Unit 10
Visualizing Geometric Relationships

Grade 8

Lesson Outline

BIG PICTURE

Students will:
- develop geometric relationships involving right-angled triangles, and solve problems involving right-angled triangles geometrically.

Day	Lesson Title	Math Learning Goals	Expectations
1	Building Squares	• Activate prior knowledge. • Explore the Pythagorean relationship, using manipulatives (tangrams, chart paper).	8m49 CGE 3c, 5a
2	Squared From All Sides *GSP®4 file:* **Pythagorean Puzzle**	• Explore and investigate, using concrete materials, the relationship between the area of the squares on the legs and the area of the square on the hypotenuse of a right-angled triangle.	8m49 CGE 3c, 4f, 5e
3	Challenges Are Shaping Up… *GSP®4 file:* **N-agon Areas**	• Investigate the relationship of the areas of semi-circles drawn on the sides of a right-angled triangle.	8m49 CGE 5g, 7b
4	Pythagoras In Proportion	• Solve problems involving right-angled triangles geometrically, using the Pythagorean relationship and proportionality. • Hypothesize and investigate the relationship between the area of similar figures drawn on the sides of a right-angled triangle.	8m50 CGE 5b
5	Instructional Jazz		
6	Mathematics in Early Greece *(lesson not included)*	• Investigate the definition and historical study of polyhedra. • Construct the five Platonic solids. http://matti.usu.edu/nlvm/nav/vlibrary.html Index → Platonic Solids → Geometry (6–8)	8m51 CGE 7f
7	What's the Connection? *(lesson not included)*	• Record and organize data consisting of the number of faces, vertices, and edges for each platonic solid. • Conjecture a possible relationship between the number of faces, vertices, and edges of a polyhedra.	8m51, 8m61, 8m68, 8m70, 8m73, 8m78 CGE 3c, 5b
8	Impossible Shapes *(lesson not included)*	• Test the hypothesis from Day 2 by constructing and examining non-Platonic solids. • Using the relationship formula developed, investigate impossible polyhedra shapes.	8m62, 8m78 CGE 3b, 3c
9	Instructional Jazz		
10	Please Move *(lesson not included)*	• Investigate and report on real-world examples of translations, reflections, and rotations.	8m53 CGE 5b, 7f
11	Shifty Business *(lesson not included)*	• Translate single points and sets of points horizontally, vertically, and through a combination of both directions. • Identify how the type of transformation affects the original point's coordinates.	8m52 CGE 3c, 4b, 5a

Day	Lesson Title	Math Learning Goals	Expectations
12	Points to Reflect Upon *(lesson not included)*	• Reflect single points and sets of points in the *x*-axis, and in the *y*-axis. • Identify how the type of transformation affects the original point's coordinates.	8m52 CGE 3c, 4b, 5a
13	A New Slant on Reflection *(lesson not included)*	• Reflect single points and sets of points in the line that forms the angle bisector of the *x*- and *y*-axes and passes through the first and third quadrants. • Identify how the type of transformation affects the original point's coordinates.	8m52 CGE 3c, 4b, 5a
14	Getting Dizzy *(lesson not included)*	• Rotate single points and sets of points through 90, 180, and 270 degrees about the origin. • Identify how the type of transformation affects the original point's coordinates.	8m52 CGE 3c, 4b, 5a
15	Summative Assessment		

Unit 10: Day 1: Building Squares

Grade 8

Math Learning Goals

- Activate prior knowledge.
- Explore the Pythagorean relationship, using manipulatives (tangrams, chart paper).

Materials

- 3 sets of tangram pairs
- chart paper
- BLM 10.1.1, 10.1.2

Assessment Opportunities

Minds On…

Whole Class → Brainstorm

List examples of, or references to, the use of right-angled triangles, e.g., bridges, trusses, flags.

Ask: How do we identify a right-angled triangle? Students use specific vocabulary in responding: e.g., *legs, hypotenuse, vertices, right angle*.

Use paper cut-outs of tangram pieces as an alternative.

Word Wall
- leg
- hypotenuse
- vertices
- right angle

Action!

Pairs → Investigation

Explain the task:

> Select one large triangular tangram piece and identify it as the central piece. Using the remaining tangram pieces, construct 3 perfect squares off the legs and hypotenuse. Trace each piece onto blank paper.

Determine if there is a relationship between the pieces of the tangram on the legs and the pieces on the hypotenuse.

Post students' work.

Learning Skills/Initiative/Rating Scale: Observe how students interact with their peers.

Consolidate Debrief

Whole Class → Discussion

Students explain their solutions.

Ask:
- What did you discover?
- Which pieces did you use?
- How are the pieces related?

Determine how the pieces of the square on the legs fit onto the pieces of the square on the hypotenuse.

Ask: What does this tell you about right-angled triangles?

Application
Exploration

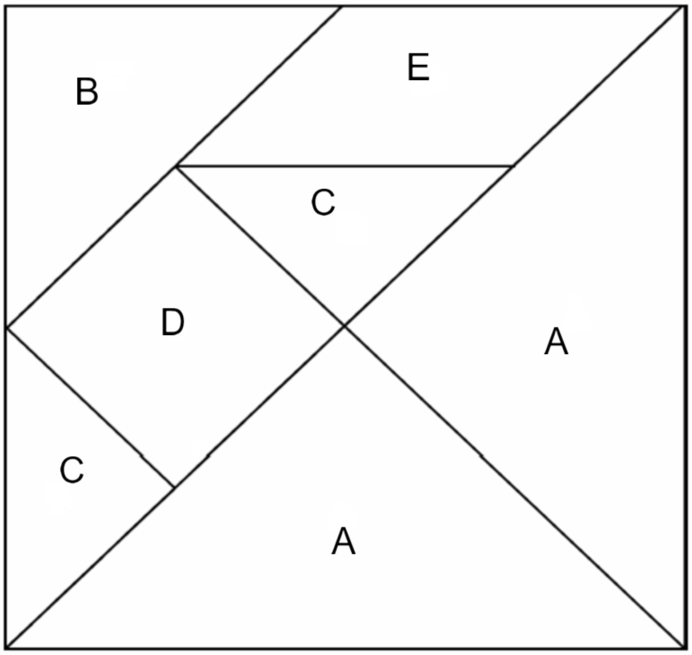
Home Activity or Further Classroom Consolidation

