Unit 7

Grade 10 Applied

Quadratic Relations of the Form $y = ax^2 + bx + c$

Lesson Outline

BIG PICTURE

Students will:

- manipulate algebraic expressions, as needed to understand quadratic relations;
- identify characteristics of quadratic relations;
- solve problems by interpreting graphs of quadratic relations.

Day	Lesson Title	Math Learning Goals	Expectations
1	The x Tiles	• Connect the algebraic representations to the graphical representations of quadratics of the forms $y = x^2 + bx + c$ and $y = (x - r)(x - s)$. • Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using algebra tiles.	QR1.01, QR2.04 CGE 4b, 5a
2	Multiply a Binomial by a Binomial Using a Variety of Methods	• Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using the chart method, and the distributive property.	QR1.01, QR2.04 CGE 5a, 5b
3	Find the y-Intercept of Quadratic Equations	• Determine the connections between the y-intercept in the graph of a quadratic relation and the c in the algebraic form $y = x^2 + bx + c$, using technology.	QR1.01, QR2.04 CGE 3c, 5a
4	Finding x-Intercepts and Introduction to Factoring Quadratic Equations	• Determine the connections between the factored form of a quadratic relation and the x-intercepts of its graph. • Factor simple trinomials of the form $x^2 + bx + c$, using a variety of tools, e.g., algebra tiles, paper and pencil, and patterning strategies.	QR1.03, QR2.04 CGE 3c, 7b
5	Factoring Quadratic Relations of the Form $y = ax^2 + bx + c$, Where a Is a Common Factor	• Determine the connections between the factored form and the x-intercepts of a quadratic relation. • Factor binomials and trinomials involving one variable up to degree two, by determining a common factor, using algebra tiles.	QR1.02, QR2.04 CGE 5b
6	Use Intercepts to Graph Quadratic Equations	• Consolidate factoring. • Connect factors to the x-intercepts of the graph of the quadratic relation. • Graph quadratic relations, using intercepts.	QR1.02, QR1.03, QR2.04 CGE 3c
7	We Have a Lot in Common	• Determine the connections between the factored form and the x-intercepts of the quadratic relation $y = x^2 + bx$.	QR1.03, QR2.04 CGE 3c, 5b
8	What's the Difference?	• Investigate the method of factoring a difference of two squares using patterning strategies and diagrams. • Use the technique of factoring the difference of two squares to determine the x-intercepts of a quadratic relation.	QR1.04, QR2.04 CGE 5b, 5e
9	Quick Sketches of Parabolas *(lesson not included)*	• Use factoring techniques to find zeros and use understanding of the connection between the equation and the y-intercept and symmetry to make "quick" sketches of the related parabola, given $y = x^2 + bx + c$.	QR1.02, QR1.03, QR1.04, QR2.04 CGE 5g

Day	Lesson Title	Math Learning Goals	Expectations
10, 11	Solve Problems Involving Quadratic Relations *(lessons not included)*	• Solve problems involving quadratic relations by interpreting a given graph or a graph generated with technology from its equation.	QR3.01, QR3.02 CGE 5g
12	Summative Assessment		

75 min

Math Learning Goals

- Connect the algebraic representations to the graphical representations of quadratics of the forms $y = x^2 + bx + c$ and $y = (x - r)(x - s)$.
- Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using algebra tiles.

Materials

- graphing calculators
- algebra tiles
- BLM 7.1.1, 7.1.2, 7.1.3

Assessment Opportunities

Minds On…

Whole Class → Demonstration

Toss a beach ball to a student. Ask the class to describe the path of the ball. Sketch the path of the ball on a graph with the independent axis representing horizontal distance from the teacher and the dependent axis representing height of the ball.

Draw another graph that represents the height of the ball over time (horizontal axis represents time, vertical axis represents the distance from the ground).

Lead a discussion that draws out the characteristics of the resulting parabola (symmetry, axis of symmetry, vertex, zeros, *x*-intercept, *y*-intercept).

Answer: arc, parabola, equation that has an x^2

Word Wall
- axis of symmetry
- vertex
- zeros
- *y*-intercept
- *x*-intercepts

Action!

Pairs → Investigation

Provide the equation of the form $y = x^2 + bx + c$, in its standard and factored forms from BLM 7.1.1.

Students graph both forms, using a graphing calculator (BLM 7.1.2), to determine that they are the same graph and thus the same equation.

Whole Class → Gallery Tour

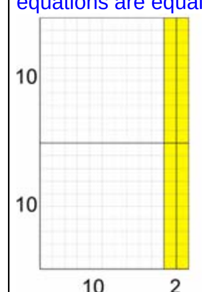
Post students' work: the two forms of the algebraic equation, a sketch of the graph, and a table of values. Students view the work of the other pairs.

Debrief:
- What did you notice about the graphs?
- What did you notice about the tables of values?
- What can you infer about the two forms of the equations?

Answer: The equations are equal.

Whole Class → Modelling

- How do we get from one form to the other?
- What is the operation between the two sets of brackets?
- Demonstrate connection to multiplication of 20×12, using base-ten materials.



20×12
$= 100 + 100 + 20 + 20$
$= 240$

Review algebra tiles: "1" tile (unit tile), *x* tile, and x^2 tile.

Using overhead algebra tiles, model the method of multiplying the factors to generate an area which represents the expanded form of the expression, e.g., $x^2 + 7x + 10$, $x^2 + 6x + 8$, $x^2 + 6x + 5$, $x^2 + 5x + 4$, using algebra tiles. Show the connection to multiplication of 20×12, using base-ten materials.

Mathematical Process/Using Tools/Observation/Mental Note: Observe students' use of a graphing calculator to graph quadratic relations.

Consolidate Debrief

Individual → Practice

Using algebra tiles, students multiply and simplify three examples with positive terms only.

$y = (x + 4)(x + 3)$ $\quad$ $y = (x + 2)(x + 5)$ $\quad$ $y = (x + 3)(x + 2)$

They make a sketch of the algebra tile solution.

Students check their answers on the graphing calculator by comparing the graphs of the two forms.

Virtual tiles are available on various websites.

Practice

Home Activity or Further Classroom Consolidation

Complete the practice questions. Use worksheet 7.1.3 to show your solutions.

Provide appropriate practice questions for students.

7.1.1: Quadratic Equations (Teacher)

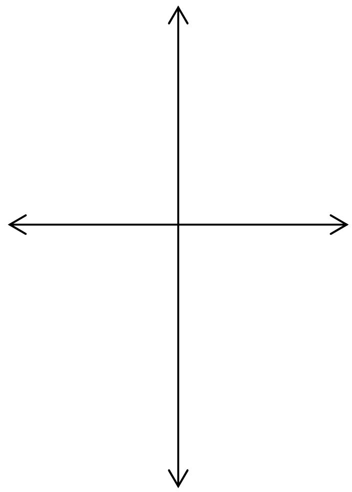
Cut out each pair of quadratic equations and distribute to the students.

Standard Form	Factored Form
$y = x^2 + 3x + 2$	$y = (x + 1)(x + 2)$
$y = x^2 + 2x - 3$	$y = (x - 1)(x + 3)$
$y = x^2 - 1x - 6$	$y = (x + 2)(x - 3)$
$y = x^2 + x - 6$	$y = (x - 2)(x + 3)$
$y = x^2 + 5x + 4$	$y = (x + 1)(x + 4)$
$y = x^2 + 2x - 3$	$y = (x + 3)(x - 1)$
$y = x^2 - x - 6$	$y = (x - 3)(x + 2)$
$y = x^2 + 6x + 5$	$y = (x + 1)(x + 5)$
$y = x^2 - 4x - 5$	$y = (x + 1)(x - 5)$
$y = x^2 - 4x - 12$	$y = (x + 2)(x - 6)$
$y = x^2 + 6x + 8$	$y = (x + 2)(x + 4)$
$y = x^2 - 5x + 6$	$y = (x - 3)(x - 2)$
$y = x^2 - 2x - 3$	$y = (x - 3)(x + 1)$
$y = x^2 + 7x + 12$	$y = (x + 3)(x + 4)$
$y = x^2 + 7x + 10$	$y = (x + 2)(x + 5)$
$y = x^2 - 7x + 10$	$y = (x - 2)(x - 5)$

7.1.2: Graphing Quadratic Equations

Name:

1. Obtain a pair of equations from your teacher.
2. Press the **Zoom** button and press **6** (for ZStandard) to set the window to make the max and min on both axes go from –10 to 10.
3. Press the ***y* =** button and key in your two equations into Y_1 and Y_2.
4. To change the graph of Y_2 to “animation”: Move the cursor to the left of Y_2. Press **Enter** four times to toggle through different graph styles available.
   You should see
5. Press **Graph**. First the Y_1 quadratic will appear, then the Y_2 quadratic will appear and be traced by an open circle.
6. Complete the three columns of the table below.

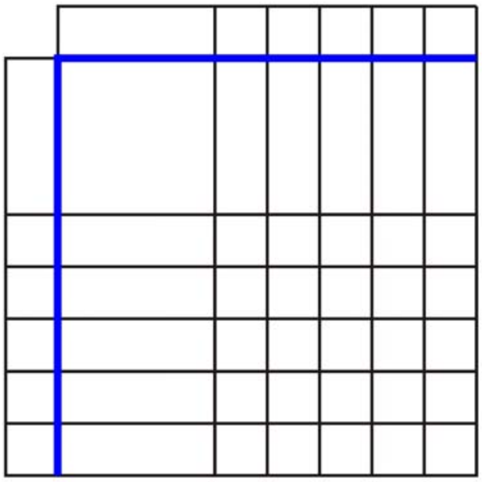
Our Two Equations	What They Look Like	What We Think It Means
		

7. Press **2nd Graph** so that you can look at the tables of values for the two curves. Discuss what you see and complete the table.

