

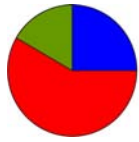
Lesson Outline

BIG PICTURE

Students will:

- develop teamwork skills through cooperative learning;
- take risks when carrying out an investigation and demonstrate perseverance;
- apply a variety of problem-solving strategies;
- apply a number of estimation strategies during problem solving;
- justify their solutions and choice of strategies;
- make connections between prior and new knowledge to draw conclusions;
- represent their thinking in a variety of ways, reflect on their learning, and communicate effectively.

Day	Lesson Title	Math Learning Goals	Expectations
1	Encouraging Others	<ul style="list-style-type: none"> • Collect and organize primary categorical data and represent as a bar graph. • Identify strategies that could be used with problems requiring estimation. 	8m70, 8m71 CGE 5a
2	Solving a Fermi Problem	<ul style="list-style-type: none"> • Use cooperative problem solving in a context requiring estimation. • Communicate mathematical thinking. 	8m16, 8m18, 8m26, 8m27 CGE 3c, 4b
3	Taking Turns	<ul style="list-style-type: none"> • Use cooperative problem solving in a context requiring estimation. • Communicate mathematical thinking. 	8m16, 8m18, 8m27 CGE 5b, 3c
4	Active Listening and Summarizing	<ul style="list-style-type: none"> • Develop problem-solving strategies through practice and comparison. • Communicate mathematical thinking. 	8m16, 8m18, 8m24 CGE 2a
5	Including All Participants	<ul style="list-style-type: none"> • Use cooperative problem solving in a context requiring estimation. • Communicate mathematical thinking. • Develop methods for effective recording of mathematics learning. 	8m16, 8m18, 8m27, 8m68, 8m73 CGE 2c, 3c
6	Analysing Math Records	<ul style="list-style-type: none"> • Examine math records and critique for improvement. • Create a graphic organizer for problem solving. 	8m18, 8m73 CGE 3b, 5b, 5e



Math Learning Goals

- Collect and organize primary categorical data and represent as a bar graph.
- Identify strategies that could be used with problems requiring estimation.

Materials

- BLM 1.1

Assessment Opportunities

Minds On ...

Whole Group → Brainstorm

Brainstorm why it is important to encourage others. Create an anchor chart using the criteria: What does encouragement look like? What does it sound like?

Display the anchor chart posters and discuss how the posters are a good reference. Point out that by encouraging others, they will be more willing to try new things and contribute to group activities.

“Learning is socially constructed; we seldom learn isolated from others.”
- Bennett and Rolheiser

LMS Library, My Professional Practice – Anchor Charts

Action!

Pairs → Gathering Data

Students refer to the anchor charts during data collection using questions like those on BLM 1.1.1 to practise the social skills they discussed.

Learning Skill/Observation/Mental Note: Circulate, observing social skills, and listening to students.

Whole Class → Data Management

Select groups for sharing based on a particularly effective statement expressed by students, clarification of the cooperative learning strategy, an interesting result on BLM 1.1, etc.

Representatives of groups share their results or report on their process for gathering data. Discuss ways to organize the data.

Select a question from BLM 1.1 and tally the responses, using four or five categories for classifying the information. Model how to create a bar graph using the data.

On BLM 1.1, include other questions appropriate for the students in your class.

Consolidate Debrief

Whole Class → Sharing

Students share strategies for problem solving in preparation for Home Activity.

Individual → Reflection

Students answer the questions in their math journal to assess their growth in social skills:

- The social skill focus of this activity was _____.
- Something I said or did to demonstrate the social skill was _____.
- I helped the group work in a positive way by _____.
- An area I should work on is _____.
- Three things I learned from data collected:

Home Activity or Further Classroom Consolidation

Write in your journal about two problems that you solved today outside math class. Describe your thinking and the strategies you used to solve the problem.

Social Skill Practice Reflection

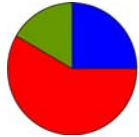
1.1: What Do We Have in Common?

Name:

Date:

Fill in the following table with your partner.

	Name:	Name:
What do you enjoy doing in your spare time?		
What school subjects do you enjoy?		
What sports do you like to watch or participate in?		
What are some activities you participate in during the summer?		



Math Learning Goals

- Use cooperative problem solving in a context requiring estimation.
- Communicate mathematical thinking.

Materials

- birdseed
- construction paper
- scales
- containers
- spoons
- graph paper

Assessment Opportunities

Minds On...