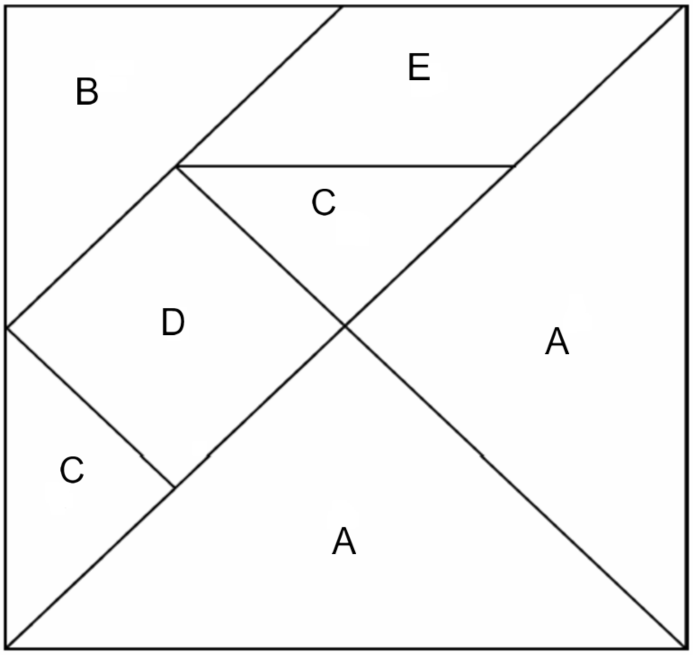
- Create your own piece-puzzles, using worksheet 10.1.1. Make a right-angled triangle and construct squares on the legs and hypotenuse of it. Cut the squares on the legs into pieces so that they fit exactly onto the square on the hypotenuse.
- Research two interesting notes about Pythagoras' life, unrelated to mathematics.

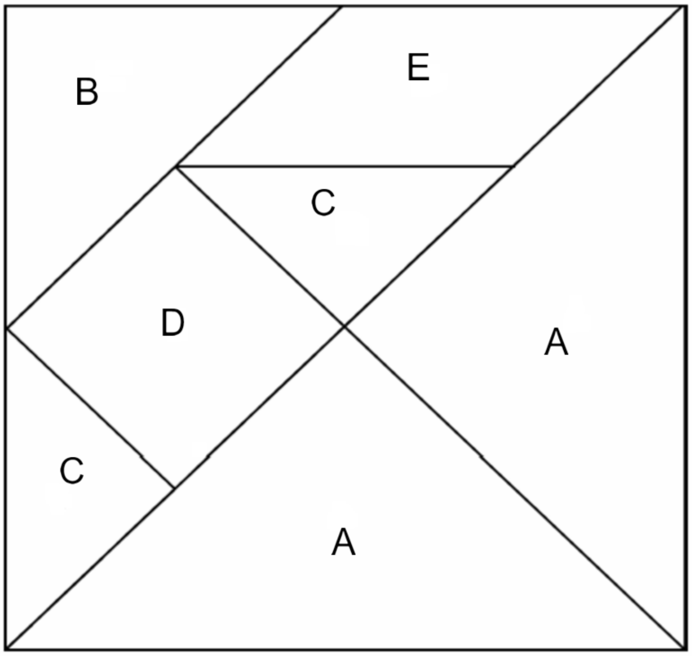
See BLM 10.1.2 for solutions.

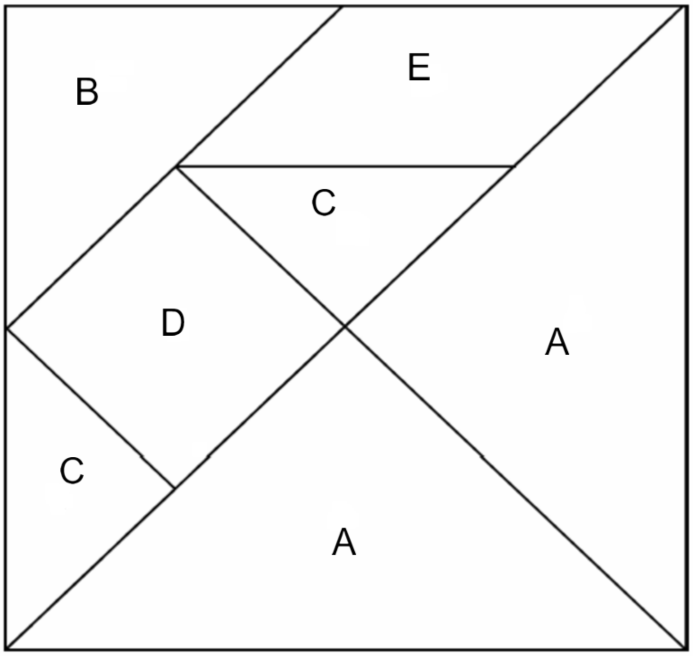
Students' research is due for Day 5.