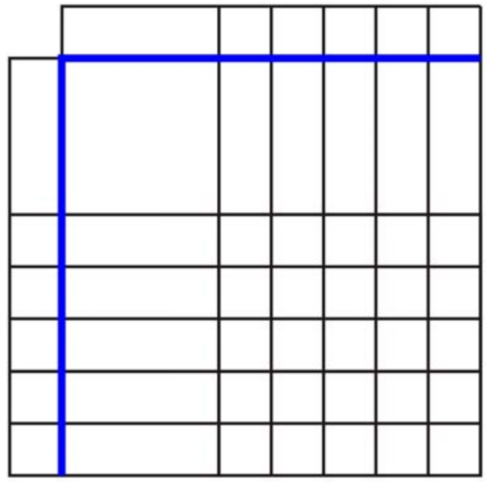
What We Noticed About the Table of Values	What We Think It Means

7.1.3: Algebra Tile Template

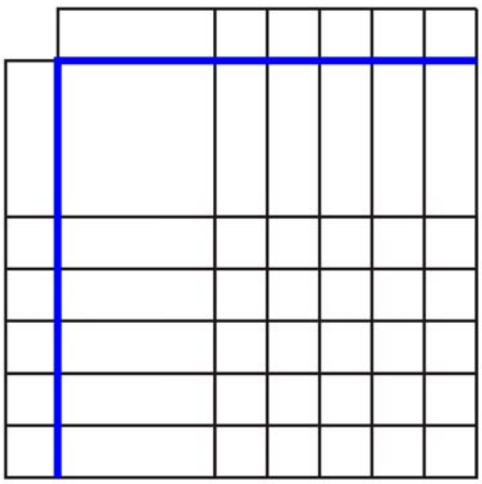
1. $y = (\qquad)(\qquad) =$ ______________



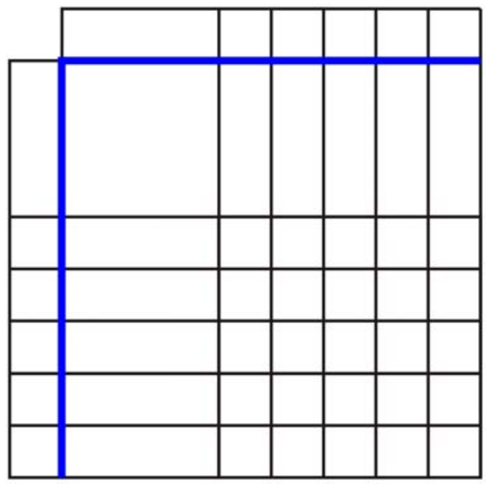
2. $y = (\qquad)(\qquad) =$ ______________



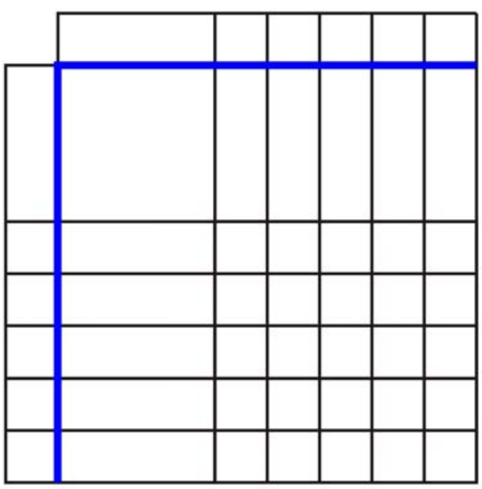
3. $y = (\qquad)(\qquad) =$ ______________



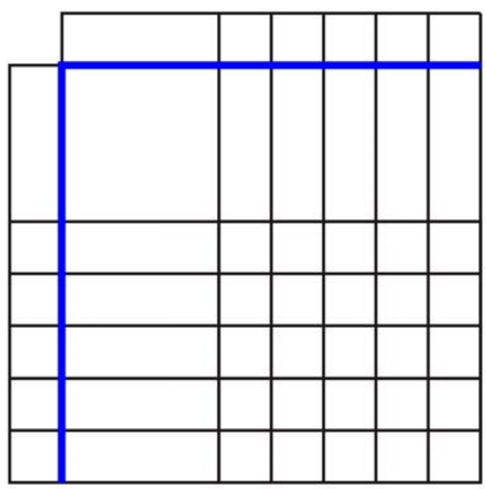
4. $y = (\qquad)(\qquad) =$ ______________



5. $y = (\qquad)(\qquad) =$ ______________



6. $y = (\qquad)(\qquad) =$ ______________



Unit 7: Day 2: Multiply a Binomial by a Binomial Using a Variety of Methods

Grade 10 Applied

75 min

Math Learning Goals

- Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using the chart method, and the distributive property.

Materials

- BLM 7.2.1, 7.2.2
- algebra tiles
- graphing calculators

Assessment Opportunities

Minds On…

Whole Class → Discussion

Students expand $(x + 1)(x + 1)$ with algebra tiles.

Discuss the meaning of repeated multiplication, i.e., $(x + 1)(x + 1) = (x + 1)^2$.

Representation with algebra tiles is best for expressions with positive terms only.

Individual → Practice

Students practise multiplication of a binomial with positives only, using BLM 7.2.1 Part A and algebra tiles.

Action!

Whole Class → Guided Instruction

Model the different combinations of multiplication, i.e., monomial × monomial and monomial × binomial with positive and negative terms, using the chart method.

Connect the use of algebra tiles to the "chart method."

Model the chart method for multiplication of a binominal × binomial. Students practise this method, BLM 7.2.1 Part B.

This representation can be used for binomials with positive and negative terms.

Use algebra tiles to show each of these and recall the distributive property.

$x(x + 3)$

$2(x + 3)$

Multiply $(x+2)(x+3)$ using tiles, and show "double distributive," (the algebraic model).

Model the steps involved in the algebraic manipulation of multiplying a binomial by a binomial. Connect to distributive property by calling it "double distributive."

Using the Distributive Property

Lead students to understand the connection between the chart method and the algebraic method of multiplying binomials.

$(x + 4)(x – 3)$

$= x (x – 3) + 4(x – 3)$

$= x^2 – 3x + 4x – 12$

$= x^2 + x – 12$

	x	$+4$
x	x^2	$4x$
$–3$	$–3x$	$–12$

Students practise this method, using BLM 7.2.1 Part C.

	x	$+3$
x	x^2	$3x$
$+2$	$2x$	6

$(x + 3)(x + 2)$

$= x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

Use visual aid to show double distributive

Learning Skills/Work Habits/Observation/Checklist: Assess how well students stay on task and complete assigned questions.

Consolidate Debrief

Individual → Journal

In your journal, write a note to a friend who missed today's class. Summarize the three methods of multiplying binomials that you worked with. Use words, diagrams, and symbols in your explanation.

Reflection

Home Activity or Further Classroom Consolidation

Solve the problems by multiplying the binomials.

Provide chart template 7.2.2 and algebra tile template 7.1.3, as needed.

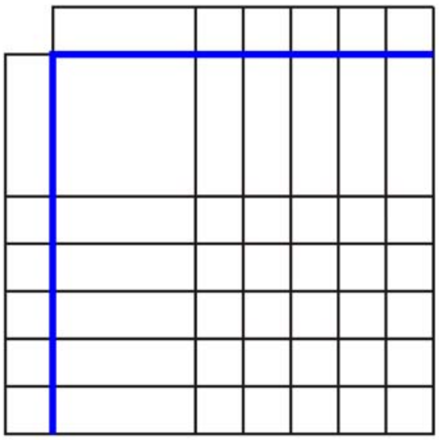
7.2.1: Multiply a Binomial by a Binomial

Name:

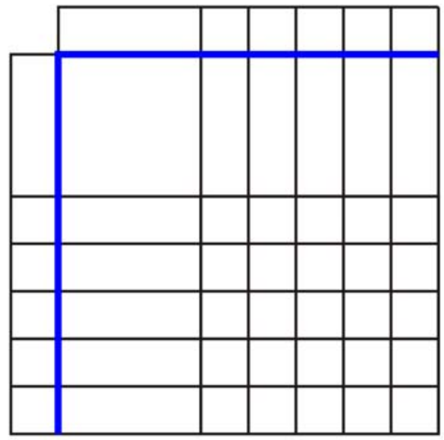
Part A

Use algebra tiles to multiply binomials and simplify the following:

1. $y = (x + 1)(x + 3) =$ ________________



2. $y = (x + 2)(x + 3) =$ ________________



Part B

Use the chart method to multiply and simplify the following:

1. $y = (x + 1)(x + 3) =$ ________________

2. $y = (x + 2)(x +3) =$ ________________

3. $y = (x + 2)(x – 1) =$ ________________

4. $y = (x – 2)(x + 3) =$ ________________

5. $y = (x – 1)(x – 1) =$ ________________

6. $y = (x – 1) (x – 2) =$ ________________

7.2.1: Multiply a Binomial by a Binomial (continued)

Part C

Multiply and simplify the two binomials, using the chart method and the distributive property.