Whole Class → Introduce the Problem

Students share their problems and solutions from the Home Activity. Describe who Enrico Fermi was and some of the famous problems for which he estimated the solutions.

Share an anecdote or a story to set the context, e.g., *Counting On Frank* by Rod Clement, ISBN: 0836803582.

Introduce the problem: How many seeds are in this large bag of birdseed?

During a brainstorm, students consider volume, mass, and counting as ways to determine a response.

A Fermi problem requires that students ask many more questions.

In groups, students receive one cup of birdseed to work with, placed on an 11 × 17 piece of construction paper and brainstorm some questions:

- How many seeds in one cup of birdseed?
- What does one cup of birdseed weigh?
- What is the volume of one cup of birdseed?

Action!

Think/Pair/Share → Problem Solving

Anonymously students write down their individual guesses about how many seeds are in the bag on a small piece of paper that is collected before the activity begins. Record all the responses and calculate the mean.

Pairs brainstorm ideas and strategies to solve the problem and share results with another pair. The group decides how to determine the estimate. Groups record on chart paper all questions they answer in their process, including any assumptions they make. Their solution will be a path of estimates.

Circulate to probe and prompt as the groups work on the problem.

Learning Skills (Working in Groups, Initiative)/Observation: Look for evidence of positive group interaction, sharing of strategies, and involvement in the problem.

Consolidate Debrief

Whole Class → Reflecting

Post the chart paper solutions. Groups share their estimates and explain their thinking, including their assumptions. Highlight the multiple ways that students estimated their path to the answer. Discuss the variety and validity of estimations. Show the class' individual guesses recorded during **Minds On...** and discuss the mean of these guesses and the difference between guessing and estimating.

Home Activity or Further Classroom Consolidation

Pose a Fermi problem in your journal and write the solution, showing the pathway of questions and estimates, and include any assumptions made.

Impact Math – Number Sense (p. 18)
LMS Library, My Professional Practice: Fermi Problems

Focus on process: Students ask themselves questions about what they need to know and what information is missing. They must construct a path of estimates that leads to the knowledge they wish to acquire.

Encourage the groups to be creative and use any tools that are available.

Share examples:

- How high would a stack of one billion loonies reach?
- How many flat toothpicks would fit on the surface of your desk?

*Application
Differentiated
Exploration*



Math Learning Goals

- Use cooperative problem solving in a context requiring estimation.
- Communicate mathematical thinking.

Materials

- BLM 3.1, 3.2
- phone books
- calculators
- chart paper and markers

Assessment Opportunities

Minds On...

Whole Class → Sharing

The class numbers off “one, two, one, two” to form two groups for an Inside Outside Circle. Students take turns sharing the Fermi problems they developed for the Home Activity.

Students in the outside circle shift and repeat the process several more times.

Review and discuss any elements of Fermi problems that may have been misunderstood.

Curriculum Expectations/Observations/Anecdotal: As students share their Fermi problems provide immediate feedback.

Whole Class → Brainstorm

Create an anchor chart for Taking Turns.

Groups of 4 → Placemat

To formalize the solution to the birdseed problem, students write a list of questions that must be asked and answered in order to make an estimate for the birdseed problem (BLM 3.1). The placemat group shares individual responses and agrees upon a response to be shared with the whole class.

Whole Class → Discussion

One person from each group shares one question from their list. Record the questions as they are provided. Students order the questions following a discussion (BLM 3.2).

Inside Outside Circle is a kinaesthetic activity that involves all the students and helps develop a positive classroom climate.



Circulate to observe students' group interactions reinforcing that they must take turns during both **Minds On...** and **Action!**

Action!

Groups of 4 → Problem Solving

Pose a Fermi problem: How many names are in the local phone book?

Recorder 1 records all the questions that need to be asked and answered to estimate the solution and completes a ranking ladder (BLM 3.2).

Two students perform the calculations on a calculator at each step and check each other's work.

Recorder 2 records the path steps and the solution on chart paper.

Students take turns explaining the solution to each other in their own words.

Consolidate Debrief

Whole Class → Discussion

Groups present their solutions using their chart paper record. Encourage the class to ask the groups questions about their method. Highlight the variety of questions and reasoning used.

Summarize strategies used to solve Fermi problems.

Share some of the positive actions observed as students were taking turns during this lesson. Add them to the anchor chart.

