Unit 6
Probability

Grade 8

Lesson Outline

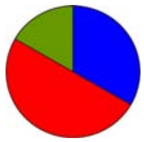
BIG PICTURE

Students will:
- investigate side length combinations for triangles;
- determine and compare theoretical and experimental probability of events;
- identify complementary events;
- calculate the probability of a complementary event;
- make predictions based on probability;
- analyse "fairness" in games of chance;
- review addition and subtraction of integers using concrete materials and drawings.

Day	Lesson Title	Math Learning Goals	Expectations
1	Number Triangles	• Discover how three side lengths must be related to create a triangle. • Count number cube combinations.	8m70 CGE 5a
2	Experimental and Theoretical Probability (Part 1)	• Represent probability in multiple ways. • Introduce concepts of theoretical and experimental probability.	8m80, 8m81 CGE 2c
3	Experimental and Theoretical Probability (Part 2)	• Compare theoretical and experimental probability.	8m18, 8m20, 8m80 CGE 3c, 5b
4	Theoretical and Experimental Probability of Events (Part 3)	• Compare theoretical and experimental probability and sample size. • Identify complementary events.	8m80 CGE 4b, 3c
5	Checkpoint	• Consolidate concepts of theoretical and experimental probability.	8m80 CGE 2b
6	Revisiting Number Triangles	• Consolidate an understanding of counting techniques required for probability problems. • Consolidate an understanding of complementary events.	8m80, 8m82 CGE 2c, 7b
7	Investigating Probability Using Integers	• Review addition and subtraction of integers. • Link probability to the study of integers.	8m22, 8m80, 8m81 CGE 5e, 7b

Unit 6: Day 1: Number Triangles

Grade 8



Math Learning Goals

- Discover how three side lengths must be related to create a triangle.
- Count number cube combinations.

Materials

- number cubes
- straws, scissors
- ruler, compass
- GSP®4
- BLM 6.1.1
- relational rods

Assessment Opportunities

Minds On…

Whole Class → Connecting

Review how to construct a triangle with side lengths 2 units, 4 units, and 5 units, using a variety of tools, e.g., ruler and compass, three straws cut to the given lengths, relational rods.

Students write their hypothesis for the question and explain their reasoning: If I roll three standard number cubes, will the three numbers that appear always form the sides of a triangle?

Do not confirm or deny their hypotheses at this time.

Action!

Small Groups → Investigation

Curriculum Expectations/Observation/Mental Note: Observe students as they work and assist groups who have trouble creating their triangles properly.

Students complete BLM 6.1.1 and record how many sets of three numbers form a triangle.

This investigation could be done as a class demonstration using GSP®4.

Consolidate Debrief

Whole Class → Summarizing

Several groups describe what they found when they tried to construct triangles. Ask:

- What relationship needs to exist among the three numbers rolled so that a triangle can be constructed? [The sum of the two shorter sides must be greater than the length of the longest side.]
- Out of 30 trials, how many triangles did you form? [Groups will have different answers. Discuss why that has happened.]
- How many possible ways do you think there are of rolling a set of three numbers that will form the sides of a triangle? (Students estimate and explain their reasoning.)

Identify groups who got the same number combination. Using one of their examples (or your own of 2, 5, 6 and 5, 2, 6), ask if the two sets of combinations represent two different rolls or if they represent the same roll.

Although order does not matter for constructing a triangle, in a later lesson students return to their math journal entry to determine the probability of forming a particular type of triangle. At that time, it will be important to notice that rolls of (2,3,4), (2,4,3), (3,2,4), (3,4,2), (4,2,3), and (4,3,2) are all different possible rolls with number cubes, but they do not form different triangles.

Small Groups → Reflection

Students reflect on rolling the following sums.

- How many ways can you get a sum of two by rolling two number cubes? [Answer: one, i.e., rolling 1, 1]
- How many ways can you get a sum of three by rolling two number cubes? [Answer: by rolling a 1 and a 2; by rolling 1, 2 or 2, 1]
- How many ways can you get a sum of four by rolling two number cubes?

Do not attempt to demonstrate counting techniques at this time.

Pairs → Summarizing

Students make individual table lists of all possible number cube combinations that will form a triangle.

They need to think about the sum of two sides compared to the longest side.

Reflection
Concept Practice

Further Classroom Consolidation and Home Activity

In your journal, respond to the question: Under what condition(s) will three given lengths form the sides of a triangle?

6.1.1: Rolling Number Cubes for Triangles

Name:
Date:

1. Roll three number cubes 30 times. The largest number should be side *c*. The other numbers are the lengths of sides *a* and *b*.
2. Construct a triangle, using one of the methods you know, and record if the roll will form a triangle by writing Yes or No.