1. $(x + 4)(x - 3)$

	x	$+4$
x		
-3		

2. $(x - 3)(x - 3)$

	x	-3
x		
-3		

3. $(x + 2)^2$

	x	$+2$
x		
$+2$		

4. $(x + 2)(x - 1)$

	x	$+2$
x		
-1		

5. $(x - 2)\ (x + 1)$

	x	-2
x		
$+1$		

6. $(x - 1)^2$

	x	-1
x		
-1		

7. $(x - 1)(x - 2)$

	x	-1
x		
-2		

8. $(x - 3)(x - 4)$

	x	-3
x		
-4		

7.2.1: Multiply a Binomial by a Binomial (continued)

Answers to Part B

1. $y = x^2 + 4x + 3$

	x	1
x	x^2	x
3	$3x$	3

2. $y = x^2 + 5x + 6$

	x	2
x	x^2	$2x$
3	$3x$	6

3. $y = x^2 + x - 2$

	x	2
x	x^2	$2x$
-1	$-x$	-2

4. $y = x^2 + x - 6$

	x	-2
x	x^2	$-2x$
3	$3x$	-6

5. $y = x^2 - 2x + 1$

	x	-1
x	x^2	$-x$
-1	$-x$	1

6. $y = x^2 - 3x + 2$

	x	-1
x	x^2	$-x$
-2	$-2x$	2

Answers to Part C

1. $x^2 + x - 12$
2. $x^2 - 6x + 9$
3. $x^2 + 4x + 4$
4. $x^2 + x - 2$
5. $x^2 - x - 2$
6. $x^2 - 2x + 1$
7. $x^2 - 3x + 2$
8. $x^2 - 7x + 12$

7.2.2: Chart Template for Distributive Property

1. $y = (\qquad)(\qquad) =$ ____________

2. $y = (\qquad)(\qquad) =$ ____________

3. $y = (\qquad)(\qquad) =$ ____________

4. $y = (\qquad)(\qquad) =$ ____________

5. $y = (\qquad)(\qquad) =$ ____________

6. $y = (\qquad)(\qquad) =$ ____________

7. $y = (\qquad)(\qquad) =$ ____________

8. $y = (\qquad)(\qquad) =$ ____________

Unit 7: Day 3: Find the *y*-Intercept of Quadratic Equations

Grade 10 Applied

75 min

Math Learning Goals

- Determine the connections between the *y*-intercept in the graph of a quadratic relation and the *c* in the algebraic form $y = x^2 + bx + c$, using technology.

Materials

- graphing calculators
- BLM 7.3.1, 7.3.2

Assessment Opportunities

Minds On…

Whole Class → Modelling

Pair students, and assign a person A and person B. Ask: What is meant by the *y*-intercept of a graph? A answers B. Ask: When we studied linear equations we looked at $y = 3x + 2$. What did the 2 represent? B answers A.

Explain the value of knowing the *y*-intercept for graphing.

Mathematical Processes/Communicating/Observation/Mental Note: Circulate as students work in pairs to note their use of mathematical language.

Action!

Pairs → Guided Investigation

Pairs complete BLM 7.3.1 to identify the *y*-intercept of quadratic relations given in standard form and factored form.

Consolidate Debrief

Whole Class → Summarizing

Students summarize how the *y*-intercept of the graph relates to the equation of linear relations and quadratic relations, including why it is valuable to know the *y*-intercept for graphing.

Connect to the understanding that the *y*-intercept occurs when $x = 0$ for any relation, referring to algebraic and graphical forms during the discussions.

Practice

Home Activity or Further Classroom Consolidation

Complete worksheet 7.3.2.

Provide algebra tiles template 7.1.3 or chart template 7.2.2, as needed.

7.3.1: Finding the *y*-Intercept of a Quadratic Equation

Name:

1. Use the graphing calculator to find the y-intercept for each of the equations:

   Note any patterns you see.

Equation	y-intercept
$y = x^2 - x - 2$	
$y = x^2 + 2x - 8$	
$y = x^2 - x + 6$	
$y = (x-1)(x-2)$	
$y = (x+4)(x+3)$	
$y = (x+3)^2$	

2. How can you determine the y-intercept by looking at a quadratic equation?

3. Which form of the quadratic equation is easiest to use to determine the y-intercept? Explain your choice.

4. Using your conclusion from question 2, state the y-intercept of each and check using a graphing calculator.

Equation	y-intercept	Does it check?	
		Yes	No
$y = x^2 - 2x - 8$			
$y = x^2 - x - 6$			
$y = x^2 + 3x + 2$			
$y = (x-4)(x-1)$			
$y = (x-2)(x+5)$			

5. Explain the connection between the y-intercept and the value of y when $x = 0$.

7.3.2: Quadratic Equations

Name:

1. Find the y-intercept for each of the following quadratic equations given in factored form. Write the equations in standard form. Show your work.

   a) $y = (x-5)(x+2)$ standard form:
   $=$ y-intercept:
   $=$

   b) $y = (x+4)(x-3)$ standard form:
   $=$ y-intercept:
   $=$

   c) $y = (x-4)^2$ standard form:
   $=$ y-intercept:
   $=$

   d) $y = (x+5)^2$ standard form:
   $=$
   $=$ y-intercept:

2. Find the y-intercept for each of the following quadratic equations:

   a) $y = (x+4)(x+2)$

   y-intercept:

   b) $y = (x-6)^2$

   y-intercept:

Unit 7: Day 4: Finding x-Intercepts and Introduction to Factoring Quadratic Equations

Grade 10 Applied

75 min

Math Learning Goals

- Determine the connections between the factored form of a quadratic relation and the x-intercepts of its graph.
- Factor simple trinomials of the form $x^2 + bx + c$, using a variety of tools, e.g., algebra tiles, paper and pencil, and patterning strategies.

Materials

- graphing calculators
- algebra tiles
- BLM 7.4.1, 7.4.2, 7.4.3

Assessment Opportunities

Minds On…

Whole Class → Demonstration

Review how to find the y-intercept for a quadratic equation given the standard form or factored form.

Introduce x-intercepts for quadratic equations. Model how to make a sketch of the parabola given its y-intercept and two x-intercepts. Place a dot on the y-intercept of 4 and x-intercept of 4 and 1. Ask a student to sketch the graph. Repeat for y-intercept of +6 and x-intercept of –3 and –2.

Each student makes a sketch of the graph y-intercept –6 and x-intercept of –2 and 3. Check the answer.

Lead students to the understanding that knowing the y-intercept and both x-intercepts can greatly assist in graphing some parabolas of the form $y = x^2 + bx + c$.

Remind students of the symmetrical nature of a parabola.

Action!

Pairs → Investigation

Students use graphing calculators to investigate the x-intercepts (BLM 7.4.1).

Lead students to realize that equations in factored form help to find the x-intercepts without a calculator. Establish the need to change equations in standard form to factored form. Tell students that the process is called factoring.

Whole Class → Demonstration

Model the use of algebra tiles and the rectangular area model to factor trinomials, using expression $x^2 + 4x + 3$.

Ask: What are the length and width of the rectangle?

Point out that there are no other ways to make rectangles with this set of tiles. Students should note that the dimensions of the rectangle are $(x + 1)$ and $(x + 3)$. These are called the factors. Identify the x-intercepts of the corresponding relation $y = (x + 1)(x + 3)$.

Students use algebra tiles to factor quadratic expressions (BLM 7.4.2). Remind them that, in building the rectangle, the "lines" or "cracks" must line up.

Algebra tiles needed to work through the activity:
one – x^2 tile
four – x tiles
three – unit tiles

It is recommended that only positive terms be modelled with algebra tiles.

Individual Practice

Students complete the chart and the questions on BLM 7.4.2. Take up as a class. Students complete BLM 7.4.3 Part A.

Once the patterns are established using positive terms, extend the patterns to include negatives.

Curriculum Expectations/Oral Question/Anecdotal Note: Assess how students make connections between the graph and the quadratic equation.

Consolidate Debrief

Whole Class → Summary

Complete BLM 7.4.3 Part B question 1, clarifying each aspect of the question.

Concept Practice

Home Activity or Further Classroom Consolidation

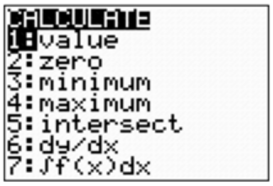
Complete worksheet 7.4.3 Part B.

7.4.1: Finding the *x*-Intercepts of a Quadratic Equation

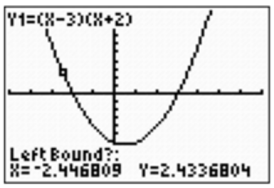
Name:

To find the *x*-intercepts:

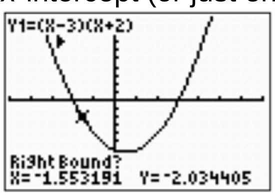
1. Enter the equation in Y_1. $y = x^2 - x - 6$
2. Press **ZOOM** and **6** (Zstandard) to set the scale for your graph. The calculator will then show the parabola.
3. Press **2nd TRACE** 1 to view the Calculate screen.



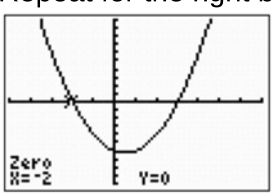
4. Select **2: ZERO**. Your screen should be similar to the following screen.



5. You will be asked to enter a left bound. You can move the cursor to the left of one *x*-intercept (or just enter an *x* value that is to the left of the *x*-intercept). Press **ENTER.**



6. Repeat for the right bound, being sure that you are to the right of the same *x*-intercept.



7. The next screen will say guess. You can guess if you want but it is not necessary. Press ENTER. You will get one *x*-intercept.

8. Repeat steps 3 through 7 to get the other *x*-intercept.

7.4.1: Finding the *x*-Intercepts of a Quadratic Equation (continued)

1. Use the graphing calculator to find the *x*-intercepts for each of the following:

Equation	First *x*-intercept	Second *x*-intercept
$y = x^2 - 4x - 12$		
$y = x^2 + 2x - 8$		
$y = x^2 - x + 6$		
$y = (x-1)(x-2)$		
$y = (x+4)(x+3)$		
$y = (x-3)(x+5)$		

2. Can you determine the *x*-intercepts by looking at a quadratic equation? Explain.

3. Which form of the quadratic equation did you find the easiest to use when determining the *x*-intercepts? Explain the connection between the factors and the *x*-intercepts.

7.4.2: Area with Algebra Tiles

Name:

Using algebra tiles create the rectangles for the following areas.
Complete the following chart.

