory, also works with the tiles. He helps to determine the shipping costs by calculating the area of each tile.

i) **Calculate** the area and record your answers in the Area column of the table.

| Length of sides (cm) | Area (cm ²) | First Differences |
|----------------------|-------------------------|-------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

ii) **Describe** what happens to the area of each tile when the side length of a tile increases by one centimetre. _____

iii) **Construct** a graph of the area vs. the length of the sides of the tiles. Include labels and titles.

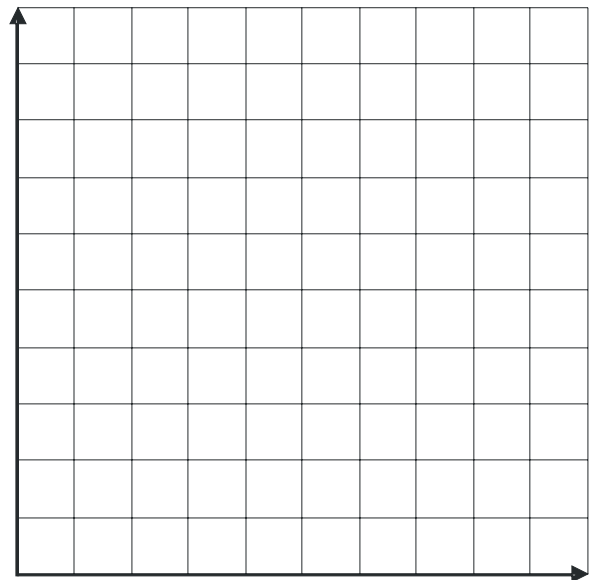
a) Which variable is the independent variable?

b) Which variable is the dependent variable?

c) Use the graph to describe the relationship between the area and the side length of the tile.

d) Describe the shape of the graph.

iv) **Calculate** the first differences in the First Differences column of the table. What do you notice about the first differences?



v) **Summarize** your observations.

a) When the side length increases by one centimetre, the area increases by _____.

b) The plotted points suggest a...

c) The first differences are...

3.7.1: First Differences (continued)

Problem 2

Chuck works on commission for sales. He earns \$12.00 for each of the first 3 boxes he sells. He earns \$24.00 each for boxes 4, 5, and 6, and \$36.00 each for selling boxes 7, 8, 9, and 10.

- i) **Calculate** Chuck's earnings for the following numbers of boxes of files and record your answers in the Earnings column of the table.

| Number of Boxes | Earnings (\$) | First Differences |
|-----------------|---------------|-------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

- ii) **Describe** what happens to his earnings when the number of boxes he sells increases by one box.

3.7.1: First Differences (continued)

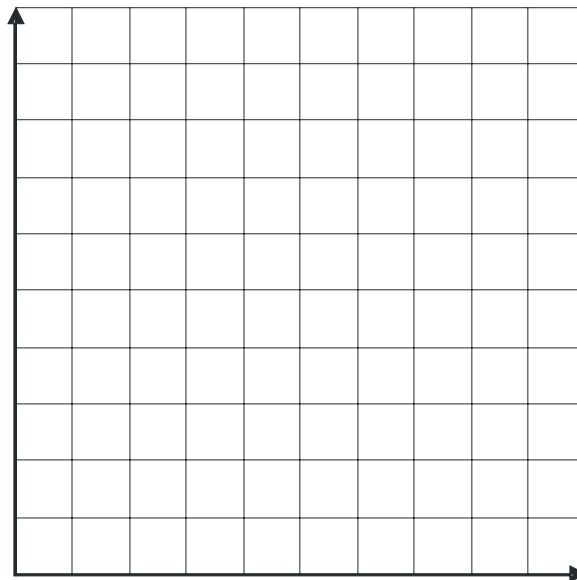
iii) **Construct** a graph of his earnings vs. the number of boxes he sells. Include labels and titles.

- a) Which variable is the independent variable?

- b) Which variable is the dependent variable?

- c) Use the graph to describe the relationship between his earnings and the number of boxes he sells.

- d) Describe the shape of the graph.



iv) **Calculate** the first differences in the third column of the table. What do you notice about the first differences?

v) **Summarize** your observations.

- a) When the number of boxes he sells increases by one box, his earnings increase by...

- b) The plotted points suggest a...

- c) The first differences are...