Length of Side *a*	Length of Side *b*	Length of Side *c*	Triangle formed? Yes or No	Type of Triangle (to be completed Day 6)

Math Learning Goals

- Represent probability in multiple ways.
- Introduce concepts of theoretical and experimental probability.

Materials

- coins
- BLM 6.2.1, 6.2.2

Assessment Opportunities

Minds On…

Whole Class → Guided Review

Curriculum Expectations/Journal: Collect and assess math journal entry.

Review the meaning of the vocabulary associated with probability situations (BLM 6.2.1). Students brainstorm, write, and share their own statements, using correct terminology. In discussion, focus on those events which students identify as "maybe" to decide whether these events are likely or unlikely to occur. Students explain their reasoning.

Probability is the mathematics of chance.

The probability of an event is a number between 0 and 1; an impossible event, 0; and an event that is certain, 1.

Action!

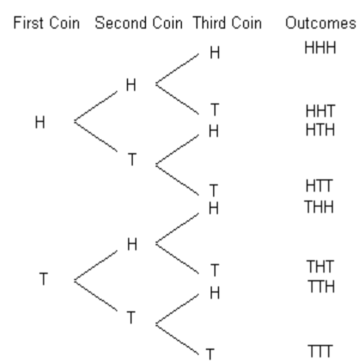
Pairs → Investigation

Students toss one coin and state the number of possible outcomes. They toss two coins and suggest possible outcomes.

Demonstrate how a tree diagram can be used to organize the outcomes of their tosses. Point out that the branches represent their choices.

Each pair of students creates a tree diagram for tossing three coins. As an example, when tossing three coins, we wish to see 1 head and 2 tails. What is the probability of this occurring?

Explain that a preference is considered to be a favourable outcome; and the probability of that event is the ratio of the number of favourable outcomes to the total number of possible outcomes.



$$P = \frac{Number\ of\ \ favourable\ \ outcomes}{Number\ of\ \ possible\ \ outcomes}$$

Each pair tosses two coins twenty times (20 is the sample size) and records each outcome.

They compare their experimental results to the theoretical results. Discuss how changing sample size (to more or fewer than 20) would affect experimental results.

Experimental results: $\frac{\#TT}{20}$ $\frac{\#HH}{20}$ $\frac{\#TH\ or\ HT}{20}$ compared to theoretical results

P (TT) = $\frac{1}{4}$ P (one of each) = $\frac{2}{4}$ P (HH) = $\frac{1}{4}$

Students prepare a presentation of their findings.

Theoretical probability applies only to situations that can be modelled by mathematically fair objects.

The coin toss provides experimental results.

Experimental probability is based on the results of an experiment and are relative frequencies, giving an estimate of the likelihood that a particular event will occur.

Experimental probabilities are often close to the theoretical probabilities especially if the sample size is large.

Consolidate Debrief

Whole Class → Presentation

One student from each pair presents their results for tossing two coins twenty times. Combine whole class data to share results with the larger sample size.

Discuss the effect of sample size on experimental outcomes. Discuss what a probability of 0 and a probability of 1 would mean in the context of coin tosses.

Curriculum Expectations/Presentations/Class Response: Assess communication skills during the student presentation.

Reflection
Concept Practice
Skill Drill

Home Activity or Further Classroom Consolidation

Complete worksheet 6.2.2.

Devise your own simulations using spinners, or a combination of coins and spinners, etc.

6.2.1: Talking Mathematically

Name:
Date:

Part A

Read each statement carefully. Choose from the terms to describe each event and record your answer in the space provided:

- certain or sure
- impossible
- likely or probable
- unlikely or improbable
- maybe
- uncertain or unsure

Part B

Consider pairs of statements and determine which of them would be:

- equally likely
- equally unlikely

1. A flipped coin will show tails.	
2. I will be in school tomorrow.	
3. It will not get dark tonight.	
4. I will have pizza for dinner tonight.	
5. I roll a 3 using a number cube.	
6. It will snow in July.	
7. The teacher will write on the board today.	
8. January will be cold in Ontario.	
9. My dog will bark.	
10. I will get Level 4 on my science fair project.	

6.2.2: Investigating Probability

Name:
Date:

Solve the following problems in your notebook:

1. Keisha's basketball team must decide on a new uniform. The team has a choice of black shorts or gold shorts and a black, white, or gold shirt.
   Use a tree diagram to show the team's uniform choices.

   a) What is the probability the uniform will have black shorts?
   b) What is the probability the shirt will not be gold?
   c) What is the probability the uniform will have the same-coloured shorts and shirt?
   d) What is the probability the uniform will have different-coloured shorts and shirt?

