

# Unit 8 Similarity, Congruency, and Transformations

Grade 7

## Lesson Outline

### BIG PICTURE

Students will:

- understand location using four quadrants of the coordinate axis;
- investigate and apply transformations and tessellations;
- investigate dilations and their relationship to the characteristics of similar figures;
- investigate and compare congruent triangles and similar triangles;
- investigate, pose, and solve problems with congruent shapes.

Day	Lesson Title	Math Learning Goals	Expectations
1	From One Point to Another  <i>Presentation file:</i> <b>Points and Their Coordinates</b>	<ul style="list-style-type: none"> <li>• Plot points on the Cartesian coordinate axis.</li> </ul>	7m54  CGE 5a
2, 3	4-Quadrant Game  <i>(lesson not included)</i>	<ul style="list-style-type: none"> <li>• Plot points on the Cartesian coordinate axis.</li> <li>• Students make a game, such as Treasure Hunt or Find My Location, that requires finding points on the 4-quadrant grid.</li> </ul>	7m54  CGE 5a
4	Investigating Transformations Using Grid Paper	<ul style="list-style-type: none"> <li>• Use various transformations to “move” a shape from one position and orientation to another on grid paper.</li> </ul>	7m56  CGE 3c
5	Investigating Transformations Using The Geometer’s Sketchpad®4	<ul style="list-style-type: none"> <li>• Explore reflections, rotations, and translations using The Geometer’s Sketchpad®4.</li> </ul>	7m56  CGE 4a, 4f
6	Pentominoes Puzzle	<ul style="list-style-type: none"> <li>• Analyse designs, using transformations.</li> </ul>	7m56  CGE 3c
7	Investigating Dilations Using a Variety of Tools	<ul style="list-style-type: none"> <li>• Investigate dilations, using pattern blocks and computer websites.</li> </ul>	7m52, 7m55  CGE 2c, 3a
8	Applying Transformations: Tessellations	<ul style="list-style-type: none"> <li>• Tessellate the plane, using transformations and a variety of tools.</li> </ul>	7m56, 7m57  CGE 5e, 5f
9	Will It Tessellate?  <i>GSP®4 file:</i> <b>Tiling</b>	<ul style="list-style-type: none"> <li>• Form and test a conjecture as to whether or not all triangles will tessellate.</li> <li>• Apply knowledge of transformations to discover whether all types of triangles will tessellate.</li> </ul>	7m56, 7m57  CGE 3c
10	Which Polygons Tessellate?	<ul style="list-style-type: none"> <li>• Form and test a conjecture as to whether or not all polygons will tessellate.</li> <li>• Identify polygons that will/will not tessellate.</li> </ul>	7m56, 7m57  CGE 3a, 3c

Day	Lesson Title	Math Learning Goals	Expectations
11	Creating Similar Figures Through Dilatations  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Plot similar triangles on the Cartesian plane.</li> <li>Investigate similar triangles by comparing longest sides of each triangle; shortest sides; remaining sides; and corresponding angles of the triangles.</li> <li>Make conclusions that dilatations create similar triangles.</li> <li>Explore dilatated figures to determine that enlargements and reductions always create similar figures.</li> </ul>	7m52, 7m54, 7m55  CGE 3c, 3e
12	Investigating Similar Figures Using Geometers Sketchpad <sup>®</sup> 4  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Draw a triangle and create several similar triangles by enlarging and reducing it using GSP<sup>®</sup>4.</li> <li>Understand relationships between the sides of similar triangles (e.g., If one side is doubled in length after enlargement, then all three sides are doubled in length. All angles remained the same measures after enlargement.).</li> </ul>	7m52, 7m54, 7m55  CGE 3c
13	Congruent and Similar Shapes in Transformations  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Investigate, using grid paper or GSP<sup>®</sup>4 which transformations (translation, reflection, rotation, dilatation) create congruent or similar shapes.</li> <li>Determine the relationship between congruent shapes and similar shapes.</li> </ul>	7m52, 7m53  CGE 3c, 4c
14	Investigating Congruency  <i>(no lessons included)</i>	<ul style="list-style-type: none"> <li>Construct congruent shapes.</li> <li>Measure and compare lengths, angles, perimeter, and area pairs of congruent shapes, and draw conclusions about congruent shapes.</li> </ul>	7m5, 7m50  CGE 3c, 4c
15	Exploring Congruency  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Pose and solve congruency problems (e.g., Are two triangles with the same areas congruent? Are triangles with equal bases and heights congruent?)</li> </ul>	7m51  CGE 3c
16	Investigating the Conditions That Make a Triangle Unique  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Determine the conditions that will make a unique triangle, e.g., Students conduct a number of investigations, using concrete materials such as: <ul style="list-style-type: none"> <li>using two different lengths</li> <li>using two lengths and one angle</li> <li>using triangles given 3 side lengths, 3 angles, two angles, and one side, etc.</li> <li>determining when 3 lengths will and will not form a triangle, e.g., straws of lengths of 4 cm, 5 cm, and 10 cm, will not form a triangle because <math>4 + 5 &lt; 10</math>, thus the sides are too short to make a closed figure.</li> </ul> </li> </ul>	7m50, 7m51  CGE 2c, 4e
17	Investigating When Triangles Will Be Congruent  <i>(lessons not included)</i>	<ul style="list-style-type: none"> <li>Determine if triangles are congruent given certain conditions, e.g., Two triangles have side lengths 3 cm and 5 cm. The triangles each have a <math>60^\circ</math> angle, but not in the same location. Are the triangles congruent?</li> </ul>	7m50, 7m51  CGE 2c, 4e
18	Summative Performance Task  <i>(lessons not included)</i>		



**Math Learning Goals**

- Plot points on the Cartesian coordinate axis.

**Materials**

- BLM 8.1.1
- graph paper
- data projector

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Demonstration**

Sketch a thermometer (vertically) on the board. Ask students where to place zero on the thermometer so there is room for positive and negative temperatures to be displayed.

A student marks zero on the thermometer; another student adds the temperatures 1, 2, 3...; and a third student labels negative temperatures -1, -2, -3...

Draw a horizontal number line through zero.

Students place the positive and negative integers on this line. The intersection of these two integer lines creates 4 quadrants and is called the Cartesian coordinate system.

Show the electronic presentation: Points and Their Coordinates to illustrate plotting points in the *xy*-plane.

**Action!**

**Whole Class → Guided Instruction**

A student describes, in words, the path taken to a point, starting at the origin, e.g., move two units to the right, then four units down. A second student plots the appropriate point on a grid. A third student writes the ordered pair. Repeat, but in a different order each time, e.g., plot the point, show the path, state the path in words.

**Pairs/Individual → Practice**

Students practise ordering pairs with positive, negative, and zero coordinates. Students complete question 1 of BLM 8.1.1 with a partner and then individually complete the remaining questions. Circulate and provide help, as needed.

**Curriculum Expectations/Observation/Mental Note:** Assess students' understanding of plotting points and movement in the *xy*-plane, using up/down; right/left to indicate the direction.

**Consolidate Debrief**

**Pairs → Sharing**

Students exchange their code from question 4 and construct each other's picture. Students assist each other, correcting any errors.

**Home Activity or Further Classroom Consolidation**

On a piece of graph paper, draw lines to make your initials.