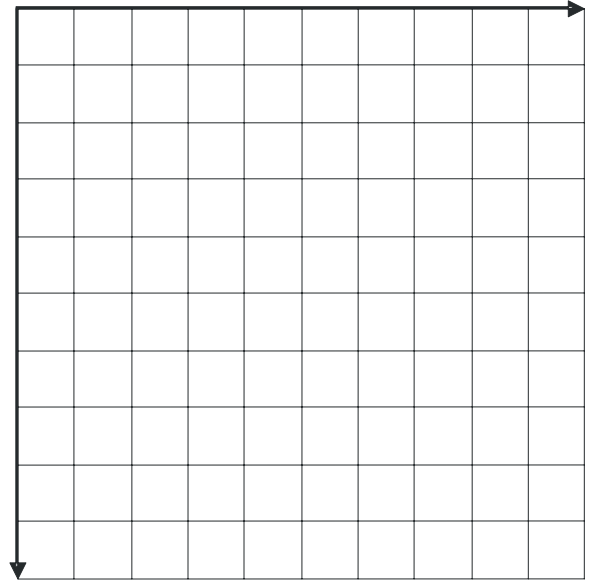
3.7.2: Using What You Have Discovered

Deep Sea Divers

The table below shows data collected as divers descend below sea level. Calculate the first differences. Use the first differences to determine if the relationship is linear or non-linear. Check your solution by graphing. Include labels and titles.

| Time (min) | Depth (m) | First Differences |
|------------|-----------|-------------------|
| 0 | -2 | |
| 1 | -4 | |
| 2 | -6 | |
| 3 | -8 | |
| 4 | -10 | |

The relationship is:

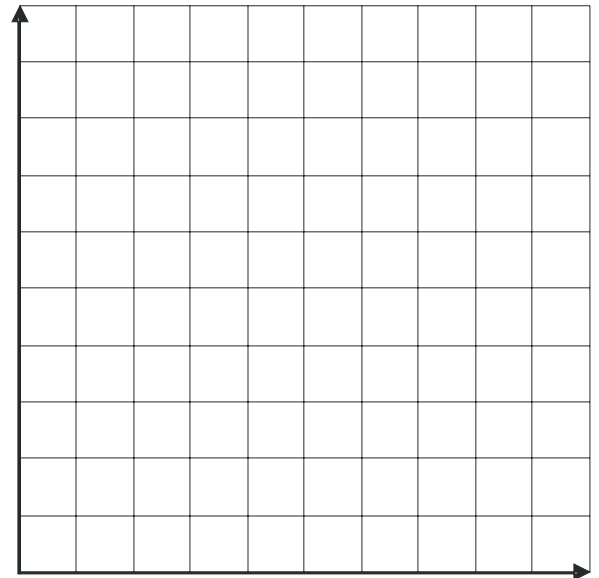


Hot Air Ballooning

The table shows data collected as a hot air balloon leaves the ground. Calculate the first differences. Use the first differences to determine if the relationship is linear or non-linear. Check your solution by graphing. Include labels and titles.

| Time (sec) | Height (m) | First Differences |
|------------|------------|-------------------|
| 0 | 2 | |
| 1 | 4 | |
| 2 | 6 | |
| 3 | 8 | |
| 4 | 10 | |


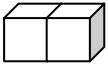
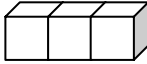
The relationship is:



3.7.3: Quiz – Describing Relationships

Use 1 cm cubes to help you.

1. Complete the following table.

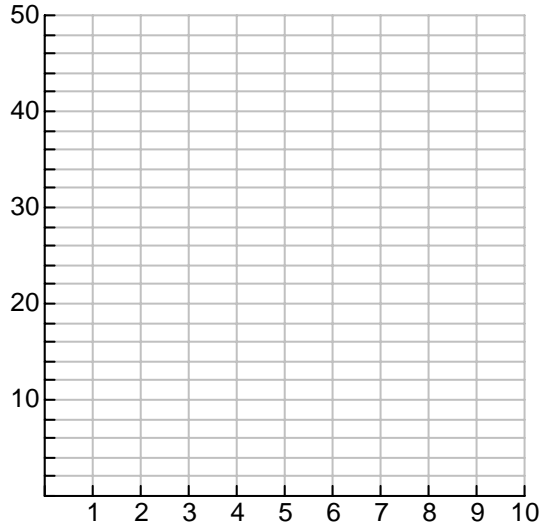
| Shape | Number of Cubes | Surface Area | First Difference |
|---|-----------------|--------------|------------------|
|  | 1 | 6 | |
|  | 2 | 10 | |
|  | 3 | | |
| | 4 | | |
| | 5 | | |
| | 6 | | |

2. Will the graph of the relationship between the number of cubes and the surface area be linear or non-linear? Use the first differences to explain your reasoning.

3. Will the graph of the relationship between the number of cubes and the surface area have a positive correlation or a negative correlation? Use the first differences to explain your reasoning.