2. Brit goes out for lunch to the local submarine sandwich shop. He can choose white or whole wheat bread, and a filling of turkey, ham, veggies, roast beef, or salami.
   Use a tree diagram to show all Brit's possible sandwich choices.

   a) How many sub choices are there?
   b) He may also choose a single topping of tomatoes, cheese, or lettuce. Now, how many possible sub choices does he have?
   c) If each possibility has an equal chance of selection, what is the probability that Brit will choose a whole wheat turkey sub topped with tomatoes?
   d) What is the probability of choosing a veggie sub topped with cheese?
   e) What is the probability of choosing a meat sub topped with lettuce on white bread?
   f) What is the probability of choosing a meat sub topped with lettuce?

3. The faces of a cube are labelled 1, 2, 3, 4, 5, and 6. The cube is rolled once.
   List the favourable outcomes for each.

   a) What is the probability that the number on the top of the cube will be odd?
   b) What is the probability that the number on the top of the cube will be greater than 5?
   c) What is the probability that the number on the top of the cube will be a multiple of 3?
   d) What is the probability that the number on the top of the cube will be less than 1?
   e) What is the probability that the number on the top of the cube will be a factor of 36?
   f) What is the probability that the number on the top of the cube will be a multiple of 2 and 3?

6.2.2: Investigating Probability (Answers)

Question 1

a) The probability the uniform will have black shorts is $\frac{3}{6}$ or $\frac{1}{2}$.

b) The probability the shirt will not be gold is $\frac{4}{6}$ or $\frac{2}{3}$.

c) The probability the uniform will have the same-coloured shorts and shirt is $\frac{2}{6}$ or $\frac{1}{3}$.

d) The probability the uniform will have different-coloured shorts and shirt is $\frac{4}{6}$ or $\frac{2}{3}$.

Question 2

a) Brit has the choice of 2 breads and 5 fillings. So, he has the choice of 2 x 5 = 10 sandwiches. This can be shown using a tree diagram that first has 2 branches (one for each of the bread types) and then 5 branches at the end of the first branches (one for each of the fillings). This will give 10 ends to the tree.

b) You can add 3 branches at the end of each branch to indicate each of 3 topping choices. This gives 30 possible outcomes.

c) Only one of these outcomes is a whole-wheat turkey sandwich topped with tomatoes. So the probability that he chooses this sandwich is $\frac{1}{30}$. It is only one of 30 possible sandwiches.

d) The probability of choosing any veggie sub topped with cheese is $\frac{2}{30}$ or $\frac{1}{15}$. The student must remember to use both the whole wheat and white bread possibility in this answer.

e) The probability of choosing a meat sub topped with lettuce on white bread is $\frac{4}{30}$ or $\frac{2}{15}$. The student must remember to use all possible meat selections for this answer.

f) The probability of choosing a meat sub topped with lettuce is $\frac{8}{30}$ or $\frac{4}{15}$. The student must remember to use all possible meat selections in this answer, and both types of bun.

Question 3

a) There are 3 odd numbers, so the probability is $\frac{3}{6}$ or $\frac{1}{2}$.

b) There is only one number greater than 5, so the probability is $\frac{1}{6}$.

c) There are two multiples of 3, i.e., 3 and 6, so the probability is $\frac{2}{6}$ or $\frac{1}{3}$.

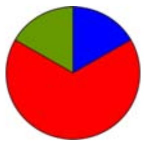
d) There is no number less than one, so the probability is zero.

e) There are 5 numbers that are factors of 36, i.e., 1, 2, 3, 4, and 6, so the probability is $\frac{5}{6}$

f) There is only one number that is a multiple of both 2 and 3, i.e., 6, so the probability is $\frac{1}{6}$.

Unit 6: Day 3: Experimental and Theoretical Probability: Part 2

Grade 8



Math Learning Goals

- Compare theoretical and experimental probability.

Materials

- BLM 6.3.1, 6.3.2
- different-colour number cubes
- coloured disks or paper squares

Assessment Opportunities

Minds On…

Pairs → Investigation

One student chooses a number and records the number of times he/she predicts the number cube would have to be rolled in order for this number to appear. The other student rolls the number cube until the partner's number comes up. Students change roles and repeat the activity.

Discuss the probability of an event using one number cube, e.g., P (rolling a 4) = $\frac{1}{6}$

- Is there a number that occurs more frequently? [No]
- How did your results compare to your predictions?
- Did your results surprise you?