10.1.1: Tangram Squares



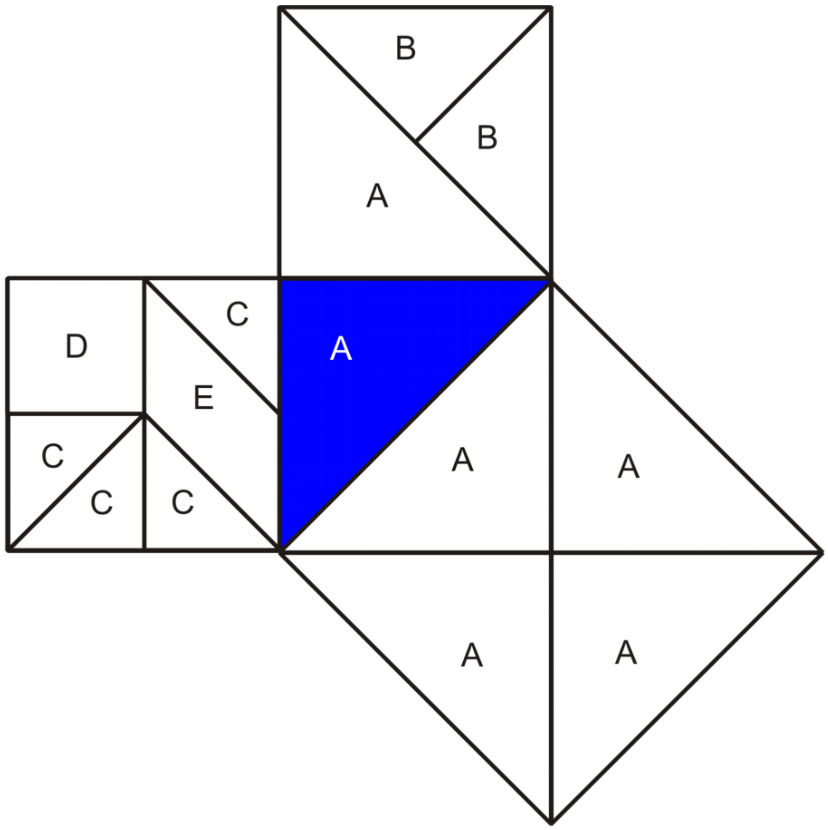




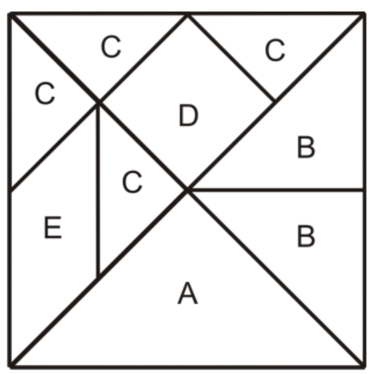


10.1.2: Tangram Squares (Solutions)

Solution 1

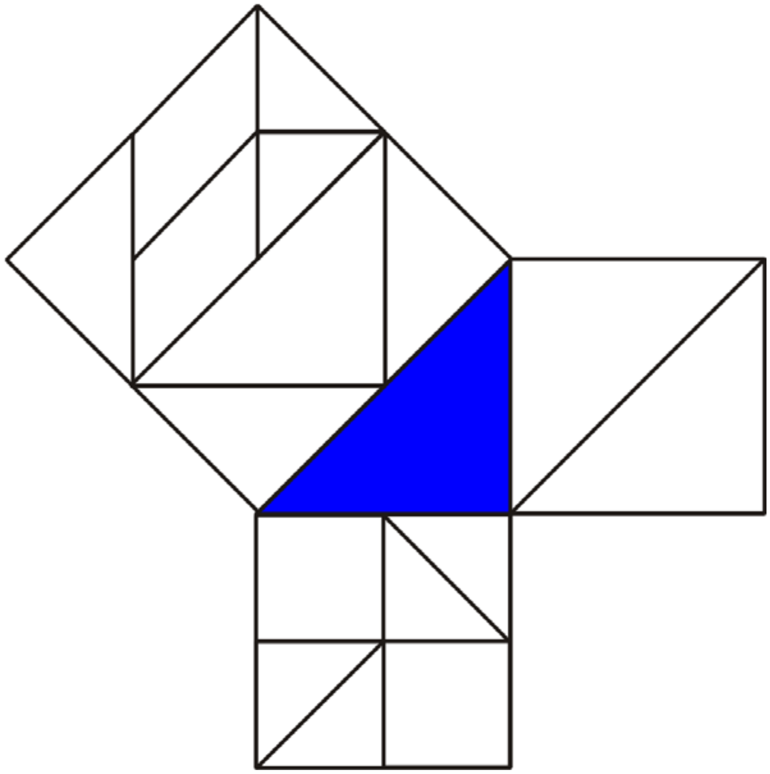


Tangrams on the legs fit on the square on the hypotenuse.



10.1.2: Tangram Squares (Solutions)

Solution 2



Unit 10: Day 2: Squared From All Sides

Grade 8

Math Learning Goals

- Explore and investigate, using concrete materials, the relationship between the area of the squares on the legs and the area of the square on the hypotenuse of a right-angled triangle.

Materials

- BLM 10.2.1, 10.2.2, 10.2.3, 10.2.4

Assessment Opportunities

Minds On…

Whole Class → Discussion

Review students' solutions to their Home Activity puzzles. Use an available representation, e.g., chart to be left up for display, poster, overhead representation using transparent tangram, Pythagorean Puzzle GSP®4. Connect results to work with tangrams on Day 1. Ask: Is there a pattern?

Individual → Journal

Draw the standard diagram for illustrating the relationship of the squares drawn on the sides of a right-angled triangle, identifying legs and hypotenuse titled Pythagorean Relationship.

Think/Pair/Share → Anticipation Guide

Students highlight key words, then complete the Before column on the Anticipation Guide (BLM 10.2.1) and explain their reasoning to a partner.