Home Activity or Further Classroom Consolidation

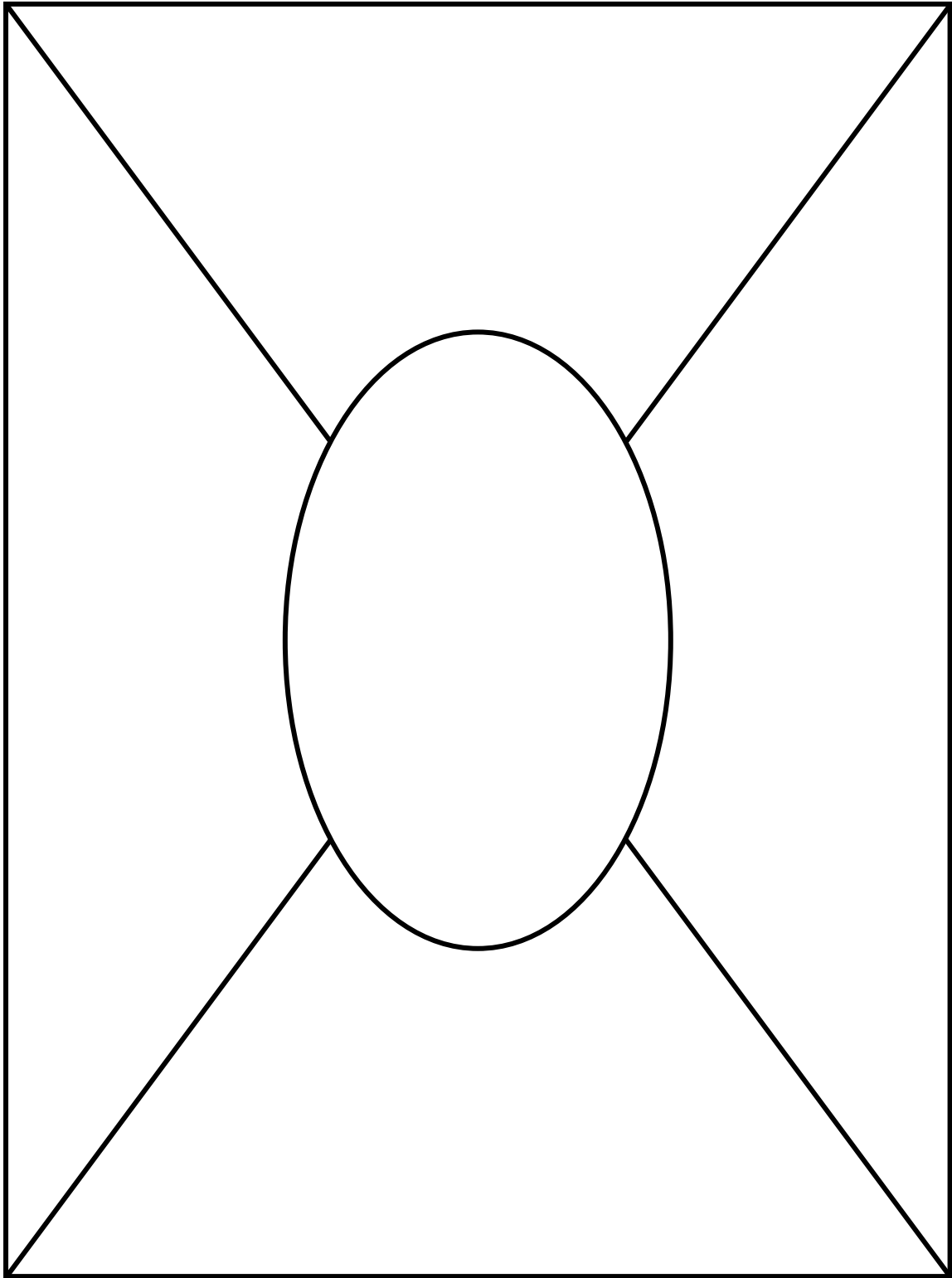
Reflection

Identify a situation where estimation is needed and describe a strategy in your journal that could be used to establish a reasonably accurate estimate.

3.1: Placemat

Names:

Date:



3.2: Ranking Ladder

Name:

Date:

Use the ranking ladder to organize the questions you used to arrive at an accurate estimation. List the first question you would ask yourself to solve the problem at the top of the ladder. List the last question at the bottom of the ladder, and use the middle rungs to put the other questions in order.