Label the coordinates of several points.

On a separate piece of paper, list the coordinates in the order that will spell out your initials.

Do not include your name on the set of ordered pairs, as one of your classmates will solve this puzzle.

**Points and Their Coordinates.ppt**

Students graphed ordered pairs in the first quadrant only in Grade 6.

Relate the story of René Descartes and why the Cartesian plane is named after him.

**Word Wall**

- x-axis
- y-axis
- origin
- horizontal axis
- vertical axis
- ordered pair
- x-coordinate
- y-coordinate
- quadrant
- Cartesian coordinate system

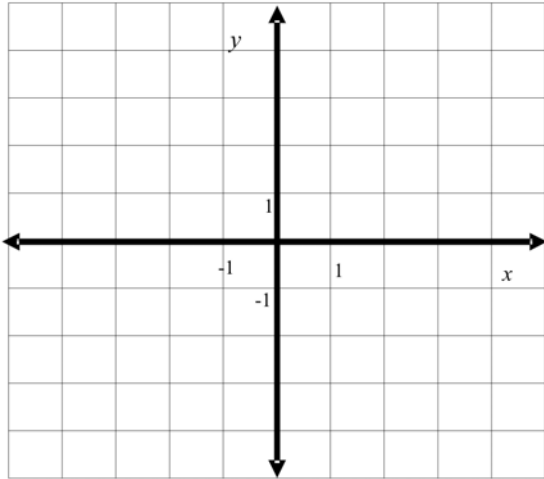
Students should consult with their partner to solve problems before they ask for assistance.

*Application Skill Drill*

## 8.1.1: Plotting Points

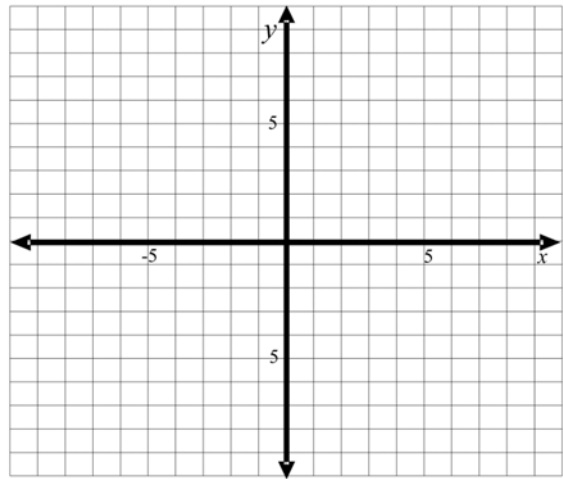
1. Plot these points.  
Connect the points in order.  
Name the polygon.

$(1, -1)$ ,  $(2, 1)$ ,  $(1, 3)$ ,  $(-1, 4)$ ,  $(-3, 3)$ ,  
 $(-4, 1)$ ,  $(-3, -1)$ ,  $(-1, -2)$ ,  $(1, -1)$



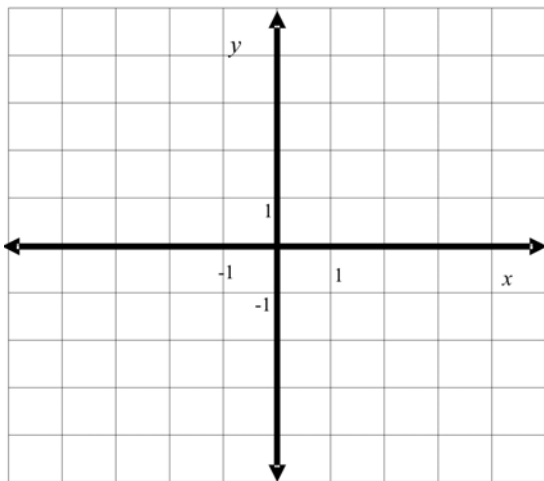
2. Plot each set of points on the grid below.  
Join the points to form a quadrilateral.  
Identify the quadrilateral.

Set 1:  $A(1, 1)$ ,  $B(1, 5)$ ,  $C(-3, 5)$ ,  $D(-3, 1)$   
Set 2:  $J(1, -3)$ ,  $K(5, 1)$ ,  $L(8, 1)$ ,  $M(4, -3)$   
Set 3:  $P(-3, 0)$ ,  $Q(-6, -2)$ ,  $R(4, -4)$ ,  $S(10, 0)$

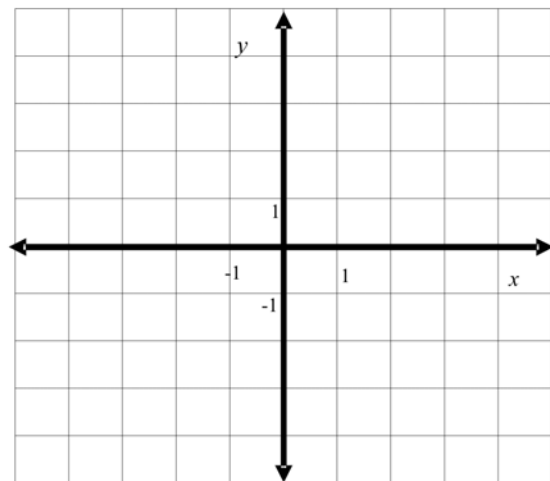


3. Plot these points.  
Connect the points in order.  
What picture do you see?

$(2, 1)$ ,  $(5, 5)$ ,  $(1, 2)$ ,  $(0, 5)$ ,  $(-1, 2)$ ,  $(-5, 5)$ ,  
 $(-2, 1)$ ,  $(-5, 0)$ ,  $(-2, -1)$ ,  $(-5, -5)$ ,  
 $(-1, -2)$ ,  $(0, -5)$ ,  $(1, -2)$ ,  $(5, -5)$ ,  
 $(2, -1)$ ,  $(5, 0)$ ,  $(2, 1)$