Pythagorean Puzzle.gsp

Demonstrate the method for determining 90°, using the corner of a sheet of paper or a grid.

Action!

Pairs → Investigation

Pairs investigate which combinations of squares will successfully form a right-angled triangle and which will not form a right-angled triangle.

Students cut out 12 different squares and arrange them in groups of three such that the side lengths create triangles. Using graph paper, they determine if the triangle is a right-angled triangle. They glue down the squares and create their own chart, using BLM 10.2.2.

Reasoning & Proving/Observation/Checklist: Observe how students talk about and record their thinking during the investigation.

Answer key for right-angled triangles:

- AYE
- WRG
- ZPF
- HSM
- PSZ

For triangles that are not right-angled, answers will vary, e.g., WHG, WHM

Consolidate Debrief

Whole Class → Summarizing

Consolidate the investigation by completing a class summary chart.

Summarize the Pythagorean relationship: In a right-angled triangle, the sum of the areas of the squares on the legs equals the area of the square on the hypotenuse.

Students copy the Pythagorean relationship into the summary (BLM 10.2.3).

Students complete the After column on the Anticipation Guide for questions 1–3.

This relationship works in only right-angled triangles.

Word Wall

- Pythagorean relationship

Practice

Home Activity or Further Classroom Consolidation

Complete worksheet 10.2.4.

Students hand in their completed worksheet for assessment.

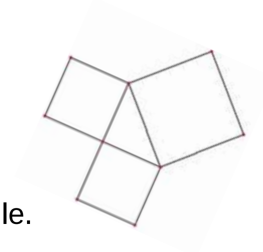
10.2.1: Anticipation Guide for Right-Angled Triangles

Instructions:

- Check Agree or Disagree beside each statement in the **Before** column.
- Compare your choice and explanation with a partner.
- Revisit your choices at the end of the task.
- Check Agree or Disagree beside each statement in the **After** column.
- Compare the choices that you would make after the task with the choices that you made before the task.

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		1. Right-angled triangles can sometimes be isosceles.		
		2. There is a special relationship between the squares of the three sides of a right-angled triangle.		
		3. There is a special relationship between the squares of the sides for any triangle.		
		4. There is a special relationship between the areas of similar shapes that fit on the sides of a right-angled triangle.		

10.2.2: Pythagorean Puzzle

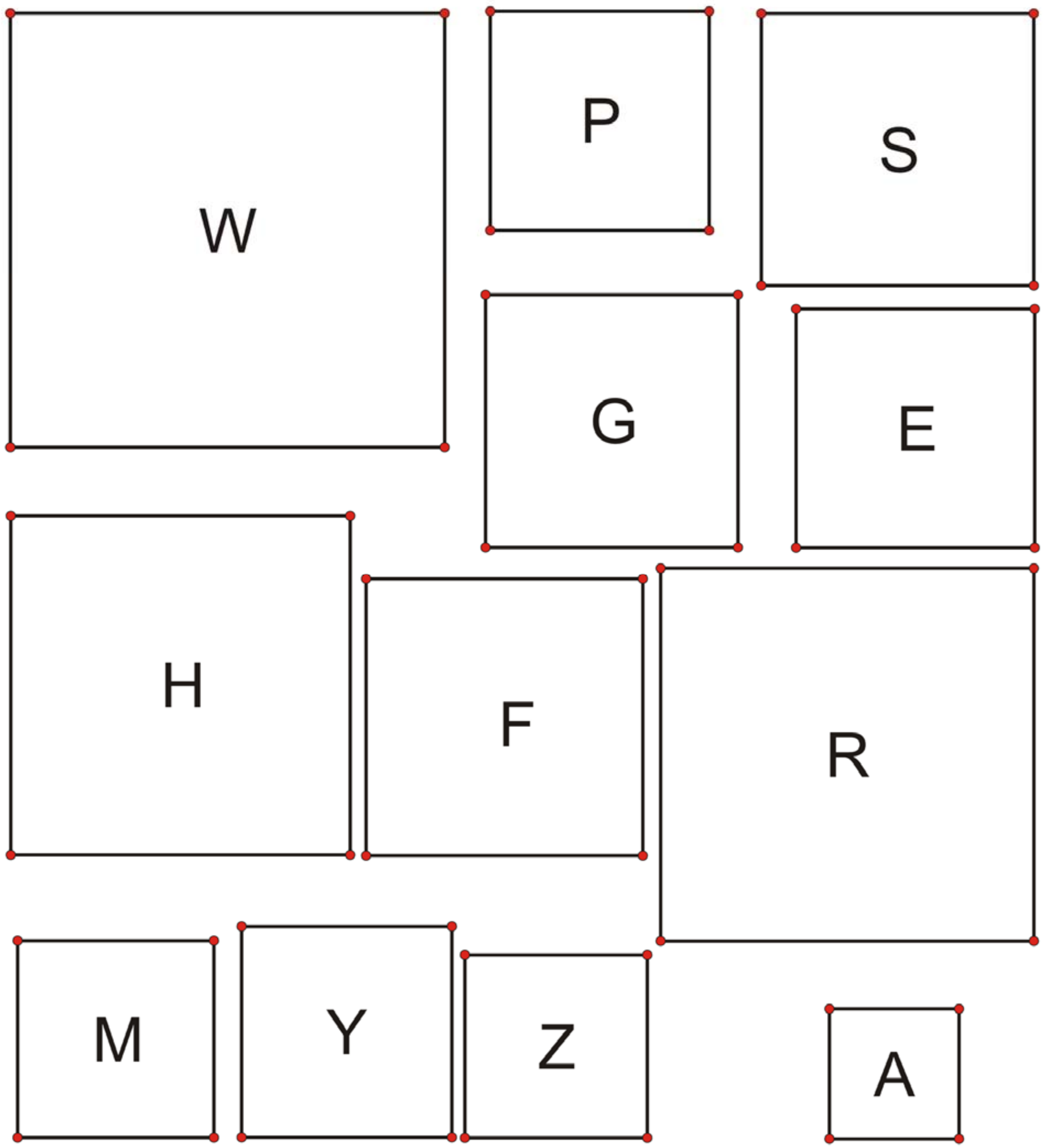


Cut out the squares.