Encourage the use of likely, *unlikely*, *probable*, and *possible*.

Students' vocabulary should be moving from 'luck' towards theoretical probability terms.

Action!

Pairs → Exploration

Explain the activity (BLM 6.3.1) and how to fill in the recording chart (BLM 6.3.2). Students predict, record, and analyse their results, using two different-coloured number cubes. Students change roles and continue the experiment until all squares on the board have at least one marker on them.

Students complete the recording charts.

Probability of a sum of 7 is 6 out of 36 or $\frac{1}{6}$.

Probability of a sum of 2 is 1 out of 36 or $\frac{1}{36}$.

Probability of a sum of 12 is also $\frac{1}{36}$.

Consolidate Debrief

Whole Class → Discussion

Learning Skills (Class Participation)/Question & Answer/Checklist: Discuss students' answers to questions 3 and 4 (BLM 6.3.1).

Relate the results to theoretical and experimental probability. Students refer to their recording chart. Which columns represent these probabilities?

$$Experimental\ Probability = \frac{number\ of\ favourable\ occurences}{total\ number\ of\ occurences}$$

$$Theoretical\ Probability = \frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$$

Collect experimental data from the whole class to show the results for increased sample size.

Ask: How can the two column totals indicate if calculation errors have been made? (BLM 6.3.2)

Remind students that experimental probabilities would be closer to the theoretical probabilities if the sample size were larger.

Theoretical probability = the predicted probability of an event based on mathematics

Experimental probability = the probability of an event based on actual trials from experiments

Application
Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

Solve this problem as an entry in your math journal:

In a game, players are asked to choose 5 numbers from 1–25, drawn at random. You choose 1, 16, 18, 24, and 25. Your friend chooses 1, 2, 3, 4, and 5. Who do you think has a better chance of winning the game? Explain.

6.3.1: Number Cube Game

Names:
Date:

1. Predict how many rolls it will take you to cover each space on the board with at least one marker.

   Our prediction is ________________________.

2. Working in pairs, one player rolls the cubes and the other player places a marker on the corresponding board space for that roll. If a combination is rolled that has already been recorded on the board, place another marker on top of the marker(s) that are already on that space.

Colour: ______

Colour: ______

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

3. When every space has at least one marker, count the markers to find your total number of rolls.

   Our total number of rolls ________________________.

4. Compare this total to your prediction. If they are different, explain why you think this happened.

6.3.1: Number Cube Game (continued)

Summary: Record the number of markers on each space.

Game 1

Colour: _______

Colour: _______

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Game 2

Colour: _______

Colour: _______

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

6.3.2: Recording Chart

Name:
Date:

Game 1

Possible total of two coloured number cubes	Number of rolls that yield this total	Theoretical probability of this total (out of 36)	Experiment: Number of rolls that did yield this total	Total number of rolls in the experiment	Experimental Fraction of total number of rolls	Experimental Percent of total number of rolls
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
				Column totals		

Game 2

Possible total of two coloured number cubes	Number of rolls that did yield this total	Total number of rolls in the experiment	Experimental Fraction of total number of rolls	Experimental Percent of total number of rolls
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
		Column totals		

Unit 6: Day 4: Experimental and Theoretical Probability: Part 3

Grade 8

Math Learning Goals

- Compare theoretical and experimental probability and sample size.
- Identify complementary events.

Materials

- BLM 6.4.1

Assessment Opportunities

Minds On…

Whole Class → Reflection

Recall the concepts of theoretical and experimental probability discussed on Day 3.

Students may refer to the results of their coin toss simulation.

A similar exploration can be completed with a spinner.

Action!

Pairs → Investigation

Introduce the game Green Is a Go (BLM 6.4.1). Students play the game and each student completes all the questions.

Curriculum Expectations/Question and Answer/Mental Note: Listen to pairs' discussions, making mental notes of all of the ideas that need to be discussed during whole-class consolidation and debriefing.

Consolidate Debrief

Whole Class → Discussion

Discuss the students' answers to BLM 6.4.1. Collect the data from each pair to build a larger sample size. Compare the probability of winning with this larger sample to the theoretical probability of winning $\left(\frac{4}{25}\right)$. Define complementary events when discussing question 5 and ask students to provide other examples.

Concept Practice
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Suppose you play the game with 2 green, 2 red, and 2 yellow tiles.

Write a summary in your math journal, explaining how to find the theoretical probability of drawing 2 green tiles from the bag, with replacement of the tile between draws.

Determine the probability of the complementary result.