Area of Rectangle	Number of x^2 Tiles	Number of x Tiles	Number of Unit Tiles	Sketch of Rectangle	Length	Width
$x^2 + 4x + 3$						
$x^2 + 5x + 6$						
$x^2 + 6x + 8$						
$x^2 + 7x + 12$						

1. Find a relationship between the number of x tiles and the numbers in the expressions for the length and width.

2. Find a relationship between the number of unit tiles and the numbers in the expressions for the length and width.

3. If the area of a rectangle is given by $x^2 + 8x + 15$, what expression will represent the length and the width?

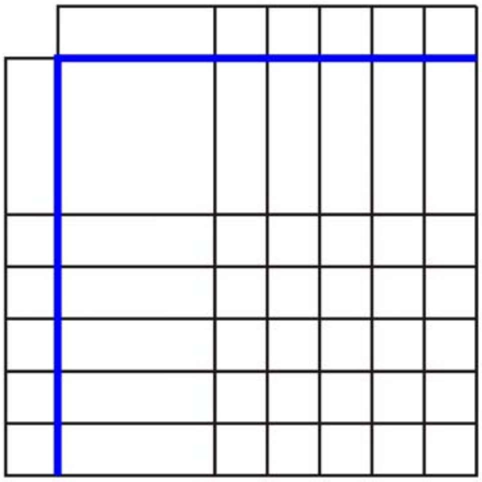
7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic

Name:

Part A

For each of the following, shade in the appropriate rectangular area. Then shade in the tiles that represent the length and width for each of those areas. Use the length and width to represent and state the factors. State the *x*-intercepts. Check using a graphing calculator.

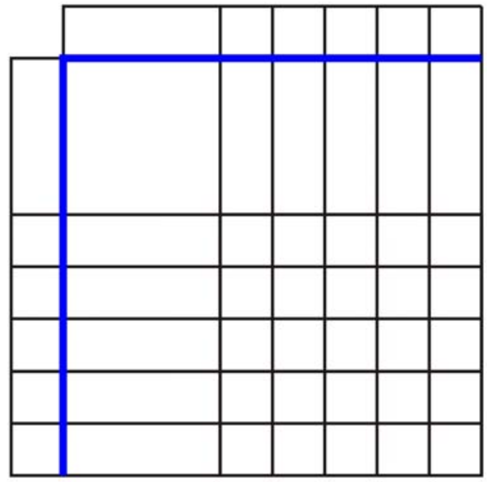
1. $y = x^2 + 3x + 2 = (\qquad)(\qquad)$



x-intercepts _____, _____

Check with calculator.

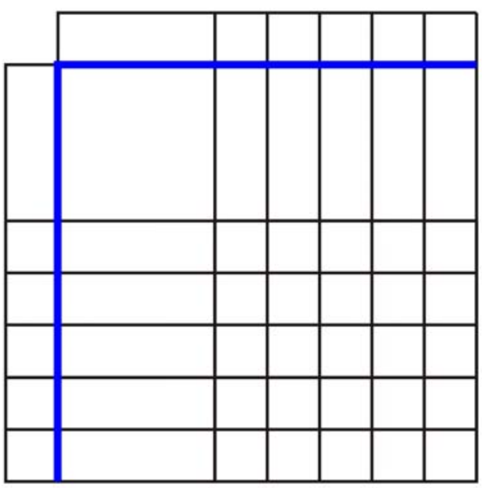
2. $y = x^2 + 5x + 4 = (\qquad)(\qquad)$



x-intercepts _____, _____

Check with calculator.

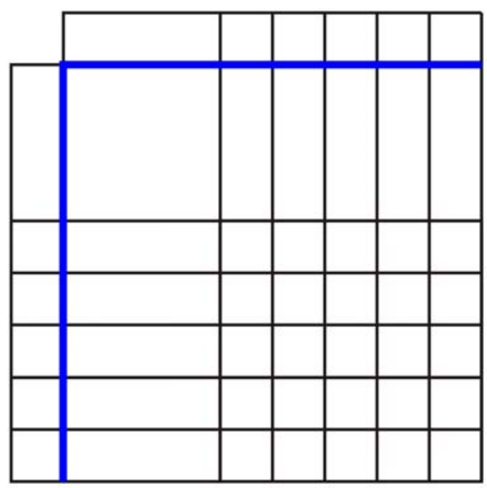
3. $y = x^2 + 6x + 5 = (\qquad)(\qquad)$



x-intercepts _____, _____

Check with calculator.

4. $y = x^2 + 4x + 4 = (\qquad)(\qquad)$



x-intercepts _____, _____

Check with calculator.

7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic (continued)

Part B

Using the diagrams in Part A, find the *x*- and *y*-intercepts for each quadratic relation. Use the information to make the sketch on the grid provided.

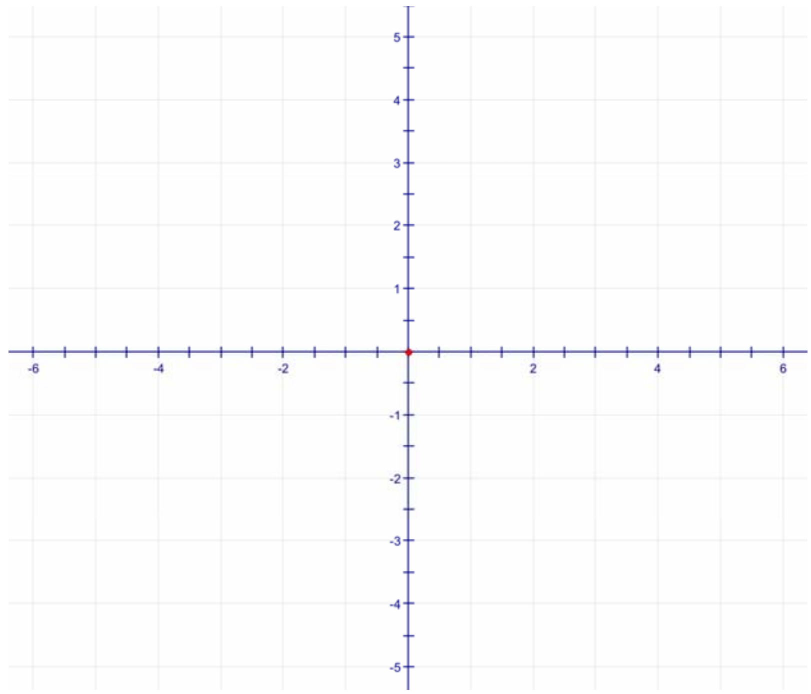
1. standard form:
   $y = x^2 + 3x + 2$

   factored form:

   y-intercept:

   first *x*-intercept:

   second *x*-intercept:



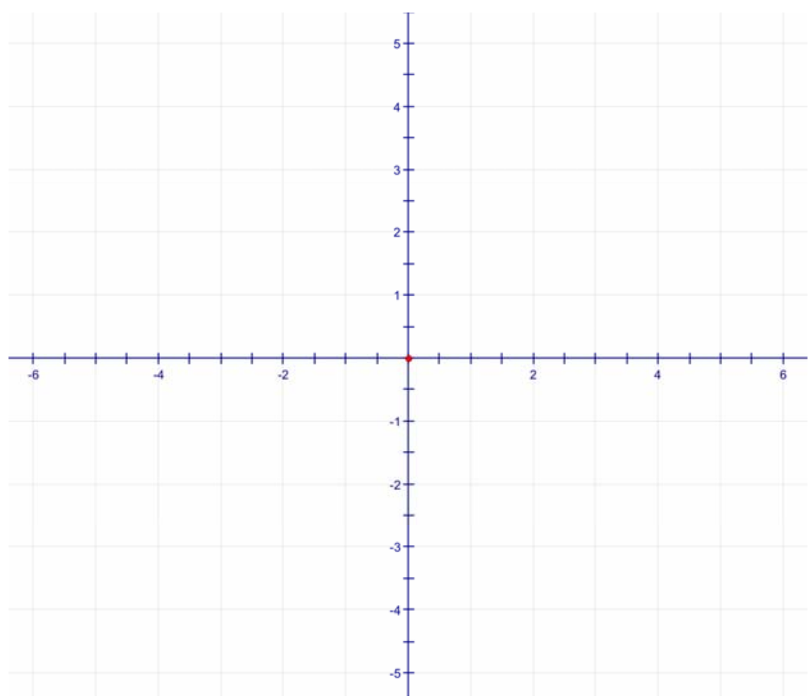
2. standard form:
   $y = x^2 + 5x + 4$

   factored form:

   y-intercept:

   first *x*-intercept:

   second *x*-intercept:



7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic (continued)

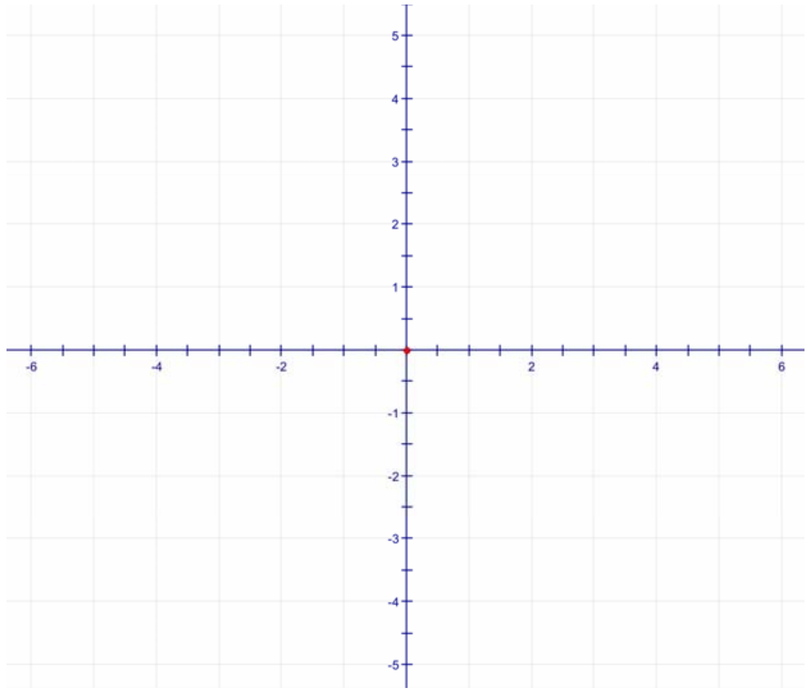
3. standard form:
   $y = x^2 + 6x + 5$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