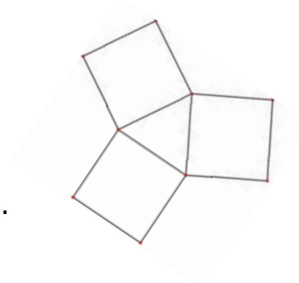
Explore which combinations of squares will successfully form a right-angled triangle.

Use graph paper to make sure your right angle is exactly 90°.

Glue down the squares.



10.2.2: Pythagorean Puzzle **(continued)**

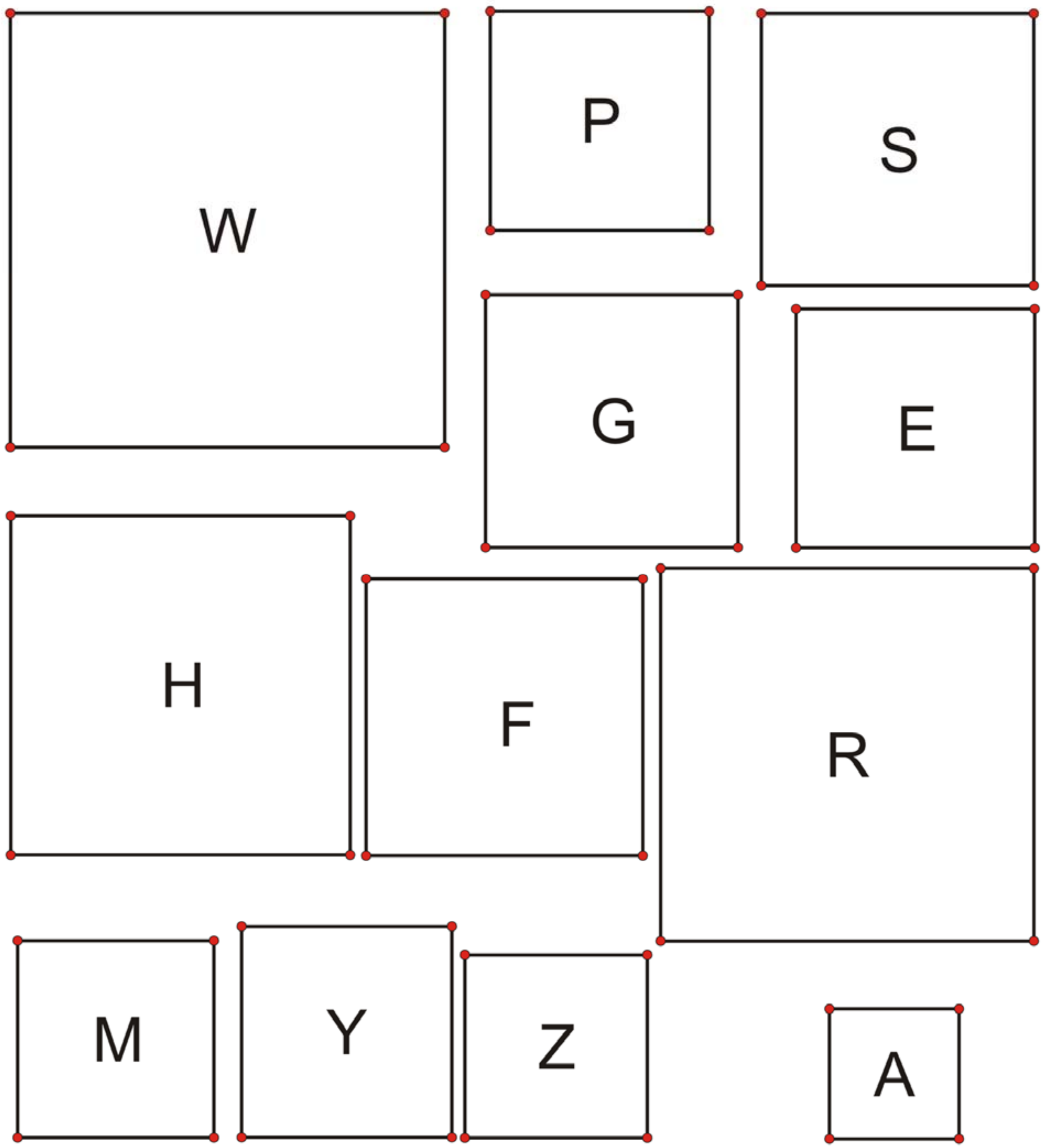


Cut out the squares.

Explore which combinations of squares will NOT form a right-angled triangle.

Use grid (graph) paper to make sure your angle is NOT 90°.

Glue down the squares.



10.2.3: Squared From All Sides Summary Chart

Name:

Part A: Squares that form a right-angled triangle

Triangle #	Label of Square on Leg 1	Area of Square on Leg 1	Label of Square on Leg 2	Area of Square on Leg 2	Label of Square on Hypotenuse	Area of Square on Hypotenuse	Area Part C

Part B: Squares that do not form a right-angled triangle

Triangle #	Label of Square on Leg 1	Area of Square on Leg 1	Label of Square on Leg 2	Area of Square on Leg 2	Label of Square on Hypotenuse	Area of Square on Hypotenuse	Area Part C

Part C: Investigate

Add the area of the square on Leg 1 to the area of the square on Leg 2.
What pattern to you notice?