If you were to play the game 40 times, what result would you expect? Suggest possible reasons to support your prediction.

6.4.1: Green Is a Go

Names:
Date:

With a partner, play a game involving 5 tiles in a bag, e.g., two red, two green, one yellow. Take two tiles from the bag during your turn.

Play the Game

You may not look in the bag. Draw one tile from the bag and place it on the table. Return the tile to the bag, draw another tile.
You win if the two tiles drawn during your turn are both green.
Predict the number of wins if you play the game 20 times. Record and explain your prediction.

1. Record your wins and losses on the tally chart. Continue this until you have played a total of 20 times.

	Green, Green, (win)	Not Green/Green (loss)
Totals		

2. Use your results to find the experimental probability of winning. (Remember that probability is the number of wins divided by the total number of times the game was played.)

3. How does this compare with your predictions? Explain.

4. Find the theoretical probability of winning. Use a tree diagram or a list to show all possible draws.

5. Compare the probability of winning to the probability of not winning, using both experimental and theoretical results. What do you notice?

6. Write a paragraph to compare the theoretical probability you just calculated to the experimental probability you found earlier. Are these results different or the same? Why do you think they are the same/different?

Math Learning Goals

- Consolidate concepts of theoretical and experimental probability.

Materials

- BLM 6.5.1, 6.5.2

Assessment Opportunities

Minds On…

Whole Class → Connecting

Invite students to ask any questions about the work from the previous class.

Review the key concepts.

Action!

Individual → Demonstrating Skills and Understanding

Students complete BLM 6.5.1.

Curriculum Expectations/Written Response/Self-Assessment: Students assess their responses and note areas where they need more help.

Consolidate Debrief

Whole Class → Sharing

Students share their responses and thinking. They assess their own understanding. Assign practice questions based on students' assessment of need. Group students so that they can help each other during the practice.

Peer tutoring would be appropriate if a student has difficulty.

Differentiated Concept Practice

Home Activity or Further Classroom Consolidation

Complete the practice questions.

Provide students with appropriate practice questions.

6.5.1: Checkpoint for Understanding Probability

Name:
Date:

Show full solutions in spaces provided. Read the questions carefully.

1. Tria had one each of five different-shaped number solids having 4, 6, 8, 12, and 20 sides. She rolled two at a time and found probabilities for the sum of the numbers that came up. She recorded the probabilities in the first column of the table. When it came time to fill in the second column, she had forgotten which number solids she had used. Figure out which number solids she must have used and explain your thinking. The first one has been done.

She found that	Using these number solids
Probability of a 6 was $\frac{5}{48}$	The total number of possible combinations was 48. Both the 4 and 12, and the 6 and 8 combinations would have given 48 possible combinations. If a 4-sided and a 12-sided number solid were rolled and the sum was 6, the possible combinations were 1 and 5, 2 and 4, 3 and 3, and 4 and 2 on the respective number solids. That gives 4 rolls totalling 6 and a probability of rolling a 6 as $\frac{4}{48}$ or $\frac{2}{24}$, not $\frac{5}{48}$. If a 6-sided and an 8-sided number solid were rolled and the sum was 6, the possible combinations were 1 and 5, 2 and 4, 3 and 3, 4 and 2, and 5 and 1 on the respective number solids. That gives 5 rolls totalling 6 and a probability of rolling a 6 as $\frac{5}{48}$. Therefore, Tria must have used the 6- and 8-sided number solids.
Probability of a 3 is $\frac{2}{80}$ or $\frac{1}{40}$	
Probability of a 3 is $\frac{1}{80}$	
Probability of a 4 is less than probability of a 5	

6.5.1: Checkpoint for Understanding Probability (continued)

2. Henry, Toshi, Lizette, Anna, and Vance were all scheduled to give oral reports in their history class on Tuesday. However, when the class met, the teacher announced that only two people would give their presentations that day. To determine which two, all of their names were placed in a hat and two names were drawn out. What is the probability that Henry and Anna were the names picked to give presentations? Show how you arrived at your conclusion.

3. Claire has two bags of coloured cubes, one marked A and the other marked B. In bag A, there are 3 yellow and 4 green cubes. In bag B, there are 2 blue and 5 red cubes. Without looking, Claire picks one cube from bag A and then one cube from bag B. Answer the questions below based on this information. Assume that after each part all cubes are replaced in their appropriate bag.
   a) What is the question, if the answer is $\frac{8}{49}$?

   b) What is the question, if the answer is 0?