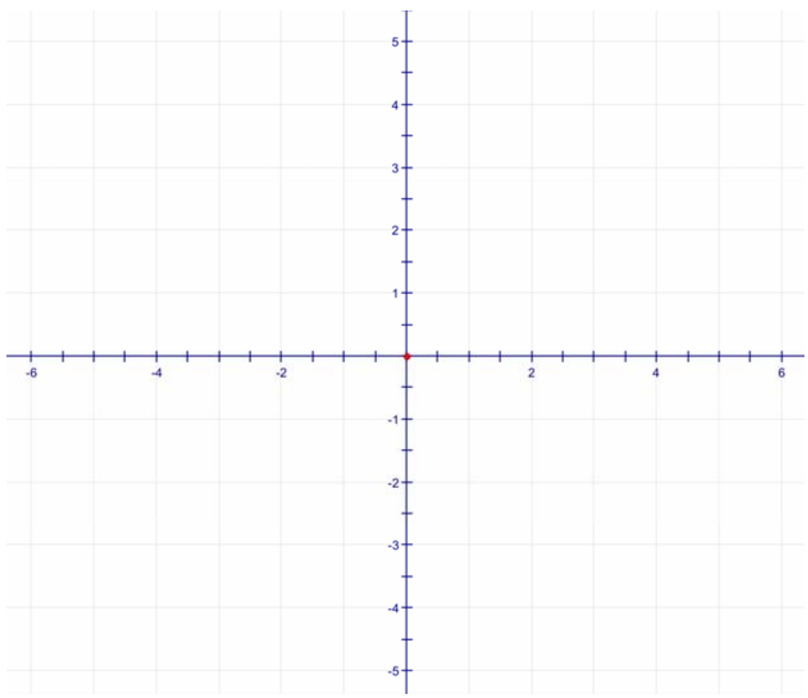
4. standard form:
   $y = x^2 + 4x + 4$

   factored form:

   y-intercept:

   first x-intercept:

   second x-intercept:



5. In what way is the last example different from the others?

Unit 7: Day 5: Factoring Quadratic Relations of the Form $y = ax^2 + bx + c$, Where a Is a Common Factor

Grade 10 Applied

75 min

Math Learning Goals

- Determine the connections between the factored form and the x-intercepts of a quadratic relation.
- Factor binomials and trinomials involving one variable up to degree two, by determining a common factor using algebra tiles.

Materials

- algebra tiles
- graphing calculators
- BLM 7.5.1, 7.5.2, 7.5.3
- glue stick

Assessment Opportunities

Minds On…

Whole Class → Four Corners

Review the forms of equations and representations of quadratics and label four stations: factored form, standard form, graphical form, and chart or algebra tiles representation. Randomly hand out the cards (BLM 7.5.1).

Students move to the corner for the form represented on their card and check that everyone has the same form. They can help one another express their representation in the other forms and jot them on the back of their cards. They then "find" the other three people holding the related cards.

Groups justify they all have different forms of representations of the same quadratic. Paste these on a single page and post them for a gallery walk.

Curriculum Expectations/Demonstration/Marking Scheme: Collect worksheets and assess students' understanding of the concepts needed to complete the Home Activity from Day 4.

Action!

Pairs → Investigation

Students complete BLM 7.5.2.

Discuss the ideas investigated and extend to relations that include negative coefficients for b and c, e.g., $y = x^2 - x - 6$. Students should understand that $(r) \times (s) = c$ and $r + s = b$, yielding factors $(x + r)(x + s)$ and x-intercepts of $-r$ and $-s$.

$r \times s = -6$
$r + s = -1$
-3 and 2
factors are
$(x - 3)(x + 2)$
x intercepts 3
and -2

Students complete BLM 7.5.3 and share their solutions on the overhead.

Whole Class → Guided Instruction

Introduce common factoring by displaying overhead tiles for $3x + 3$.

Demonstrate that this is 3 groups of $(x + 1)$, thus this can be expressed as $3(x + 1)$. The "common factor" is 3. Repeat for $2x + 4$ and $5x - 10$.

In Grade 10 Applied, students only factor $ax^2 + bx + c$ when a is a common factor.

Display the overhead tiles needed to factor $2x^2 + 2x + 2$.

Ask:

- Can these be placed into groups where each type of tile is equally represented in all groups? Why or why not?
- How many groups did you get? What are the contents of each group?

Answer:
$y = 2x^2 + 2x + 2$
$y = 2(x^2 + x + 1)$
i.e., 2 sets of
$x^2 + x + 1$

Show how the factored answer would be expressed.

Complete several more examples, such as: $3x^2 + 6x + 12$, $2x^2 + 6x - 8$, $2x^2 - 8x + 10$, $4x^2 + 8x + 8$.

For $4x^2 + 8x + 8$, some students may think that 2 groups of $(2x^2 + 4x + 4)$ would be appropriate when 4 groups of $(x^2 + 2x + 2)$ is the most appropriate answer. Use this example to discuss the greatest common factor.

Another similar example:
$y = 4x^2 + 12x + 12$

Consolidate Debrief

Individual → Journal

Students summarize how to factor a trinomial, including conditions that make it possible for a trinomial to be factored and how to determine a common factor.

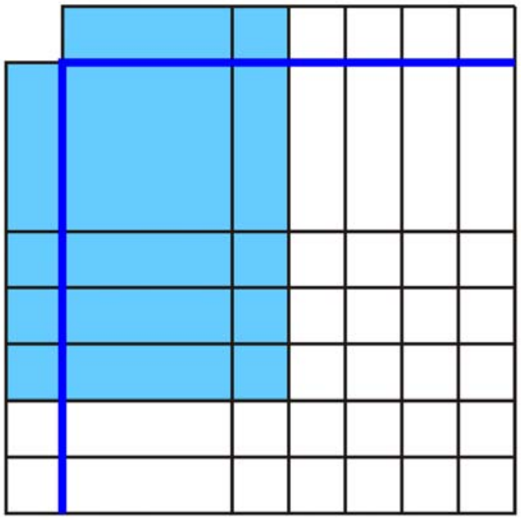
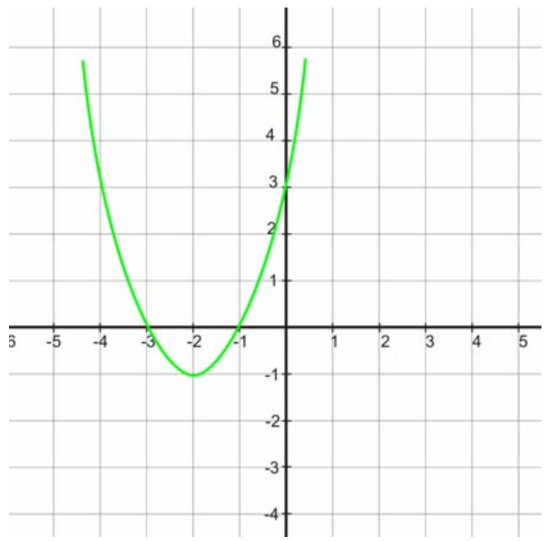
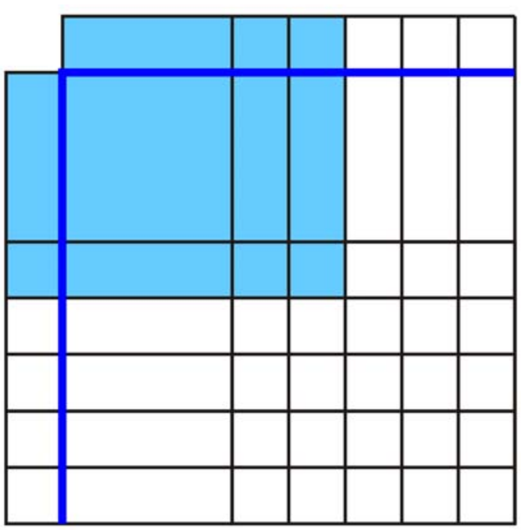
Home Activity or Further Classroom Consolidation

Concept Practice

Complete the practice questions.

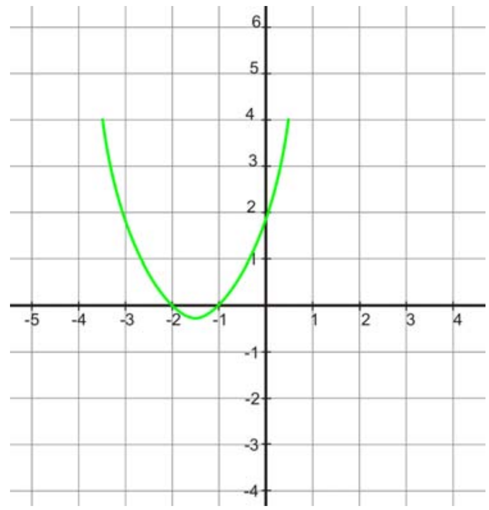
Provide questions that require checking answers by expanding or graphing with technology.