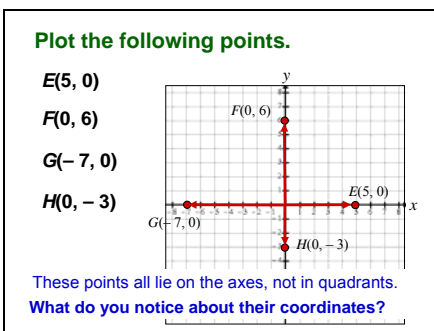
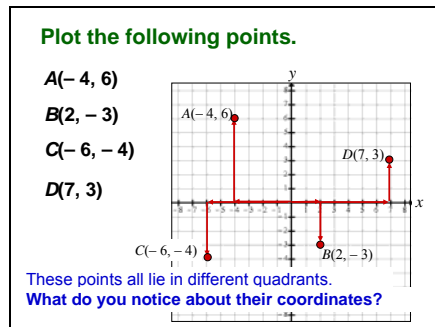
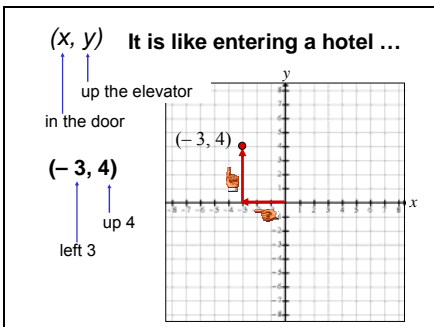
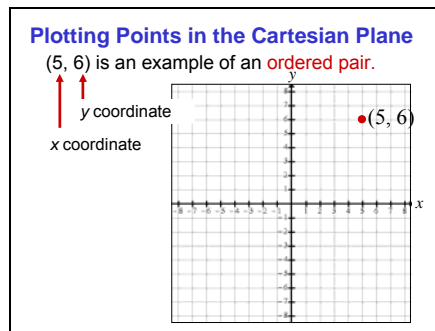
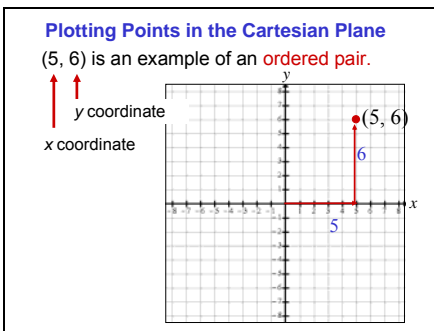
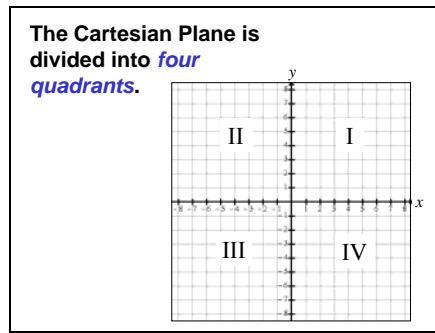
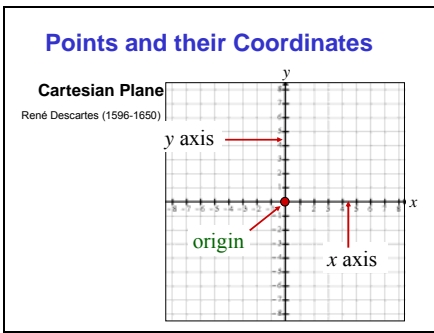


4. Make your own picture. Record the points in order. Exchange your picture code with a classmate and construct each other's picture.



# Points and Their Coordinates (Presentation software file)

## Points and Their Coordinates.ppt





**Math Learning Goals**

- Use various transformations to “move” a shape from one position and orientation to another on grid paper.

**Materials**

- BLM 8.4.1

**Assessment Opportunities**

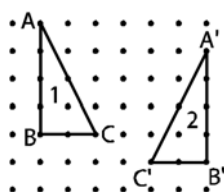
**Minds On ... Whole Class → Activating Prior Knowledge**

Place two congruent triangles ( $\Delta 1$  and  $\Delta 2$ ) on overhead dot paper. Ask: What different types of transformations could be used to move  $\Delta 1$  to the orientation shown by  $\Delta 2$ ? List transformation names on the board.

Students move  $\Delta 1$  onto  $\Delta 2$  several times using different combinations and/or sequences of transformations each time.

Review the precision needed for describing transformations. For example, Reflect  $\Delta 1$  in the vertical line going through C, then translate it 2 units right and 1 unit down.

OR



Translate  $\Delta 1$  down 1 unit and right 6 units, then reflect it in side AB.

Demonstrate that different types of transformations can result in the same image.

**Action!**

**Pairs → Exploration**

Students transform  $\Delta 1$  onto  $\Delta 2$  using translations, reflections, and rotations and record all transformations (BLM 8.4.1).

Prompt students’ thinking:

- What different types of transformations are there?
- Which combinations have you tried?
- How can you perform the transformation in [*specific number*] moves?

**Curriculum Expectations/Oral Questioning/Mental Note:** Assess students’ understanding of transformations.

**Consolidate Debrief**

**Pairs → Making Connections and Summarizing**

Student pairs check another pair’s descriptions by following the description to see whether the intended image results.

Students record more than one way to describe at least two of the examples on BLM 8.4.1.

**Home Activity or Further Classroom Consolidation**

Create your own transformation challenge.

Complete your transformations in several ways.

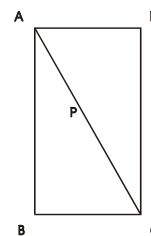
*Application  
Concept Practice*

Shapes other than triangles could be used.

If a shape is rotated, the description should identify the centre of rotation.

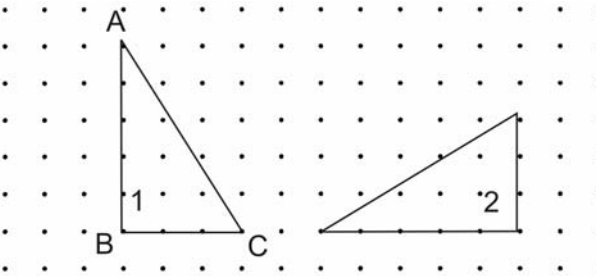
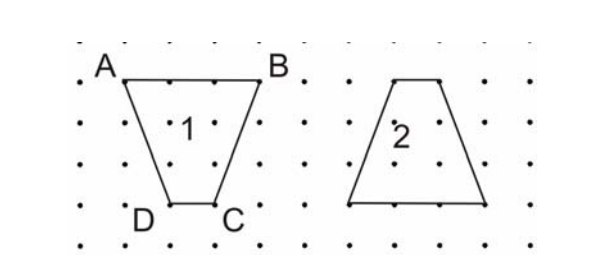
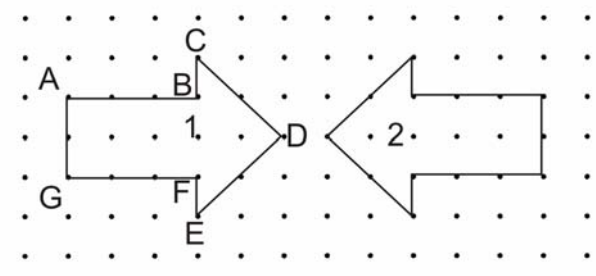
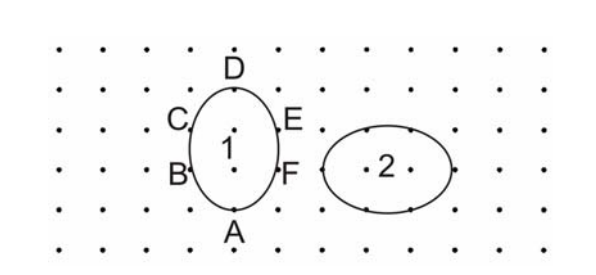
If a shape is reflected, the description should identify the line of reflection.

The diagram below illustrates  $\Delta ABC$  being rotated  $180^\circ$  about point P to  $\Delta CB'A$



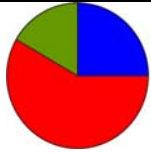
Cut-outs of Shape 1 for each question BLM 8.4.1 may be helpful for students to do this activity.

## 8.4.1: Transformations Recording Chart

<p style="text-align: center;"><u>Transformation Diagram</u></p> <p>Sketch each step in the transformation from figure 1 to figure 2.</p>	<p style="text-align: center;"><u>Description of Transformations</u></p> <p>Describe in words the transformation used for each sketch.</p>
	
	
	
	

# Unit 8: Day 5: Investigating Transformations Using The Geometer's Sketchpad® 4

Grade 7



### Math Learning Goals

- Examine reflections, rotations, and translations using the Geometer's Sketchpad® 4.