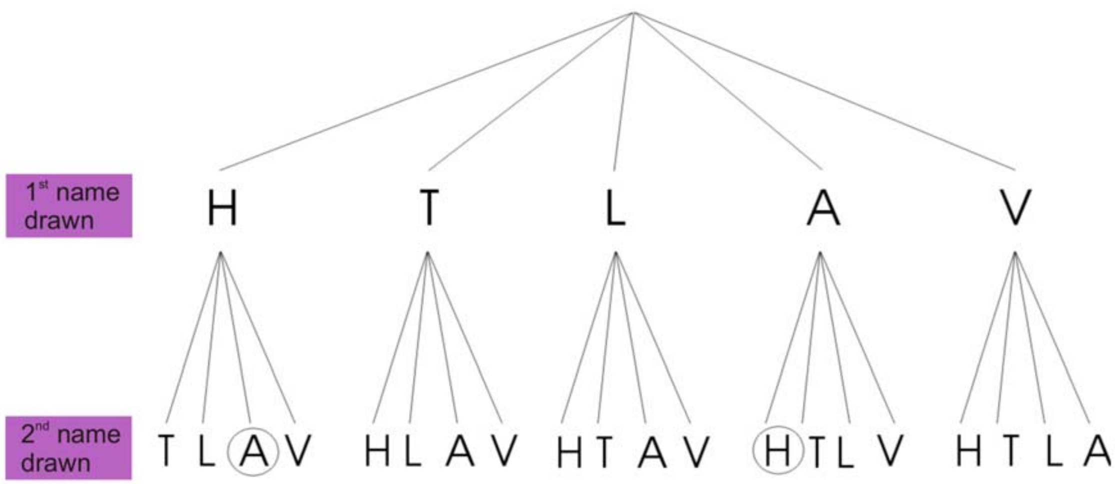
   c) What is the question, if the answer is $\frac{3}{7}$?

6.5.2: Checkpoint for Understanding Probability (Answers)

1.

Probability of a 3 is $\frac{2}{80}$ or $\frac{1}{40}$	The 4 and 20 combination gives 80 possible outcomes. A sum of 3 results from rolling 1 and 2, or 2 and 1. That gives 2 rolls totalling 3 and a probability of rolling a 3 as $\frac{2}{80}$ or $\frac{1}{40}$. Therefore, Tria must have used the 4- and 20-sided number solids.
Probability of a 3 is $\frac{1}{80}$	The sum of 3 results from rolling 1 and 2, or 2 and 1. For these 2 rolls to yield a probability of $\frac{1}{80}$, we must have had $\frac{2}{160}$ probability. Only the combination of 8 and 20 gives 160 possibilities. Therefore, Tria must have used the 8- and 20-sided number solids.
Probability of a 4 is less than probability of a 5	A sum of 4 results from rolling 1 and 3, 2 and 2, or 3 and 1. A sum of 5 results from rolling 1 and 4, 2 and 3, 3 and 2, or 4 and 1. All of these rolls are possible using any of the number solids. Since the sum of 4 can occur in fewer ways then a sum of 5 for any pair of number solids, probability of a 4 is less than probability of a 5 for any pair of these number solids. Therefore, Tria cannot tell from this information which number solids she used.

2. Students may list all possible outcomes using a tree diagram.



There are 20 outcomes in the tree diagram and the two circled outcomes represent Henry and Anna being picked. Therefore the probability of Henry and Anna being picked is $\frac{2}{20}=\frac{1}{10}$

OR

Students may reason that there are 5 choices for the first pick and 4 left for the second so there are 5 × 4 or 20 possible outcomes. However, only 2 of the outcomes are favourable: HA or AH. Therefore the probability of Henry and Anna being picked = $\frac{2}{20}=\frac{1}{10}$.

6.5.2: Checkpoint for Understanding Probability (continued)

3. a) Each of the 7 cubes from A could be picked along with each of the 7 cubes from B. This gives 7 × 7 = 49 possibilities in all. If 8 of these 49 outcomes are favourable, then we want the 4 green cubes from A with the 2 blue cubes from B. Therefore, the question is: What is the probability of picking 1 green and 1 blue cube?

   b) If the probability is 0, the outcome is impossible. There are many possible answers to this question, e.g., What is the probability of picking a purple cube? What is the probability of picking a yellow and a green cube?

   c) Since the answer is and there are 49 possible outcomes, I'll think of as.
   To get 21 favourable outcomes, I could pick yellow from A and any colour from B in 3 × 7 = 21 ways. Therefore, the question could be: What is the probability of picking 1 yellow cube?