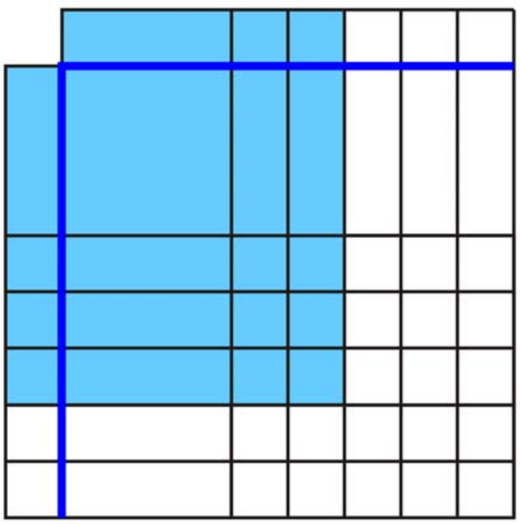
7.5.1: Four Corners

	$y = x^2 + 4x + 3$
$y = (x + 1)(x + 3)$	
	$y = x^2 + 3x + 2$

7.5.1: Four Corners **(continued)**

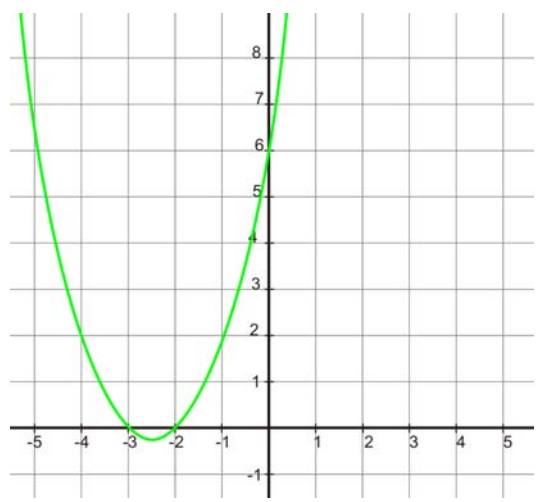
$y = (x + 2)(x + 1)$



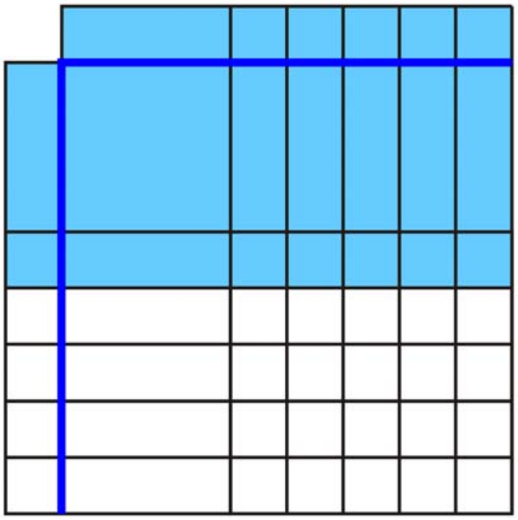


$y = x^2 + 5x + 6$

$y = (x + 3)(x + 2)$

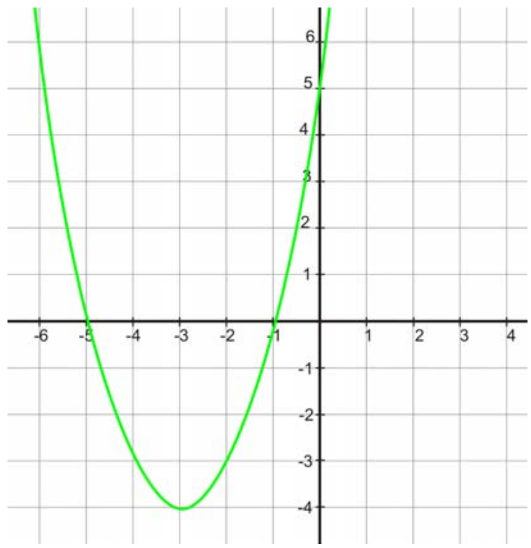


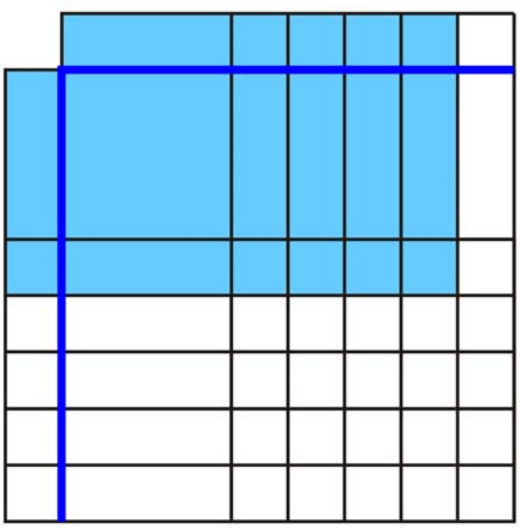
7.5.1: Four Corners (continued)



$y = x^2 + 6x + 5$

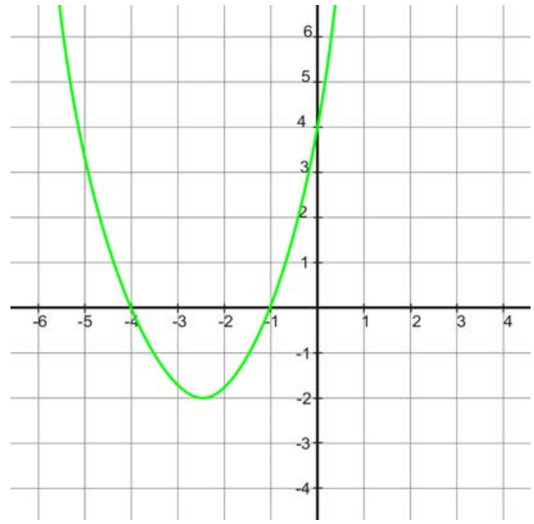
$y = (x + 5)(x + 1)$





$y = x^2 + 5x + 4$

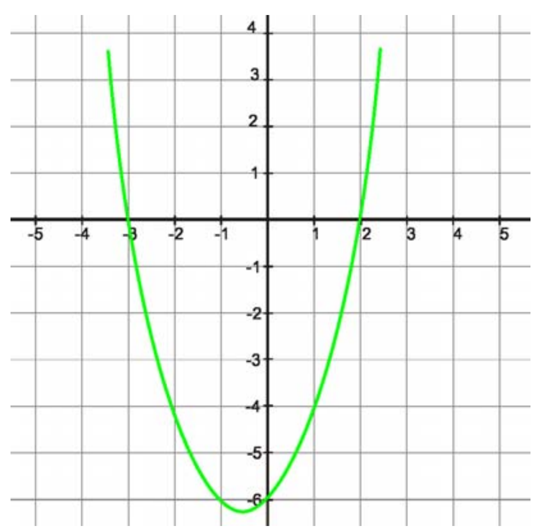
$y = (x + 1)(x + 4)$



$y = x^2 + 1x - 6$

	x	$+3$
x	x^2	$+3x$
-2	$-2x$	-6

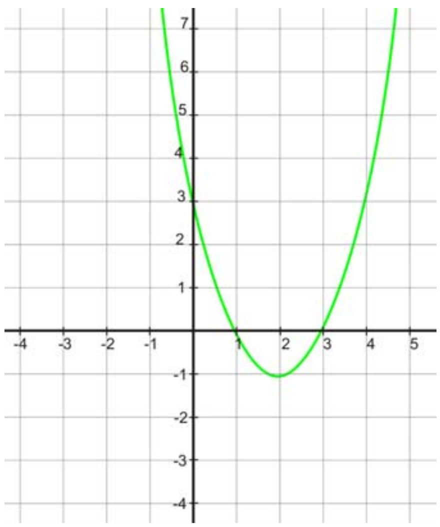
$y = (x + 3)(x - 2)$



7.5.1: Four Corners (continued)

	x	-1
x	x^2	$-1x$
-3	$-3x$	3

$y = (x - 1)(x - 3)$

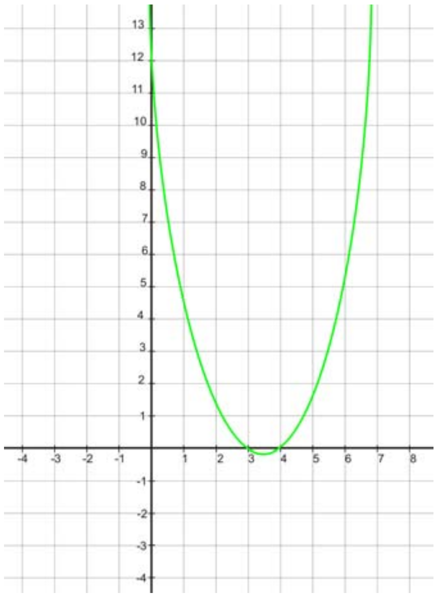


$y = x^2 - 4x + 3$

$y = (x - 3)(x - 4)$

	x	-3
x	x^2	$-3x$
-4	$-4x$	$+12$

7.5.1: Four Corners (continued)



$y = x^2 - 7x + 12$

7.5.2: Factored Form and x-Intercepts

Name:

Use algebra tiles to find the length and width for each given area. Use the graphing calculator to find the x-intercepts of the corresponding quadratic relation. Graph both the area model and factored form of the quadratic relation to check that these are the same before finding the x-intercepts.

Area	Length	Width	Factored Form	First x-intercept of corresponding relation	Second x-intercept of corresponding relation
$x^2 + 4x + 3$					
$x^2 + 5x + 6$					
$x^2 + 6x + 8$					
$x^2 + 7x + 12$					

1. What do you notice about the constant term in the length and width expressions and the coefficient of the x term in the area expressions?

2. What do you notice about the constant term in the length and width expressions and the constant term in the area expressions?

3. If an area is expressed as $x^2 + 10x + 21$, what must be true of the constant terms in the length and width expressions?