### Materials

- The Geometer's Sketchpad® 4
- BLM 8.5.1, 8.5.2

### Assessment Opportunities

#### Minds on ...

#### Pairs → Teacher Guided

Mixed-ability pairs work together at a computer with The Geometer's Sketchpad® 4 (GSP® 4) to construct a regular pentagon using BLM 8.5.1. Pairs share their understanding.

#### Action!

#### Pairs → Guided Exploration

In mixed-ability pairs, students use Geometer's Sketchpad® 4 to work with various types of transformations (BLM 8.5.2, Parts A–D). Students who complete the task can do Part E on BLM 8.5.2.

**Learning Skills (Cooperation)/Observation/Anecdotal Note:** Assess students' cooperation.

#### Consolidate Debrief

#### Whole Class → Demonstrating Understanding

Discuss student responses to the questions.

#### Home Activity or Further Classroom Consolidation

Create and describe several different designs based on transformations of a single shape.

Name your design to suggest the type of transformation(s) used in developing the design, e.g., Tilted Tiles, Spun Petals.

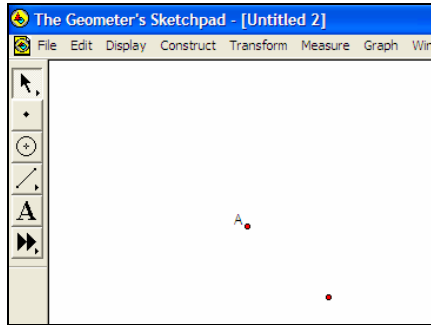
*Application  
Concept Practice*

Refer to *Think Literacy: Cross Curricular Approaches - Mathematics, Grades 7–9*, Following Instructions pp. 70–72.





## 8.5.1: Constructing a Regular Pentagon

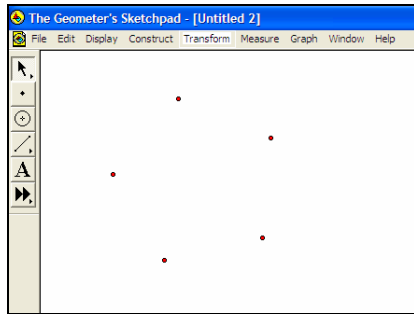


Using the Point Tool, construct 2 points on the screen as shown.

Select the Arrow Tool, click on any white space to deselect everything.

Click on point A; from the Transform Menu, select Mark Center.

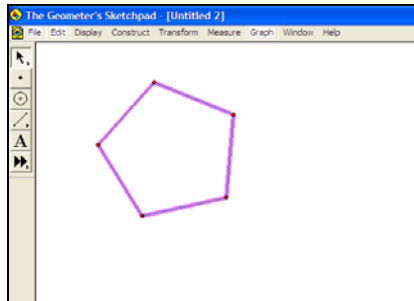
Click on the other point.



In Transform Menu, select Rotate; type 72 into the degree box, click on Rotate.

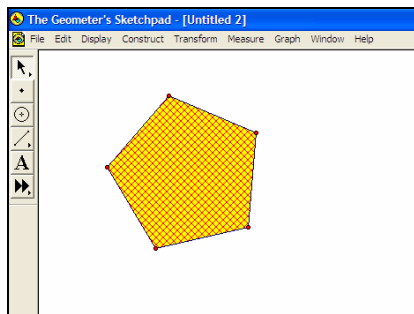
Select Rotate again; in the Rotate Box, click on Rotate, repeat 2 more times.

Click on point A; in the Display Menu, select Hide Point.



Select each of the 5 points in order clockwise.

In the Construct Menu, select Segments.



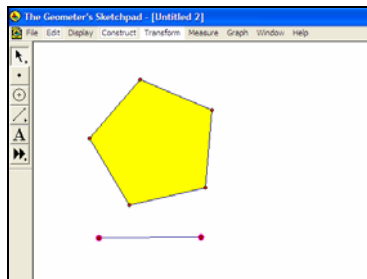
Deselect the segments and select each of the 5 points.

In the Construct Menu, select Interior.

Save your sketch.

## 8.5.2: Transformations in The Geometer's Sketchpad® 4

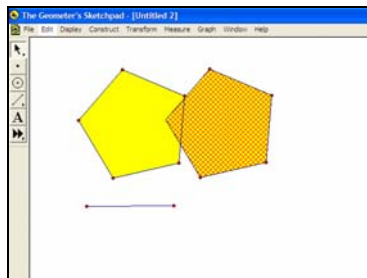
### Part A: Translations



Open the Pentagon Sketch that you saved, select the Segment Tool and draw a segment under your Pentagon. Select the Arrow Tool; deselect the segment, then select each of the two end points. From the Transform Menu, select Mark Vector.

Select the entire Pentagon. From the Transform Menu, select Translate. In the Translate Box click Translate.

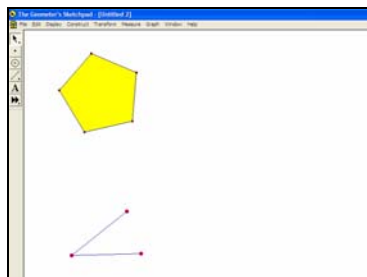
Describe in your notes what happened.



Select the right most point on your line and drag the point.

Compare the Translated image to the original by describing how they are the same and how they are different.

### Part B: Rotations

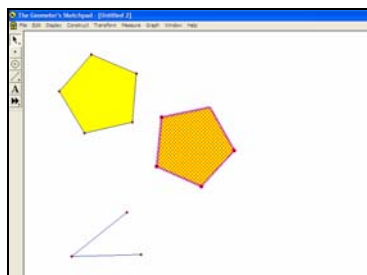


Open the Pentagon Sketch that you saved.

Using the Segment Tool, construct an angle below your Pentagon. Select the Arrow Tool, deselect the segment.

Select the 3 points of the angle in order counter-clockwise. In the Transform Menu, select Mark Angle.

Select the vertex of the angle and in the Transform Menu select Mark Center.



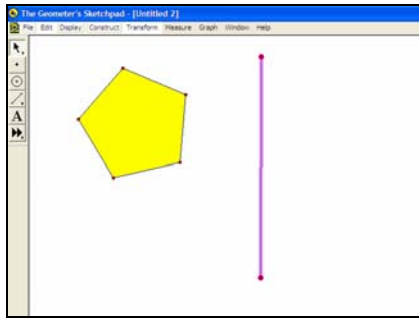
Select the entire Pentagon. In the Transform Menu, select Rotate. In the Rotate box, click Rotate.

Describe in your notes what happened.

Select one of the end points on your angle and drag it.

Compare the translated image to the original by describing how they are the same and how they are different.

## Part C: Reflections

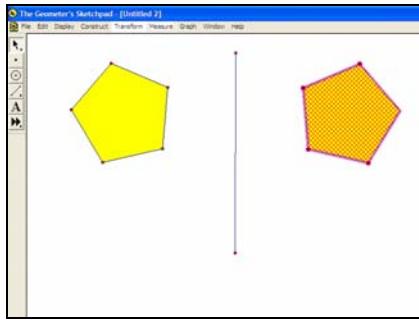


Open the Pentagon Sketch that you saved.

Using the Segment Tool, construct a segment to the right of your Pentagon. In the Transform Menu, select Mark Mirror.