Summary

Pythagoras was a famous Greek philosopher, Olympic coach, and mathematician. He was born on the island of Samos sometime in the sixth century B.C.E. He is credited with discovering the Pythagorean relationship, which states:

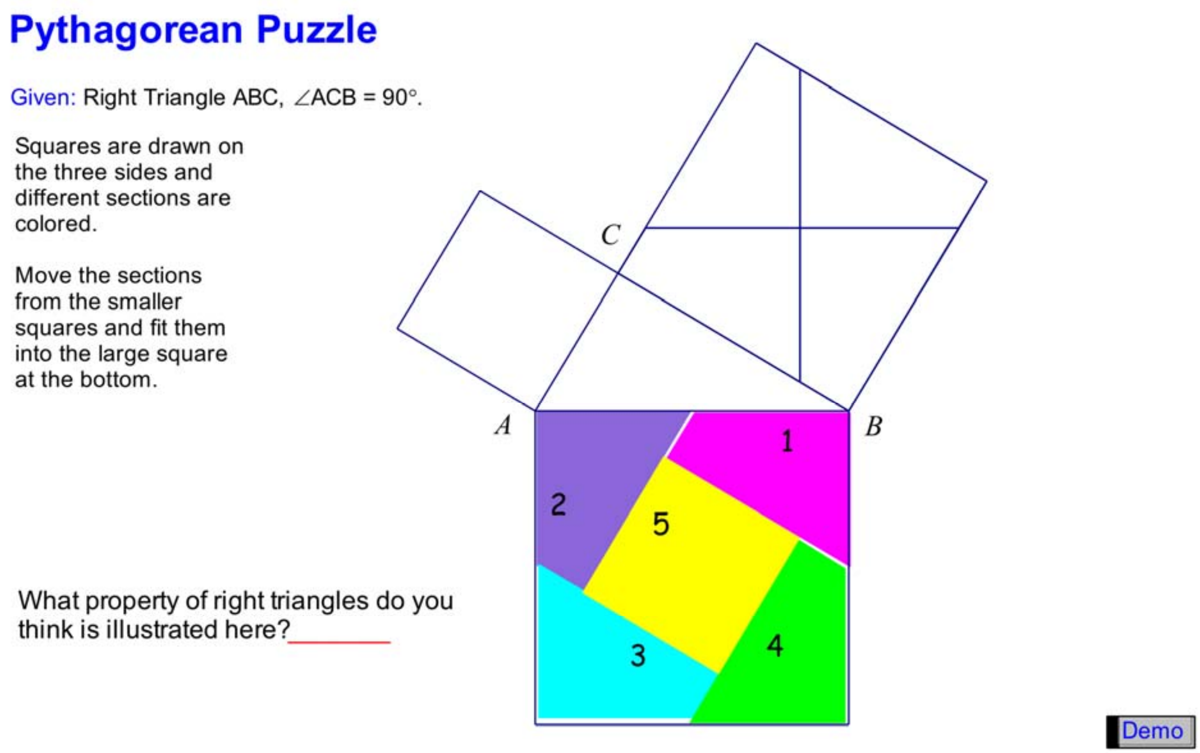
10.2.4: Follow-Up Chart for Pythagorean Relationship Investigation

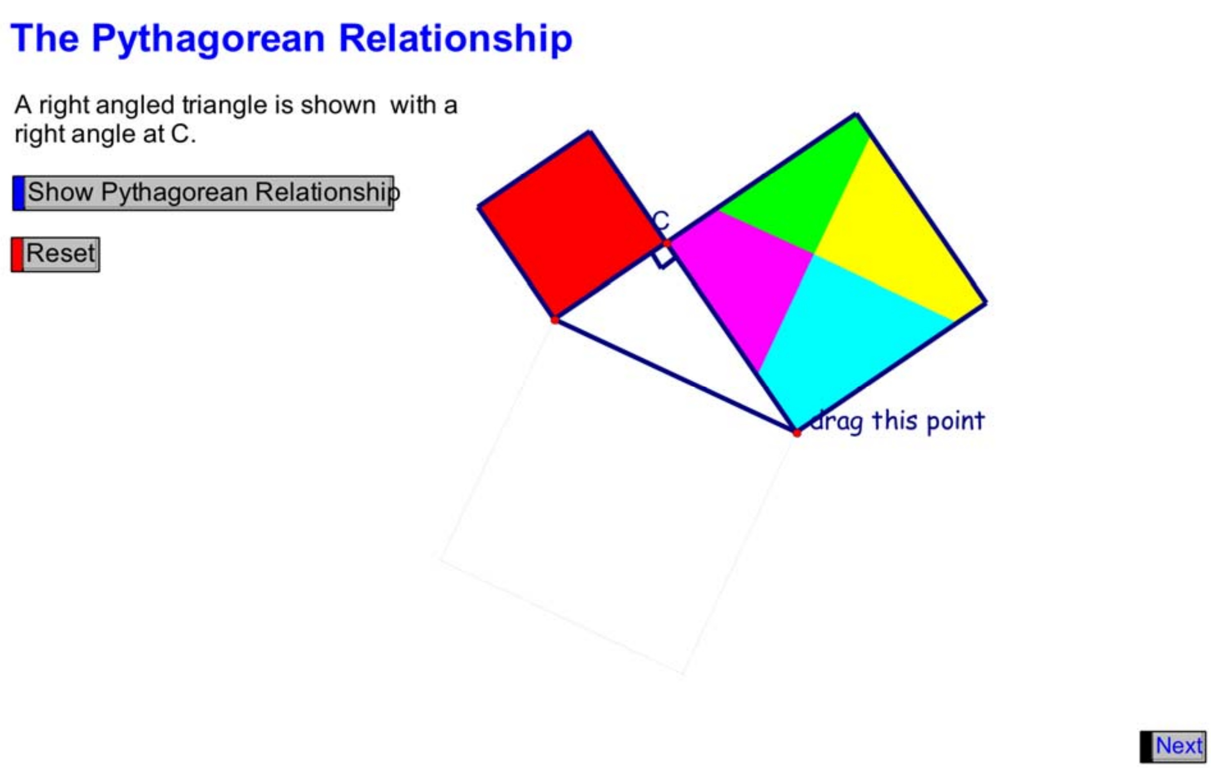
Fill in the blanks on the chart.