4. If the standard form of a quadratic relation is $y = x^2 + bx + c$, and it has x-intercepts of r and s, then the same relation would then be $y = (x – r)(x – s)$. How would you find the value of r and s?

BLM 7.5.3: Match It!

Name:

Match each pair of numbers on the left with the correct product and sum on the right.

Pair of Numbers		Product and Sum	
1.	$r = –1, s = –5$	a.	$r \times s = 5$ $r + s = 6$
2.	$r = 1, s = 6$	b.	$r \times s = –5$ $r + s = –4$
3.	$r = 2, s = –3$	c.	$r \times s = –5$ $r + s = 4$
4.	$r = 1, s = –5$	d.	$r \times s = 5$ $r + s = –6$
5.	$r = –3, s = 4$	e.	$r \times s = 6$ $r + s = 7$
6.	$r = –1, s = 5$	f.	$r \times s = 6$ $r + s = –7$
7.	$r = –6, s = 2$	g.	$r \times s = –6$ $r + s = –1$
8.	$r = 1, s = 5$	h.	$r \times s = 12$ $r + s = 13$
9.	$r = –1, s = –6$	i.	$r \times s = –12$ $r + s = –4$
10.	$r = 1, s = 12$	j.	$r \times s = –12$ $r + s = 1$

75 min

Math Learning Goals

- Consolidate factoring.
- Connect factors to the x-intercepts of the graph of the quadratic relation.
- Graph quadratic relations, using intercepts.

Materials

- BLM 7.6.1
- large chart (grid) paper

Assessment Opportunities

Minds On...

Pairs → Review

Students discuss their journal entry from Day 5 and refer to their answers on BLM 7.5.1 and 7.5.2.

Pairs factor the following quadratic expressions, discuss what is the same and different about each, and how the results would correspond to the x-intercepts of the corresponding relation:

$x^2 + 11x + 30$, $x^2 - 11x + 30$, $x^2 - x - 30$, $x^2 + x - 30$.

Curriculum Expectation/Observation/Checklist: Listen to students' conversations to identify level of understanding of factoring trinomials and the connection between the factors and the x-intercepts.

Include some examples where the parabola is opening downward, and where there is a double root.

Action!

Whole Class → Demonstration

Using large chart (grid) paper, place a dot on the x-axis at –4 and 2, and a dot on the y-axis at –8. Point out the –4 and 2 are the x-intercepts and –8 is the y-intercept of a parabola.

Ask volunteers to sketch the parabola that goes through these points. Point out that, at this time, they are focusing on the symmetry of the parabola, and that the parabola intersects correctly on the y-axis, and the x-axis. Repeat this several times using other values for the intercepts.

Discuss question A on BLM 7.6.1 and do question B together. Refer to question F to be sure students understand what is different in this question.

Note the use of the word *sketch*. The graph is not intended to be exact, but to show the symmetry, the intercepts, and the general movement of the curve.

Individual → Practice

Students complete BLM 7.6.1.

Students who finish early can assist students who are having difficulty, and/or put an answer on the overhead and make a sketch of the relation on large chart (grid) paper.

Students can visualize and draw their solution in the air also.

Some students may still need algebra tiles to factor.

Consolidate Debrief

Whole Class → Discussion

Students check their answers using the response on the overhead and on the chart paper grid.

Lead a discussion that compares graphing linear relations using intercepts and quadratic relations using intercepts. Discuss how factoring provides an efficient method for making a sketch of the graph, recognizing that it is a sketch only.

Differentiated
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Complete the practice questions.

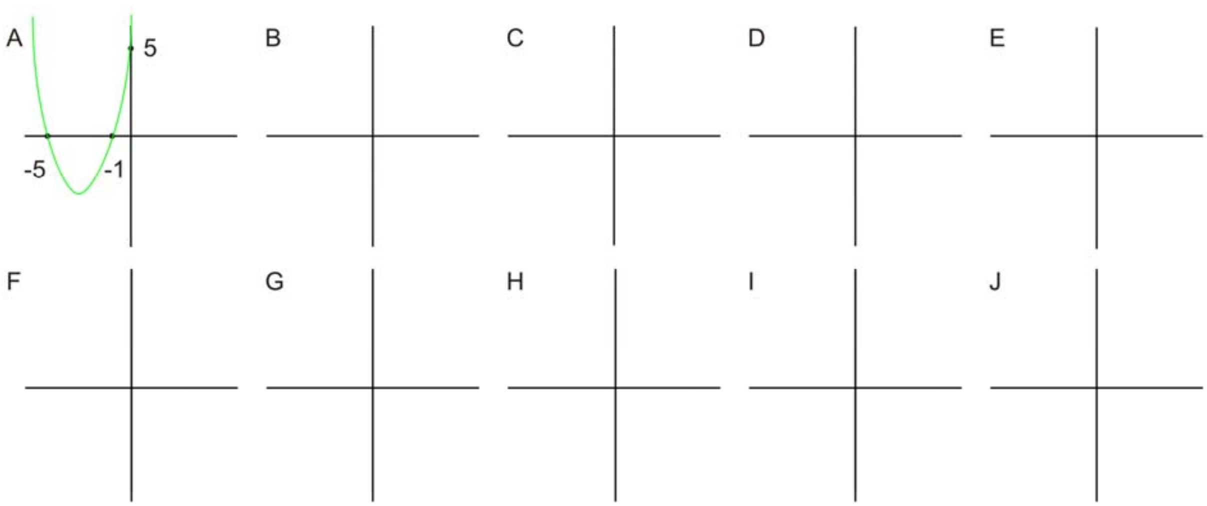
Provide students with appropriate practice questions.

7.6.1: Use Intercepts to Graph It!

Given the standard form of the quadratic relation, identify the value of the sum and product needed to factor. Express the relation in factored form, identify the *x*-intercepts and *y*-intercept, and use these results to make a sketch of each parabola.

	Standard Form	Product and Sum	Pair of Numbers	Factored Form	*x*-intercepts	*y*-intercept
A	$y = x^2 + 6x + 5$	r × s = 5 r + s = 6	$r = 1$, $s = 5$	$y = (x+1)(x+5)$	–1 and –5	5
B	$y = x^2 - 4x - 5$	r × s = –5 r + s = –4				
C	$y = x^2 + 4x - 5$	r × s = –5 r + s = 4				
D	$y = x^2 - 6x + 5$	r × s = 5 r + s = –6				
E	$y = x^2 + 7x + 6$	r × s = 6 r + s = 7				
F	$y = x^2 - 6x + 9$	$r \times s =$ $r + s =$				
G	$y = x^2 - x - 6$	$r \times s =$ $r + s =$				
H	$y = x^2 + 13x + 12$	$r \times s =$ $r + s =$				
I	$y = x^2 - 4x - 12$	$r \times s =$ $r + s =$				
J	$y = x^2 + x - 12$	$r \times s =$ $r + s =$				

Sketch of the relation



7.6.1: Use Intercepts to Graph It! (Answers)

	Standard Form	Product and Sum	Pair of Numbers	Factored Form	*x*-intercepts	*y*-intercept
A	$y = x^2 + 6x + 5$	$r \times s = 5$ $r + s = 6$	$r = 1$ $s = 5$	$y = (x+1)(x+5)$	–1 and –5	5
B	$y = x^2 - 4x - 5$	$r \times s = -5$ $r + s = -4$	$r = 1$ $s = -5$	$y = (x+1)(x-5)$	–1 and 5	–5
C	$y = x^2 + 4x - 5$	$r \times s = -5$ $r + s = 4$	$r = -1$ $s = 5$	$y = (x-1)(x+5)$	1 and –5	–5
D	$y = x^2 - 6x + 5$	$r \times s = 5$ $r + s = -6$	$r = -1$ $s = -5$	$y = (x-1)(x-5)$	1 and 5	5
E	$y = x^2 + 7x + 6$	$r \times s = 6$ $r + s = 7$	$r = 1$ $s = 6$	$y = (x+1)(x+6)$	–1 and –6	6
F	$y = x^2 - 6x + 9$	$r \times s = 9$ $r + s = -6$	$r = -3$ $s = -3$	$y = (x-3)(x-3)$	3	9
G	$y = x^2 - x - 6$	$r \times s = -6$ $r + s = -1$	$r = 2$ $s = -3$	$y = (x+2)(x-3)$	–2 and 3	–6
H	$y = x^2 + 13x + 12$	$r \times s = 12$ $r + s = 13$	$r = 1$ $s = 12$	$y = (x+1)(x+12)$	–1 and –12	12
I	$y = x^2 - 4x - 12$	$r \times s = -12$ $r + s = -4$	$r = -6$ $s = 2$	$y = (x-6)(x+2)$	6 and –2	–12
J	$y = x^2 + x - 12$	$r \times s = -12$ $r + s = 1$	$r = -3$ $s = 4$	$y = (x-3)(x+4)$	3 and –4	–12

75 min

Math Learning Goals

- Determine the connections between the factored form and the x-intercepts of the quadratic relations $y = x^2 + bx$.

Materials

- graphing calculators
- algebra tiles
- overhead algebra tiles
- BLM 7.7.1, 7.7.2, 7.7.3

Assessment Opportunities

Minds On…

Whole Class → Discussion

Use the context of kicking a soccer ball. Depending on the kick, the ball may travel different heights and distances. Lead a discussion as to how to draw a height vs. horizontal distance graph for three scenarios in which there are different height and distance measures.

Ask:

- If you were to represent this scenario graphically:
  - what are appropriate labels for the axes? [height above ground in metres; horizontal distance from the kicker's toe in metres]
  - where would the point be that represents the kick?
  - what would one of the x-intercepts of the graph be?
  - what might the general equation of the graph look like?

Action!