Select the Arrow Tool. Select your entire Pentagon. In the Transform Menu, select Reflect.

Describe in your notes what happened.



Select your mirror line and drag it.

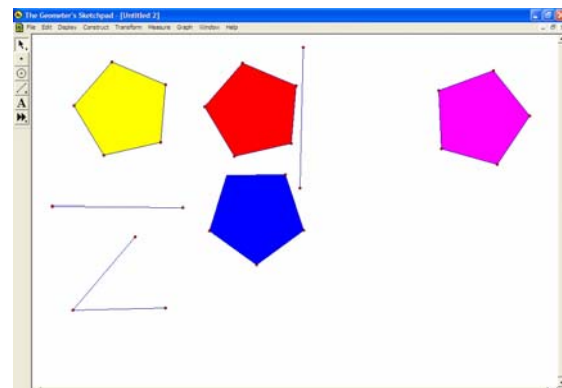
Compare the translated image to the original by describing how they are the same and how they are different.

## Part D: Put It All together

Open the Pentagon Sketch that you saved.

Repeat each of the above transformations, this time not opening a new sketch each time. For each transformation select the image and change it to a different colour.

Your final sketch may have the images overlapping. You may need to drag your mirror line to achieve something similar to the screen shown.



Considering the three images, explain whether it is possible for any of the three images to lie directly on top of one another. Experiment by dragging different parts of your sketch.

## Part E: Explore More

Use various combinations of transformations to create a design.

Reflect an image over a line.

Create a second reflection line parallel to the first line.

Reflect the image over the second line.

Describe a single transformation that alone would have created the second reflected image.

Repeat using two mirror lines that intersect.

Make up your own combination of transformations that could also be created by a single transformation.



**Math Learning Goals**

- Analyse designs using transformations.

**Materials**

- square tiles
- BLM 8.6.1, 8.6.2
- dot paper

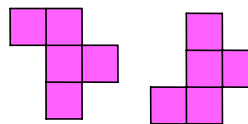
**Assessment Opportunities**

**Minds On ...**

**Whole Class → Demonstration**

Create different shapes using five square tiles or five linking cubes. Demonstrate one shape that can be made using the five squares.

Construct a congruent shape as it would appear under a transformation, explaining that these two shapes are considered to be the same shape.



If square tiles are not available, copy BLM 8.6.1 onto card stock and have students cut out the squares to be used.

**Action!**

**Pairs → Exploration**

Students find different pentomino shapes and draw them on the dot paper.

**Whole Class → Guided Investigation**

Use an enlarged overhead copy of BLM 8.6.2 to guide students in identifying A and 1 as a congruent pair under a rotation of 90° clockwise, followed by a translation.

Students fill in the chart. Students complete the chart in the following sequence:

- identify all of the pairs of congruent pentominoes;
- identify whether a rotation or reflection is needed;
- describe the amount and direction of rotation or the type of reflection;
- mark the centre of rotation or the reflection line, and the direction of translation.

**Pairs → Problem Solving**

Students work on the next 11 pairs individually and in pairs share their results and discuss any differences.

There are 12 different pentomino shapes.

The translation will vary depending on the centre of the rotation.

Using cut-outs of the pentomino shapes may help students during their task.

**Consolidate Debrief**

**Whole Group → Presentation**

Students explain a pair that they found, using an overhead of BLM 8.6.2. One student could write the description while the other student demonstrates the transformation with cut-outs.

For each pair of pentomino shapes, students describe the transformation differently, and explain it.

**Communicating/Presentation/Checklist:** Assess student’s ability to demonstrate, orally and visually, their understanding of transformations.



Encourage students to communicate clearly, using mathematical terminology.

*Concept Practice*

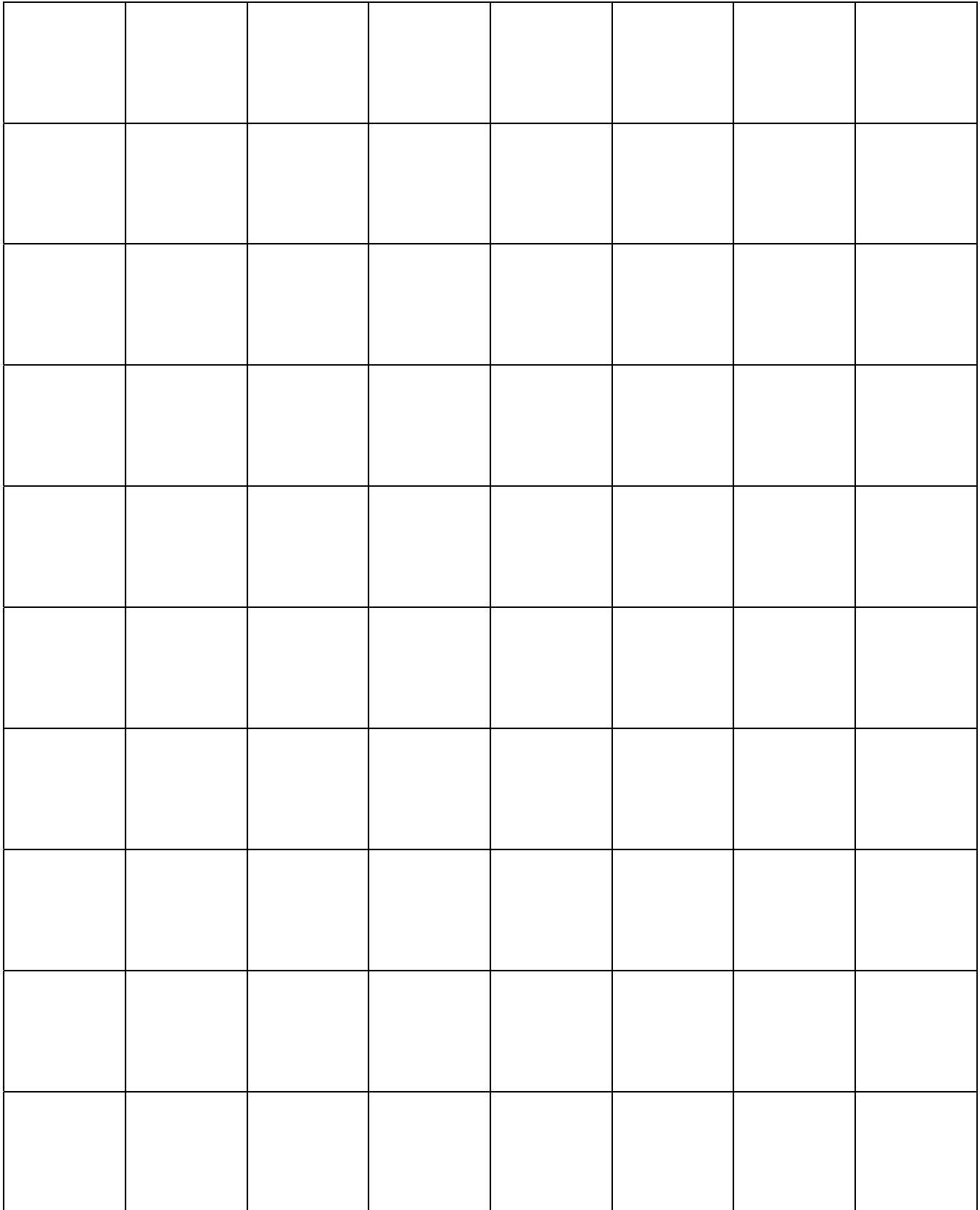
**Home Activity or Further Classroom Consolidation**

Find another arrangement of pentominoes that form a rectangle.