Unit 6: Day 6: Revisiting Number Triangles

Grade 8

Math Learning Goals

- Consolidate an understanding of counting techniques required for probability problems.
- Consolidate an understanding of complementary events.

Materials

- BLM 6.6.1
- coloured number cubes

Assessment Opportunities

Minds On…

Whole Class → Connecting

Recall the concepts from Day 1. How must three side lengths be related to form a triangle?

Review types of triangles – equilateral, isosceles, scalene. Could you consider 1, 1, and 2, or 3, 4, and 7 to be the sides of a triangle? Why or why not?

Does the order of the numbers matter in this context?

Individual → Applying Knowledge

Using BLM 6.1.1, students complete column entitled Type of Triangle and explain their decision.

Action!

Whole Class → Investigation

Students consider the following question and record their answers: How many outcomes are possible when rolling three number cubes? Explain.

Identify students who can demonstrate that: 6 × 6 × 6 = 216 is the number of possibilities if the order of the numbers matters and students who can show that there are: (6 ways to get all 3 numbers the same) + (30 ways to get 2 numbers the same) + (120 ways to get all 3 numbers different) = 156 (if the order of the numbers in this context does not matter but some rolls will lead to the same triangle).

Pairs → Exploring

Students complete BLM 6.6.1.

Answers:

$$P\ (equilateral) = \frac{6}{216} = \frac{1}{36}$$

$$P\ (isosceles\ but\ not\ equilateral) = \frac{48}{216} = \frac{2}{9}$$

$$P\ (scalene) = \frac{42}{216} = \frac{7}{36}$$

$$P\ (impossible\ triangle) = \frac{120}{216}\ by\ counting$$

$$OR\ by\ 1 - (\frac{6}{216} + \frac{48}{216} + \frac{42}{216})$$

Consolidate Debrief

Whole Class → Presentations

Students share their responses (BLM 6.6.1).

Define complementary events in this context, i.e., making a triangle/not making a triangle.

Curriculum Expectations/Presentation/Checkbric: Assess students on the clarity of their presentations.

Concept Practice
Exploration

Home Activity or Further Classroom Consolidation

Roll three number cubes 50 times and record the results of each roll. Create a tally chart of the outcomes according to "no triangle possible" or "the type of triangle" that could be formed.

Calculate the experimental probability of each type of triangle.

Compare the theoretical probabilities to the experimental probabilities and explain differences.

For two numbers the same, there are 6 choices for the repeated number and 5 choices for the different number, making 6 × 5 = 30 ways.

Of these 30 ways, 16 generate isosceles triangles and 14 generate impossible triangles. Each of the 16 combinations could be rolled in three ways, making 48 ways to roll.

For all three numbers different there are 6 × 5 × 4 = 120 possible combinations.

Challenge some of the strongest students to explain the counting in two ways.

For all three numbers to be different, it does not matter what number is chosen first. After that, there are 5 ways for the second number to be different, then 4 ways (for each of those 5 possible second numbers) for the third number to be different from the first and second. This makes 5 × 4 = 20 ways for all three numbers to be different.

6.6.1: Analysing the Number Cube Data

Name:
Date:

1. What is the total number of possible outcomes when rolling three number cubes? Explain.

2. Fill in the chart using the data from the Home Activity on Day 1.

Type of triangle	Number of rolls that resulted in this type of triangle
Equilateral	
Isosceles	
Scalene	
No triangle	

3. a) When rolling three number cubes to determine the three possible side lengths for a triangle, what is the theoretical probability of forming:

   i) an equilateral triangle?

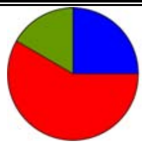
   ii) an isosceles triangle?

   iii) a scalene triangle?

   iv) no triangle?

   b) When rolling three number cubes, what is the theoretical probability of being able to form a triangle of any type?

   c) When rolling three number cubes, what is the theoretical probability of **not** being able to form a triangle of any type? Calculate this answer two ways.



Math Learning Goals

- Review addition and subtraction of integers.
- Link probability to the study of integers.

Materials

- integer tiles
- number cubes
- BLM 6.7.1

Assessment Opportunities

Minds On…

Whole Class → Connecting

Calculate experimental probabilities for more trials using all student data from the Day 6 Home Activity. [Generally, the larger the number of trials, the closer experimental probability should approach theoretical probability.]

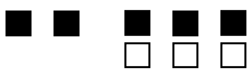
Review the representation of integers, using integer tiles. Identify opposites, several models of zero, and several models of +2.

Students model adding and subtracting integers, using integer tiles.

For (+3) + (−2) show 

Using the zero principle, the result is +1.