The diagram is a ranking ladder, which is a trapezoidal shape formed by two parallel slanted lines on the left and right sides, connected by five horizontal rungs. The rungs are evenly spaced and extend from the left slanted line to the right slanted line, creating five distinct rectangular sections for writing questions.



Math Learning Goals

- Develop problem-solving strategies through practice and comparison.
- Communicate mathematical thinking.

Materials

- BLM 3.2
- community maps
- chart paper

Assessment Opportunities

Minds On...

Whole Class → Brainstorm

Create an anchor chart for active listening and summarizing. Define paraphrasing and brainstorm how this technique applies to active listening and summarizing.

The interview role-play process provides an opportunity to apply the skills of active listening and paraphrasing.

Action!

Groups of 3 → Problem Solving/Role Play

Pose a Fermi problem: How many times does the wheel of your bicycle turn on a trip from the school to a local destination, e.g., arena, movie theatre, community centre?

Students, in the role of an interviewer, pose questions that they would ask the persons attempting to solve this problem. Sample questions might include:

- What questions do you need answered to estimate the solution?
- What strategies will you use?
- What information do you need to know?

In their group, students decide on a role: one interviewer and two interviewees. They reflect for a few minutes and then conduct the interview to collectively determine an appropriate strategy to solve the problem. Paraphrase and summarize the interview.

Groups share summaries of their interview role play with the class.

The groups complete the solution of the problem, recording on chart paper each question asked, the calculations, and the pathway to the final estimate. They include all assumptions made and the ranking ladder of questions.

Do not solve the problem at this time.

Consolidate Debrief

Whole Class → Reflection

Students share their estimates using the chart paper solutions and reflect on the problem-solving steps and strategies they used to solve Fermi problems. Demonstrate that there are different methods to solve the same problem. Use a think-aloud strategy to model and paraphrase solutions.

Curriculum Expectations/Communication/Observation: Observe both the written and oral presentations to provide immediate feedback. Highlight instances of active listening, good summaries, and clear paraphrasing.

Debrief the problem-solving strategies used for the last four days, referring to the poster Problem-Solving Strategies. Show that logical reasoning was used to develop and order questions for the ranking ladder.



LMS Library, My Professional Practice

Home Activity or Further Classroom Consolidation

Reflection

Create a summary in your journal of the steps and strategies you used to solve problems, providing examples that you have already worked on.



Math Learning Goals

- Use cooperative problem solving in a context requiring estimation.
- Communicate mathematical thinking.
- Develop methods for effective recording of mathematics learning.

Materials

- BLM 5.1
- chart paper

Assessment Opportunities

Minds On...

Whole Class → Brainstorm

Share an anchor chart for including all participants.

Learning Skill/Self-Assessment/Anecdotal: In their journal students describe the social skills focused on in the first four lessons and assess their own development and use of the social skills.

Groups of 4 → Graffiti

Locate the graffiti charts on the walls around the room: with chart questions:

- Why write and record in mathematics?
- What should written explanations include?
- Besides words, what other ways could you use to show your thinking?
- What organizational formats help you to understand someone else’s thinking?

Groups rotate through the stations to write their ideas. They read what previous groups have written, then add new ideas to the chart.

Use a think-aloud strategy to debrief collective knowledge and important ideas. Summarize findings and post for reference.



LMS Library, My Professional Practice – Posters:

- Teaching Through Problem Solving
- Representations Make Our Thinking Visible
- Understand the Problem
- Problem-Solving Strategies

During the think-aloud, refer to writing for different audiences.

Ask students to pay attention to the learning skills that they have worked on during this unit.

Action!

Groups of 4 → Problem Solving

Pose a Fermi problem: How many hours do students in Grade 8 in Ontario talk on the telephone in one year? (BLM 5.1)

Once they complete their solutions, groups discuss and create a record of their problem solving on chart paper. Refer to the graffiti summary for criteria.

Learning Skills/Observation/Checklist: Observe the problem-solving process and select groups to share their recordings and strategies.



It may be helpful for students to conduct a class survey to get started.

Consolidate Debrief

Whole Class → Reflection

Selected groups display their recordings and explain their problem-solving strategies. Encourage the class to ask group members to clarify by asking questions and requesting that they paraphrase some aspects of their presentation.

Discuss the use of estimation for solving Fermi problems. Students reflect on the improvement of their estimation skills since the birdseed problem.