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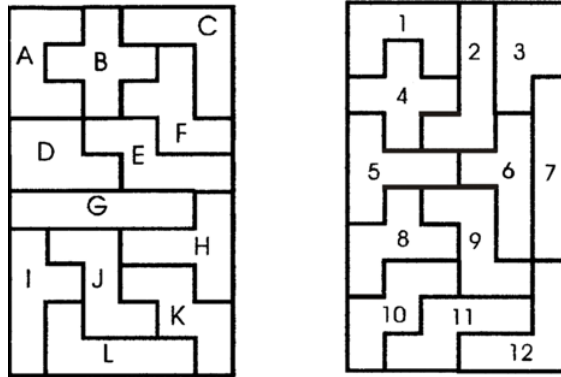
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## 8.6.1: Square Tiles



## 8.6.2: Pentominoes

Two different rectangular arrangements of the 12 pentomino pieces:



Find all pairs of congruent pentomino pieces and describe the transformation that you applied.

Pairs	Description of Transformation
A and ___	
B and ___	
C and ___	
D and ___	
E and ___	
F and ___	
G and ___	
H and ___	
I and ___	
J and ___	
K and ___	
L and ___	



**Description**

- Investigate dilatations using pattern blocks and computer websites.

**Materials**

- pattern blocks
- BLM 8.7.1
- data projection unit

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Presentation**

Draw a triangle or polygon on an overhead acetate. Shine the overhead onto the board and trace the shape with chalk. Students suggest what will happen when the overhead is moved closer/farther from the board, using the vocabulary of dilatations. Move the overhead and trace the new shape.

**Pairs → Discussion**

Students make predictions about the relationships between the original and the image angles and the orientation of the original and image sides. They record their predictions in their journals.

**Whole Class → Guided Exploration**

Project examples of enlargements and reductions, e.g., maps found on the Internet.

Examine parallel streets and right-angled intersections. Point out that the angles keep the same measurements throughout dilatations and that parallel and perpendicular sides remain.

**Action!**

**Pairs → Investigation**

Students investigate the predictions they made about angles and orientation of sides, using pattern blocks (BLM 8.7.1).

They should note that angles remain congruent throughout a dilatation. Some students might notice the relationship between side length and number of pattern blocks used.

**Curriculum Expectations/Observation/Anecdotal Note:** Assess students' ability to identify, analyse, and describe dilatations.

- Word Wall
- enlargement
  - reduction
  - dilatation
  - similar figures
  - congruent angles
  - image

<http://www.mapquest.com>

<http://earth.google.com>



Challenge those students who finish early to create their own dilatations to further verify their findings.

**Consolidate Debrief**

**Whole Class → Discussion**

Students describe their findings and note if their original predictions were accurate.

Discuss where they have seen dilatations used in daily life.

**Home Activity or Further Classroom Consolidation**


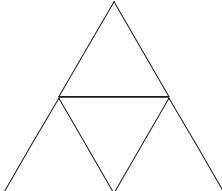
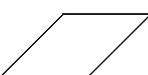
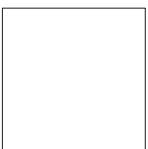
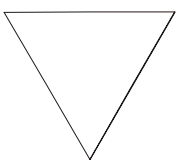
Complete the practice questions.

*Application*

Provide students with appropriate practice questions.

## 8.7.1: Investigating Dilatations

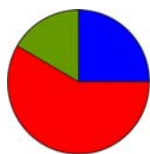
Use pattern blocks to enlarge each of these shapes. Examine and measure the image shape and complete the table.

Shape	Dilatation to be performed	Compare original angles to image angles	Compare original sides to image sides
	Side length enlarged by two times 		
	Enlarge the rhombus' sides by three times		
	Enlarge the square's sides by five times		
Draw the original shape	This shape's sides have been reduced by four times 		

What conclusions can you make about the effect of dilatations on a shape's angles and sides?

What other interesting patterns do you notice?





**Math Learning Goals**

- Tessellate the plane using transformations and a variety of tools.

**Materials**

- The Geometer’s Sketchpad®4
- BLM 8.8.3 or BLM 8.8.1, 8.8.2
- coloured pencils

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Demonstration**

Explain the meaning of *tessellate* and *tessellation*. Ask: Will all shapes tessellate? Demonstrate how to use rotations to create an object and tessellate it using translations. Use a design completed in The Geometer’s Sketchpad®4.

Alternatively, explain how they are to create a shape on dot paper and then tessellate it on the dot paper (BLM 8.8.1, 8.8.2).

Have one completed GSP®4 design on a computer for students to see.

Students working collaboratively on one computer take turns manipulating the tessellation and giving advice or feedback.

**Action!**

**Pairs → Exploration**

Students explore tessellations, using GSP®4 (BLM 8.8.3). Alternately, students create the tessellation illustrated on BLM 8.8.3, using dot paper.

Challenge those students who finish early to create their own tessellation in GSP®4.

**Consolidate Debrief**

**Whole Class → Discussion**

Initiate discussion by asking: What are some criteria needed for a geometric shape to tessellate the plane?

Students describe some of the “problems” and “fun” they had creating the tessellations.

Discuss where they have seen tessellations used.

**Communicating/Oral Questioning/Mental Note:** Assess students’ ability to communicate, using geometric language.



**Home Activity or Further Classroom Consolidation**

Create a tessellation using more than one shape.

Colour the results to create an interesting piece of art.