For (−2) – (−5) show  and ask if it is possible to take away −5. Ask for a different model of –2 that would make it possible to take away −5.



Once –5 is removed, the result of +3 is obvious.

Ask:

- What is the result of adding an integer and its opposite?
- Does the order matter when we add integers? When we subtract?

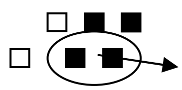
Model a series of questions: (+2) – (+5) and (+2) + (−5); (−1) – (+4) and (−1) + (−4) to show that subtracting an integer is like adding the opposite integer.

Ask: What possible addition question and answer are modelled by…?

e.g.,

[giving (+ 1) + (−2) = −1]

[giving (−1) – (−2) = +1]



What subtraction question and answer are modelled by…?

Use integer tiles cut out of coloured transparencies to visually reinforce concepts.

Students should be able to use the zero principle where simple matching and removing "zeros" is required. They may require extra practice with situations involving addition of one or more zeros to facilitate an operation.

Students can use the integer tiles to assist in finding sums.

Action!

Pairs → Exploration

Students conduct a simple probability experiment with integers and two different-coloured number cubes, recording results on a tally chart (BLM 6.7.1).

Learning Skills/Observation/Checklist and Curriculum Expectations/ Observation/Mental Note: Circulate while students analyse the experimental and theoretical probabilities up to and including question 6 on BLM 6.7.1.

Note students who need to review integer skills.

Consolidate Debrief

Whole Class → Making Connections

Several students describe and compare their experimental results. Lead a discussion to compare the experimental probabilities with each of the theoretical probabilities. What have the students found? Are the results close?

What do you think would happen to your experimental probabilities if you did more than 25 trials or if you combined the trials from every group in the class? How might expertise with integers have affected findings?

Reflection

Home Activity or Further Classroom Consolidation

Complete worksheet 6.7.1.

6.7.1: Integer Number Cubes

Name:
Date:

Part A

Use two different-coloured number cubes. Choose one cube to be negative numbers and the other to be positive numbers. Record all possible results in the table.

1. Roll the number cubes and **add** the two numbers together. Note the sum in the chart.
2. Repeat rolling the number cubes and finding the sum until you have a variety of sums.
3. Complete the chart to record all 36 possible outcomes.

Colour: _______

Colour: _______		1	2	3	4	5	6
	–1						
	–2						
	–3						
	–4						
	–5						
	–6						

Part B

1. Roll the number cubes 25 times and record each outcome of the sum in the tally column of the table.
2. Total the tallies to find the frequency of the various sums.
3. What sum did you get the most? Why do you think this is so?
4. What sum did you get the least? Why do you think that is?
5. The **experimental probability** of an event happening is given by the fraction

$$\frac{\textit{number of times the event happened}}{\textit{total number of trials}}$$

For example, if you rolled the number cubes 25 times, and you got a sum of 3 five times, then the experimental probability of getting a sum of 3 is $\frac{5}{25} = \frac{1}{5}$.

Find the experimental probability of each of the sums and enter these experimental probabilities in the table.

6. Fill in the 5th column of the table with all of the outcomes that you could roll the cubes to yield each sum. For example, a sum of 4 could be 6 and −2, or 5, and −1.
7. Fill in the 6th column with the number of outcomes in the 5th column.
8. The **theoretical probability** of an event is given by the ratio

$$\frac{\textit{number of possible ways of the event happened}}{\textit{total possible outcomes}}$$

For example, there are two possible ways of getting a sum of 4 (see the first chart you completed). There are a total of 36 possible outcomes of the number cubes, so the theoretical probability of getting a sum of 4 is $\frac{2}{36} = \frac{1}{18}$.

Find the theoretical probability of rolling each of the possible sums. Enter your results in the last column of the table.

	Experimental			**Theoretical**		
Sum	**Tally**	**Frequency**	**Probability**	**Possible Outcomes**	**Number of Possible Outcomes**	**Probability**
–5						
–4						
–3						
–2						
–1						
0						
1						
2						
3						
4				6, –2; 5, –1	2	$\frac{2}{36}$
5						
Totals						

6.7.1: Integer Number Cubes (continued)

Part C

1. What is the theoretical probability of:
   a) rolling a negative sum?

   b) rolling an even sum (positive or negative?)

   c) not rolling a sum of 3?

   d) rolling anything other than a sum of 0?

2. What extra considerations would be needed if the roll of the number cubes were subtracted rather than added?

3. What other words do we use to indicate positive and negative? Suggest a variety of situations where these words might be used.

4. How might addition and subtraction of integers be shown using a number line?