Right-Angled Triangle	Area of Square on Leg 1	Area of Square on Leg 2	Area of Square on Hypotenuse
ABC	9 cm^2	16 m^2	
DEF		15 mm^2	64 mm^2
STU	121 cm^2	36 cm^2	
XYZ	100 cm^2	30.25 cm^2	
LMN	16 cm^2		100 cm^2

Pythagorean Puzzle (GSP®4 file)

Pythagorean Puzzle.gsp





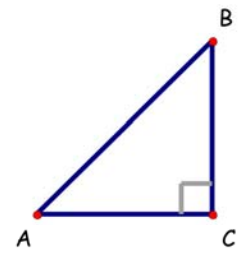
Pythagorean Relationship (GSP®4 file continued)

The Pythagorean Relationship

Given: Right Triangle ABC, ∠ACB = 90°.

Squares will be appear on each of the three sides when you click on the button:

Show Objects Hide Objects



Click in the colored section of each square then under *Measure* choose *Area.*

Under the *Measure* tab choose *Calculate* and determine the sum of the areas of AEFC and CGHB.

What do you notice about the sum of these two areas? _____

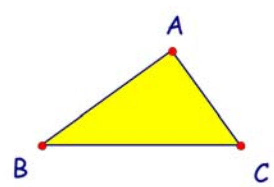
Next

The Pythagorean Relationship

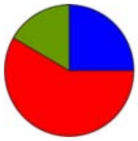
A right angled triangle is shown with a right angle at A.

Follow the steps below.

1) Show Squares of Sides
2) Show Altitude
3) Show Quadrilaterals
4) Show Area Measurements



Beginning



Math Learning Goals

- Investigate the relationship of the areas of semi-circles drawn on the sides of a right-angled triangle.

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Collect the Home Activity for assessment.

Using a sketch, reinforce the concept of the Pythagorean relationship. Stress that the relationship is true for right-angled triangles only.

Ask: Does this relationship work with shapes other than squares drawn on the right sides of a right-angled triangle?

N-agon areas.gsp
This GSP®4 sketch can be used to explore or consolidate.

Action!

Pair/Share → Investigation

Using grid paper, students draw a right-angled triangle. They construct semi-circles on the legs and hypotenuse of the triangle and calculate the areas of each semi-circle to determine the relationship the same way they did with squares on Day 2. Students share their work with another pair and explain their reasoning.

Reasoning & Proving/Observation/Checklist: Observe students as they explain their reasoning.

Review how to determine the area of a circle.

Consolidate Debrief

Whole Class → Discussion/Brainstorm

Summarize the findings of their investigation. The sum of the area of the semi-circles on the legs is equal to the area of the semi-circle on the hypotenuse. Pythagorean relationship works for a right-angled triangle using squares and semi-circles drawn on the sides.

Ask:
- What other shapes will work?
- Under what conditions will other shapes work?

Students complete the After column for question 4 of the Anticipation Guide (Day 2 BLM 10.2.1).

Do not answer these questions – this is a brainstorm only.

Exploration Practice

Home Activity or Further Classroom Consolidation

Draw a right-angled triangle with the length of legs being whole numbers. On each side of the triangle draw a rectangle (no squares are allowed!). Calculate the areas of the three rectangles. Does this demonstrate the Pythagorean relationship? Explain. Repeat with two more triangles.

N-agnon Areas (GSP®4 file)

N-agon Areas.gsp

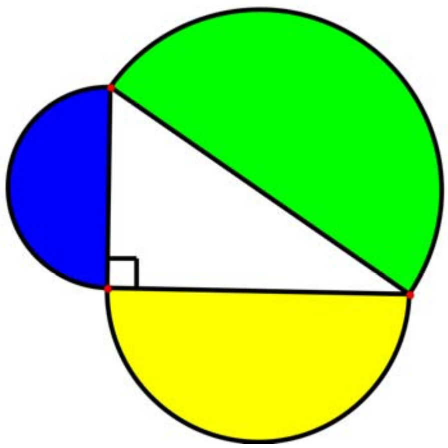
Green Area = 15.10 cm^2

Yellow Area = 10.45 cm^2

Blue Area = 4.64 cm^2

(Yellow Area)+(Blue Area) = 15.10 cm^2

Green Area	(Yellow Area)+(Blue Area)
3.91 cm^2	3.91 cm^2
7.21 cm^2	7.21 cm^2
44.16 cm^2	44.16 cm^2
24.95 cm^2	24.95 cm^2
15.10 cm^2	15.10 cm^2



Double click here to change the number of sides on the polygon = 4.00

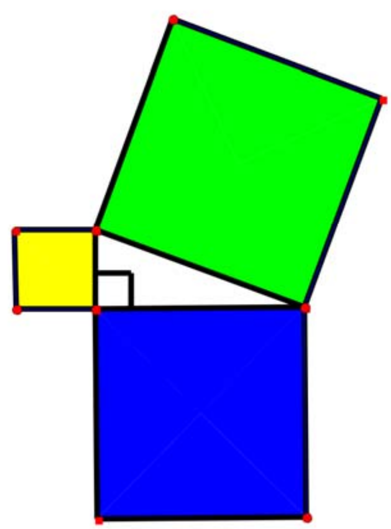
Green Area = 9.88 cm^2

Blue Area = 8.63 cm^2

Yellow Area = 1.25 cm^2

(Blue Area)+(Yellow Area) = 9.88 cm^2

Green Area	(Blue Area)+(Yellow Area)
72.42 cm^2	72.42 cm^2
76.50 cm^2	76.50 cm^2
35.90 cm^2	35.90 cm^2
9.88 cm^2	9.88 cm^2