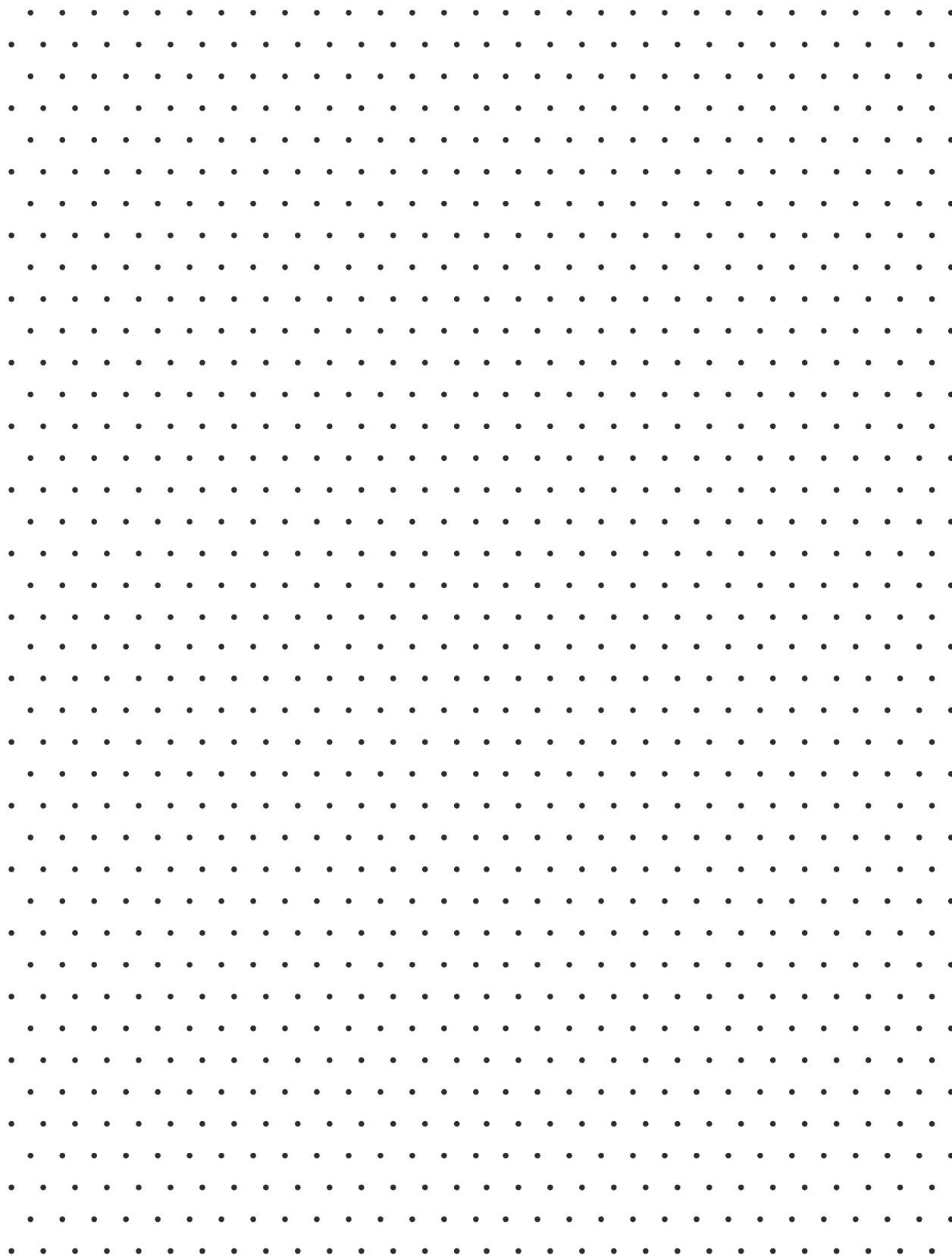
Find examples where tessellations have been used around your home, neighbourhood, in art, at school.

*Application  
Concept  
Practice*

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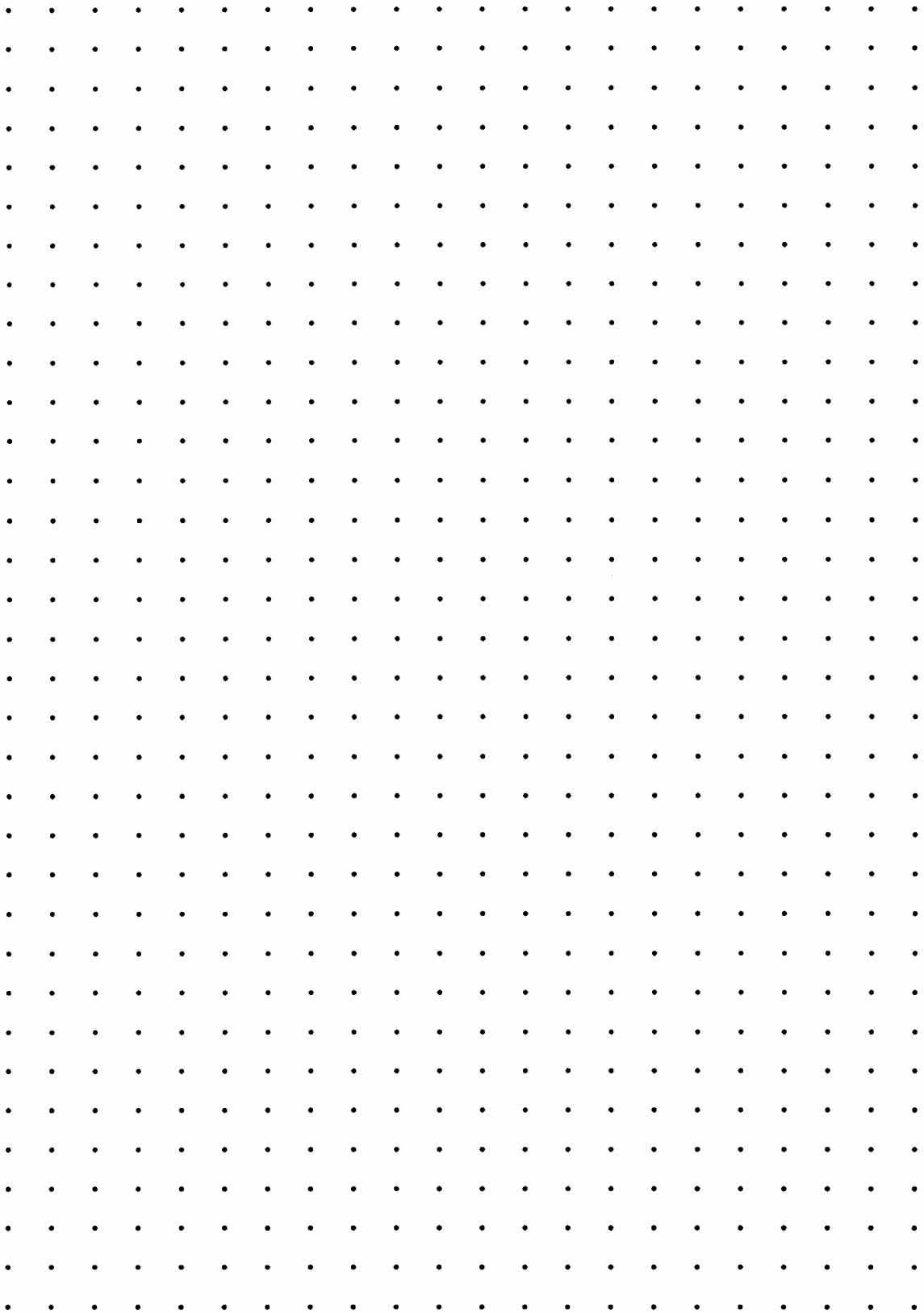
## 8.8.1: Isometric Dot Paper



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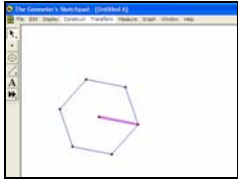
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## 8.8.2: Orthographic Dot Paper



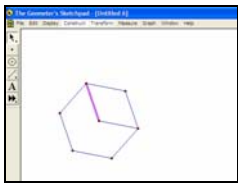
## 8.8.3: Tessellations with The Geometer's Sketchpad® 4

Create a regular hexagon: Using the Point Tool construct two points on the screen. Select the Arrow Tool, click on any white space to deselect everything. Click on the leftmost point. Using the Transform Menu, select Mark Center. Click on the other point. In the Transform Menu, select Rotate and type 60 into the degree box; click on Rotate. Select Rotate again and click on Rotate; repeat three more times.

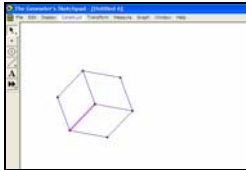


Select each of the 6 points in order clockwise. In the Construct Menu, select Segments.