3.7.3: Quiz – Describing Relationships (continued)

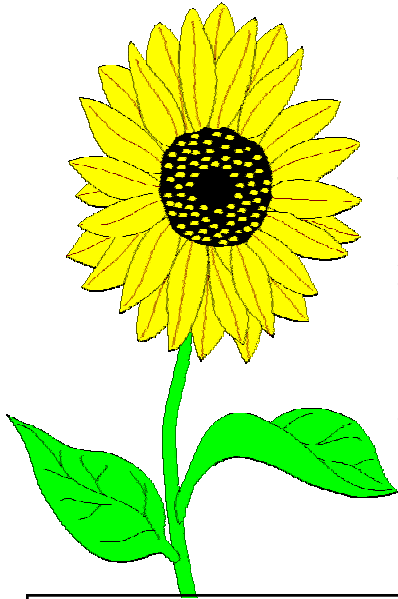
4. Make a scatter plot of the relationship between the number of cubes and the surface area. Construct a line of best fit.



5. Use your line of best fit to determine the surface area of the shape made from 10 cubes.
6. What is the surface area of the shape made from 100 cubes? Explain your reasoning.

| Mathematical Process | Criteria | Level 1 | Level 2 | Level 3 | Level 4 |
|-----------------------|------------------------------------|---|--|--|--|
| Reasoning and Proving | Use of critical thinking processes | - uses critical thinking processes with limited effectiveness | - uses critical thinking processes with some effectiveness | - uses critical thinking processes with considerable effectiveness | - uses critical thinking processes with a high degree of effectiveness |

3.10.1: Sunflower Performance Task



Roxanne, Jamal, Leslie, and Ada did a group project on sunflower growth for their biology class. They investigated how different growing conditions affect plant growth. Each student chose a different growing condition.

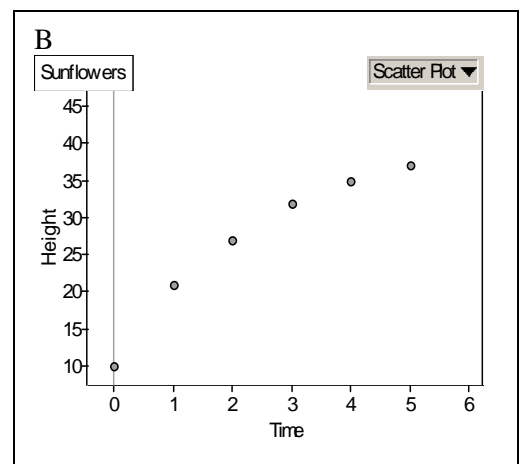
The students started their experiment with plants that were 10 centimetres tall. They collected data every week for five weeks. At the end of five weeks, they were to write a group report that would include a table, a graph, and a story for each of the four growing conditions.

Unfortunately, when the students put their work together, the pages were scattered, and some were lost. This page shows the tables, graphs, and written reports that were left.

Find which table, graph, and written report belong to each student. Create the missing tables, graphs, and reports.

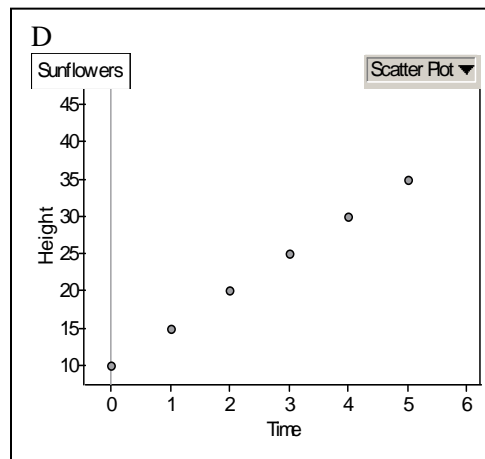
A

| | | | | | | |
|----------------------|----|------|------|----|----|------|
| Time (weeks) | 0 | 1 | 2 | 3 | 4 | 5 |
| Height (centimetres) | 10 | 12.5 | 17.5 | 25 | 35 | 47.5 |



C

| | | | | | | |
|----------------------|----|----|----|----|----|----|
| Time (weeks) | 0 | 1 | 2 | 3 | 4 | 5 |
| Height (centimetres) | 10 | 10 | 10 | 10 | 10 | 10 |



E
I planted my sunflower in a shady place. The plant did grow, but not so fast. The length increased every week by equal amounts.
Roxanne

F
I put my plant in poor soil and didn't give it much water. It did grow a bit, but every week less and less.
Leslie

G
I treated my sunflower very well. It had sun, good soil, and I even talked to it. Every week it grew more than the week before.
Jamal