N-agnon Areas (GSP®4 file continued)

Double click here to change the number of sides on the polygon = 6.00

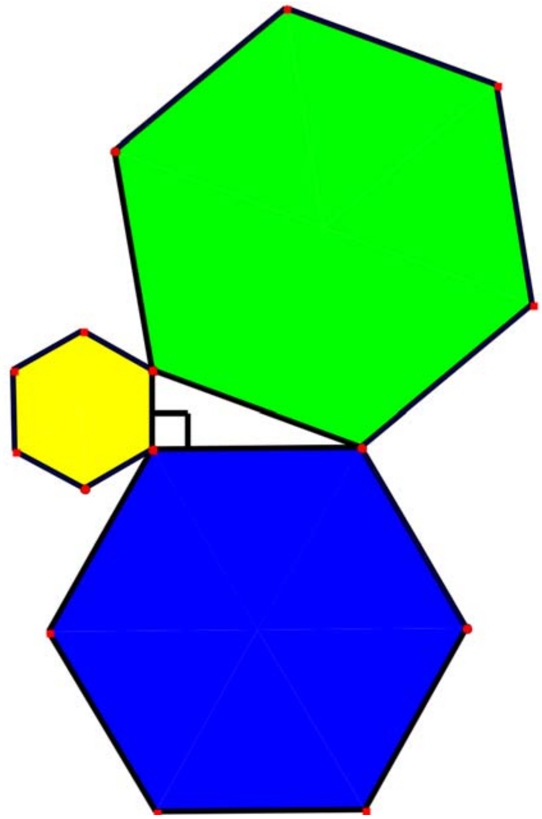
Green Area = 25.67 cm^2

Blue Area = 22.41 cm^2

Yellow Area = 3.25 cm^2

(Blue Area)+(Yellow Area) = 25.67 cm^2

Green Area	(Blue Area)+(Yellow Area)
72.42 cm^2	72.42 cm^2
76.50 cm^2	76.50 cm^2
35.90 cm^2	35.90 cm^2
25.67 cm^2	25.67 cm^2



Unit 10: Day 4: Pythagoras In Proportion

Grade 8

Math Learning Goals

- Solve problems involving right-angled triangles geometrically, using the Pythagorean relationship and proportionality.
- Hypothesize and investigate the relationship between the areas of similar figures drawn on the sides of a right-angled triangle.

Materials

- BLM 10.4.1, 10.4.2

Assessment Opportunities

Minds On…

Whole Class → Discussion

Students share the results of their Home Activity. Volunteers record their area measurements and sketches on the board. Discuss why their investigation did not show a Pythagorean relationship. Why do squares and semi-circles work? Stress similar shapes.

N-agon areas.gsp
(See Day 3.)

Action!

Pairs → Investigation

Pairs use a GSP®4 sketch to investigate the hypothesis: Areas of similar figures drawn on the sides of a right-angled triangle show a Pythagorean relationship.

They record observations and patterns, and explain their reasoning.

Reasoning & Proving/Exploration/Checklist: Observe as students' investigate and look for opportunities to probe for generalization of the relationship.

Consolidate Debrief

Small Group → Reflection

Identify which type of polygon can be used on the sides of a right-angled triangle to create the Pythagorean relationship. Guide students to discover that only similar polygons fulfill the relationship.

Students include one of the GSP®4 sketches they investigated, along with a general statement about the Pythagorean relationship and similar polygons.

Create a class Frayer Model on the Pythagorean relationship. Post titles in four different locations of the room: Definition, Facts/Characteristics, Examples, Non-examples. Working in small groups, students respond at each station, phrasing or rephrasing and adding to the previous group's work. Assemble a large poster to display as a Frayer Model (BLM 10.4.2).

Exploration

Home Activity or Further Classroom Consolidation

Complete worksheet 10.4.1.

10.4.1: Test a Triangle – Is It a Right-Angled Triangle? (Teacher)

Test each of the following triangles and determine if the triangle is a right-angled triangle:

Side 1	Side 2	Longest side	Areas	Is this a right-angled triangle?	How do you know?
3	4	5			
4	6	7			
5	12	13			
8	15	17			
7	10	13			
8	12	15			
9	40	41			

10.4.1: Test a Triangle – Is It a Right-Angled Triangle? (continued)

Side 1	Side 2	Longest side	Areas		Is this a right-angled triangle?	How do you know?
3	4	5	3^2+4^2 $=9+16$ $=25$	5^2 $=25$	**yes**	$3^2+4^2=5^2$
4	6	7	4^2+6^2 $=16+36$ $=52$	7^2 $=49$	**no**	$4^2+6^2\neq 7^2$
5	12	13	5^2+12^2 $=25+144$ $=169$	13^2 $=289$	**yes**	$5^2+12^2=13^2$
8	15	17	8^2+15^2 $=64+225$ $=289$	17^2 $=169$	**yes**	$8^2+15^2=17^2$
7	10	13	7^2+10^2 $=49+100$ $=149$	13^2 $=169$	**no**	$7^2+10^2\neq 13^2$
8	12	15	8^2+12^2 $=64+144$ $=208$	15^2 $=225$	**no**	$8^2+12^2\neq 15^2$
9	40	41	9^2+40^2 $=81+1600$ $=1681$	41^2 $=1681$	**yes**	$9^2+40^2=41^2$