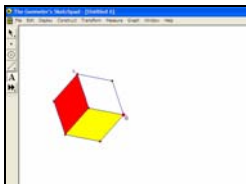
Create a cube.  
Select the centre point and one other point. Construct the segment.



Select the centre point and not the next point but the one after it; construct the segment.

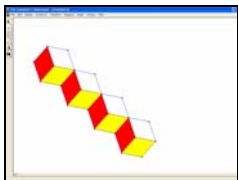


Repeat to complete the cube.

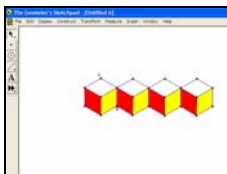


Select the four points that form the vertices of one of the square faces you have represented. In the Construct Menu, select Quadrilateral Interior. Repeat for another square. Change the colour of the second square using the Display Menu.

Select the two diagonal points of the top square shown on the screen as A and B; in the Transform Menu select Mark Vector.

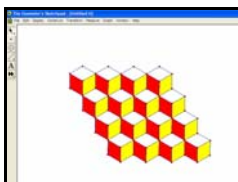


Select the entire cube and from the Transform Menu, select Translate. Repeat a few more times.



Drag the leftmost corner of the original cube and “straighten” out the line of cubes.

Mark a vector by selecting the two points indicated on screen as A and B.



Select all the cubes and translate them. Repeat a few times.

Describe the different shapes and effects you see in your diagram.



**Description**

- Form and test a conjecture as to whether or not all triangles will tessellate.
- Apply knowledge of transformations to discover whether all types of triangles will tessellate.

**Materials**

- BLM 8.9.1
- geoboards,
- dot paper,
- grid paper
- pattern blocks
- The Geometer’s Sketchpad®4

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Anticipation Guide**

Present the problem: Is this statement true? – All types of triangles will tile a plane.

Students complete the Before column of an Anticipation Guide before beginning the discussion and activity.

**Groups of 4 → Brainstorm**

Students brainstorm various ways in which they can solve the problem with materials and tools that are available.

**Learning Skills/Observation/Mental Note:** Circulate, noting students’ contributions to the discussion.

See *Think Literacy Mathematics, Grades 7–9, Anticipation Guide* pp. 10–14

GSP®4 demonstration of tessellations: [Tiling.gsp](#)

**Action!**

**Pairs → Problem Solving**

Students solve the problem several ways using the ideas generated during their brainstorm.

Students complete the After column of their Anticipation Guide.

Possible manipulatives: pattern blocks, polydron, grid paper, The Geometers’ Sketchpad®4.

**Consolidate Debrief**

**Whole Class → Discussion**

Students present their conclusions. Students should realize that any triangle will tile the plane, and should understand why this is the case.

**Mathematical Processes/Self-Reflection/Checklist:** Students reflect on their problem-solving process, using the criteria of the checklist (BLM 8.9.1).

To fill the plane, the sum of the angles that meet must add to 360°.

*Reflection*

**Home Activity or Further Classroom Consolidation**

Reflect on your solution to the problem and complete worksheet 8.9.1.

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## 8.9.1: Will All Types of Triangles Tile a Plane?

Name:

Date:

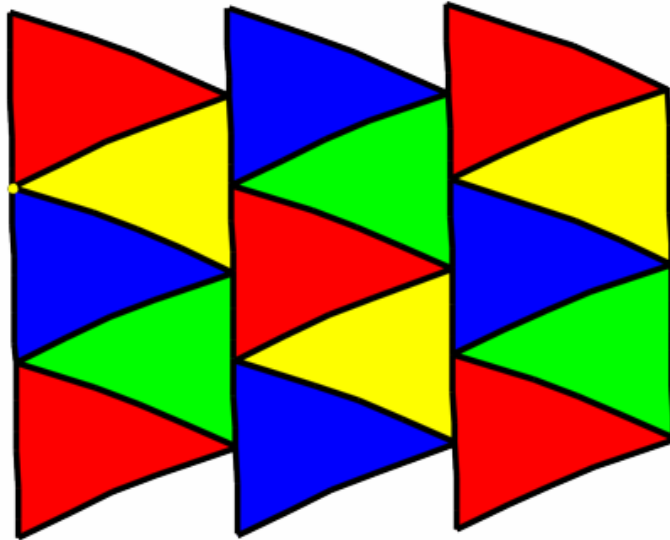
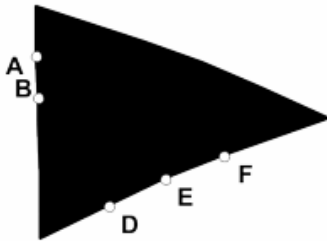
Criteria	Yes	No	Comments
<b>Selecting Tools:</b>			
I used different tools to test for tiling (e.g., pattern blocks, grid paper, computer software, polydron, concrete materials).			
<b>Reasoning and Proving:</b>			
I tested several different types of triangles (e.g., scalene, isosceles, equilateral, right angled, obtuse angled, acute angled).			
I made a convincing argument, explaining and justifying my conclusions.			
<b>Communicating:</b>			
I used mathematical language (including words, pictures, diagrams, charts, etc.), to clearly explain the process I used.			
<b>Reflecting:</b>			
I monitored my thinking (e.g., by assessing how effective my strategy was, by proposing an alternate approach, by verifying my solution).			

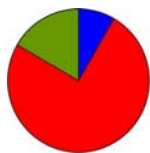
# Tiling (GSP®4)

[Tiling.gsp](#)

The black polygon is constructed to tile the plane by glide reflection (a reflection about one axis combined with a translation). Drag its visible vertices.

From an idea by D. Schattschneider





**Description**

- Form and test a conjecture as to whether or not all polygons will tessellate.
- Identify polygons that will/will not tessellate.

**Materials**

- geoboards
- dot paper
- grid paper
- pattern blocks
- The Geometer’s Sketchpad® 4

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Posing the Problem**

Recall the problem from Day 9: Will all types of triangles tile a plane? Present today’s problem: Will all polygons tessellate?

Students write down their conjecture. Ask If they don’t all tile the plane, which types of polygons (or combinations of two or three polygons) will tile a plane? Consider convex polygons only.

**Groups of 4 → Brainstorm**

Students brainstorm various ways in which they can solve the problem with materials and tools that are available.

**Action!**

**Individuals → Problem Solving**

Students work independently to solve the problem in several ways, using the ideas generated during their brainstorm.

**Curriculum Expectations/Demonstration/Checkbric:** Assess students’ ability to investigate polygons that tile a plane.

Teachers may need to explain *convex polygon*.

Possible manipulatives: pattern blocks, polydron, grid paper, The Geometer’s Sketchpad® 4.

**Consolidate Debrief**

**Whole Class → Discussion**

Students present their original conjectures and their conclusions after the investigation. Students should realize that not all polygons will tile the plane.

Comment on the students’ strengths and next steps that they can take to improve performance.

*Reflection*

**Home Activity or Further Classroom Consolidation**

Reflect on your solution to the problem and answer the following questions in your math journal:

- Did you have a plan before you started the task? What was it?
- Did you make an hypothesis? How can an hypothesis help you to plan your strategy?
- Did you reflect on your thinking to examine how effective you were being? Did this cause you to select an alternative strategy?