Home Activity or Further Classroom Consolidation

Describe today’s Fermi problem in your journal and how your group solved it for a student who has been absent during today’s lesson.

Remind students that good records include some of: words, diagrams, numbers, symbols, tables, representations of thinking, etc.

*Application
Concept Practice*

5.1: Thinking to Solve Problems

Name:

Date:



What do you predict? Why?

What question will you use to begin estimating?

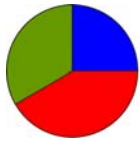
How will you decide how many students there are in Grades 7 and 8?

What surprises you? Why?

What do you find interesting? Explain.

Describe any trends you see in the data.

Why do you think these trends are happening?



Math Learning Goals

- Examine math records and critique for improvement.
- Create a graphic organizer for problem solving.

Materials

- BLM 6.1

Assessment Opportunities

Minds On...

Whole Class → Exploration

Students work in pairs to discover the social skill represented (BLM 6.1). Brainstorm to create an anchor chart for disagreeing in an agreeable way. Using a combination of several examples of the previous day’s records on problem-solving strategies, help students look for evidence of good mathematics communication. Students look first for the strengths shown in the examples, e.g., clear, concise statements; helpful graphic organizers, and then focus on some positive suggestions for improvement. Review all the social skills worked on in this unit, referring to the anchor charts and stressing that all students are a vital part of the classroom.

Be sensitive to the fact that in some cultures it is considered disrespectful to maintain direct eye contact with another person.

Action!

Small Groups → Discussion/Sharing

Individually, students examine the examples and write down at least three instances of strengths shown and at least two positive ways that the examples can be improved. Pairs compare their examples and agree or disagree with each other’s suggestions in a positive way. In groups of four, they record the examples where the members of the group agree with each other and the instances where they disagree with each other. Ask each group to share one example of good communication that the group agreed on and one example of communication that the group did not agree on, explaining the rationale behind the disagreement. Repeat the process so that each group shares an example of a positive suggestion that they agreed on and one that they did not agree on. Groups share an example that started as a disagreement but once the group talked about it, it ended with an agreement. Point out any evidence of good communication or positive suggestions for improvement that students may have missed. Create a concept map to review problem solving generally, including solving Fermi problems. Students should agree on the items in the concept map.

Discussion items could be recorded on chart paper.

Learning Skills/Observation/Checklist: Circulate to observe and assess team member interactions and contributions to the task.

Consolidate Debrief

Whole Class → Sharing

Groups post and share concept maps. Look for similarities, differences, and creative techniques. Review lessons learned by solving Fermi problems. Students share ways that they learn mathematics best. Invite students to share some past successes.

Differentiated Reflection

Home Activity or Further Classroom Consolidation

In your journal, write a letter to the teacher explaining what you have learned during this unit. Describe your goals for mathematics this year. Also describe ways in which you learn mathematics best and examples where learning mathematics may have been challenging.

6.1: What Is the Social Skill?

Name:

Date:

Decide which social skill is being demonstrated by the positive examples:

Positive Examples	Negative Examples
<p><i>Looks like ...</i></p> <ul style="list-style-type: none">• Eye contact with a slight shake of the head• Listening to someone's entire idea before speaking• Smiling at the speaker• Puzzled or questioning look	<p><i>Looks like ...</i></p> <ul style="list-style-type: none">• Listener interrupts the speaker• Shaking the head rapidly back and forth• Impatiently challenging the speaker• Rapidly tapping the fingers• Angry challenging look
<p><i>Sounds like ...</i></p> <ul style="list-style-type: none">• I understand what you are thinking but have you considered ...?• Your idea is important but have you thought about ...?• I think I understand what you are saying but have you thought about ...?• Calm, quiet, controlled voices	<p><i>Sounds like ...</i></p> <ul style="list-style-type: none">• No way! I disagree, my idea is much better than that.• So what. Who cares? I have a different idea.• I totally disagree with everything you just said.• Loud, angry, or aggressive voices

Look at the examples below and decide whether each is a positive or negative example for the social skill. Discuss why this skill is important for successful learning and for getting along in your teams:

Examples

1. That's what you think...my idea is much better!
2. Something else to consider is _____, which is a little different than your idea.
3. My idea is fine. I'm not changing anything.
4. Is there anything we can add to _____'s idea?
5. You think only your ideas are important. What about mine?