Pairs → Group Investigation

Students work through the graphing calculator investigation (BLM 7.7.1).

Provide each pair with one of the following equations: $y = x^2 + 4x$, $y = x^2 + 6x$, $y = x^2 – 2x$, $y = x^2 – 5x$. Pairs of students complete questions 1–3 on BLM 7.7.1.

Form groups of four where each student in the group has a different equation. Students complete questions 4–9 (BLM 7.7.1). They compare what is the same about their graphs and what is different and comment on the x-intercept, y-intercept, and the general shape. Lead a discussion about the algebraic representations, e.g., $y = x(x + 4)$, $y = (x – 0)(x + 4)$ and $y = (x + 4)(x – 0)$ being equivalent ways of expressing the equation $y = x^2 + 4x$ in factored form.

Answers
Same: zero as a y-intercept, zero as one x-intercept, shape, and size, opening up

Different: the other x-intercept

Use algebra tiles to demonstrate common factoring of $y = x^2 + 6x$, then do Example 1 and 2 of BLM 7.7.2. Students complete BLM 7.7.2. (See Teacher BLM 7.7.3.)

Mathematical Process/Using Tools/Observation/Anecdotal Note: Assess students' use of graphing calculators and algebra tiles to identify the factors.

Whole Class → Demonstration

Relate the algebra tile method of factoring to the algebraic method. Demonstrate the algebraic method using several examples, including some with negatives.

Consolidate Debrief

Individual → Journal

Students describe the three methods of common factoring that were investigated in the lesson, and describe the connections of the factors to the x-intercepts of the corresponding relation.

Home Activity or Further Classroom Consolidation

Concept Practice

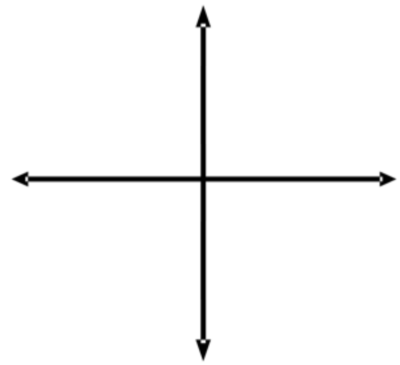
Complete the practice questions.

Provide students with appropriate practice questions.

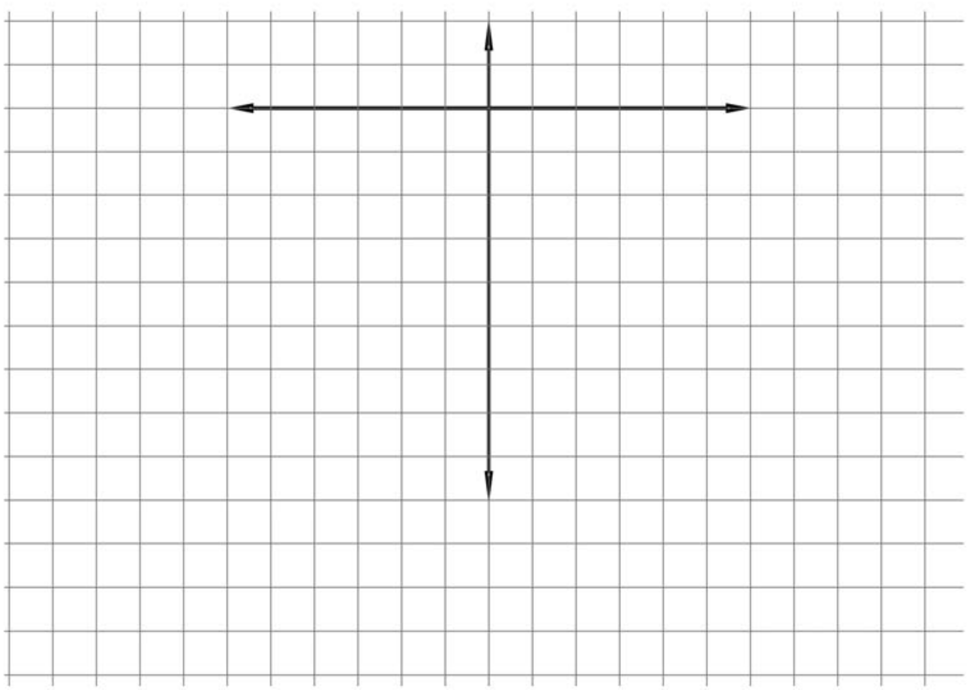
7.7.1: Investigate Relations of the Form $y = ax^2 + b$

Name:

1. Obtain a graphing calculator and equation from your teacher.
2. Type in the equation (using the **Y =** button on your calculator). Key in **zoom 6** to get the max and min from –10 to 10 on your window.
3. Fill in the table.

Your equation	A sketch of your quadratic equation (including the *x*- and *y*-intercepts)
$y =$ Coordinates of *x*-intercepts (______, ______) and (______, ______) and the coordinate of the *y*-intercept (______, ______)	

4. In your group, sketch all four graphs.



5. Identify what is the same and what is different in these four graphs.

7.7.1: Investigate Relations of the Form $y = ax^2 + b$ (continued)

6. Fill in the following table.

Standard form	From the graph, identify the x-intercepts r and s	Write each equation in factored form $y = (x - r)(x - s)$
$y = x^2 + 4x$	r = _____ s = _____	
$y = x^2 + 6x$	r = _____ s = _____	
$y = x^2 - 2x$	r = _____ s = _____	
$y = x^2 - 5x$	r = _____ s = _____	

7. Clear the y = screen on one of the four calculators, enter the factored form of the four equations, and graph. Are these graphs the same as the ones in your sketch?
If yes, continue to question 8. If no, revise and check. Ask your teacher for assistance, if needed.

8. Can the equations in the third column of your table be simplified? Explain.

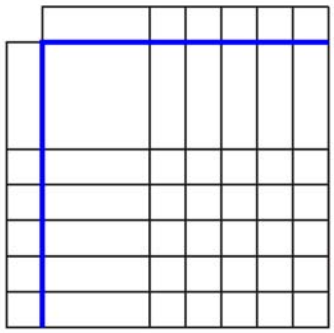
9. Record the simplified versions of your relation in factored form.

Standard Form	Factored Form
$y = x^2 + 4x$	
$y = x^2 + 6x$	
$y = x^2 - 2x$	
$y = x^2 - 5x$	

7.7.2: Factoring $x^2 + bx$

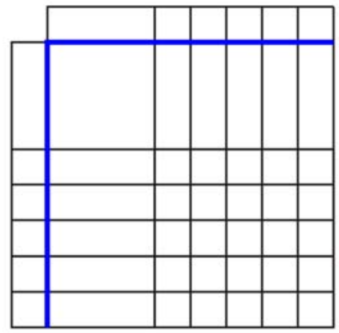
Consider the outer portion of the algebra tile representation as the length and width of a room. The rectangle is the carpet. Colour in as many cells as required for each example to form a rectangle. To factor, form a rectangle using the tiles, then determine the length and width of the room.

Example 1: $x^2 + 2x$



factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + 2x$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

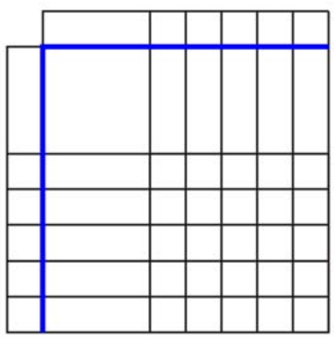
Example 2: $x^2 + 3x$



factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + 3x$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

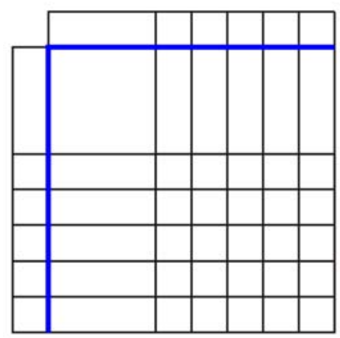
Use your algebra tiles to factor the following:

1. $x^2 + 4x$



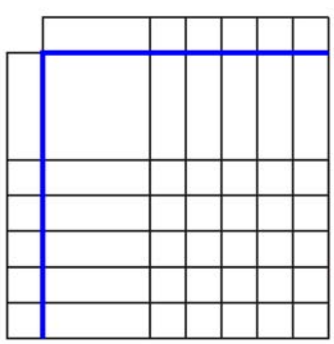
factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + 4x$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

2. $x^2 + 1x$



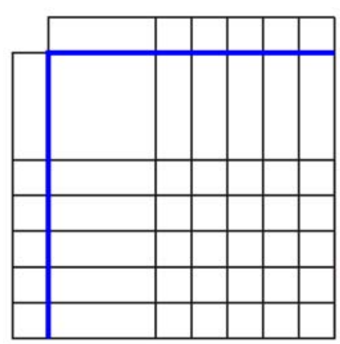
factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + 1x$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

3. $x^2 + 5x$



factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + 5y$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

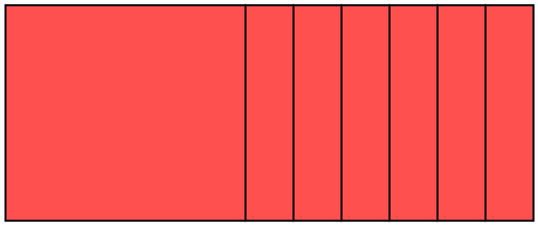
4. $x^2 + x$



factored form (______) (______)
the coordinates of the x-intercepts of $y = x^2 + x$
(______, ______) and (______, ______)
the coordinate of the y-intercept (______, ______)

7.7.3: Two Methods for Factoring $x^2 + bx$ (Teacher)

1. **Algebra Tile Model for Common Factoring**
   Use overhead tiles to demonstrate the concept of common factoring for the example $x^2 + 6x$.

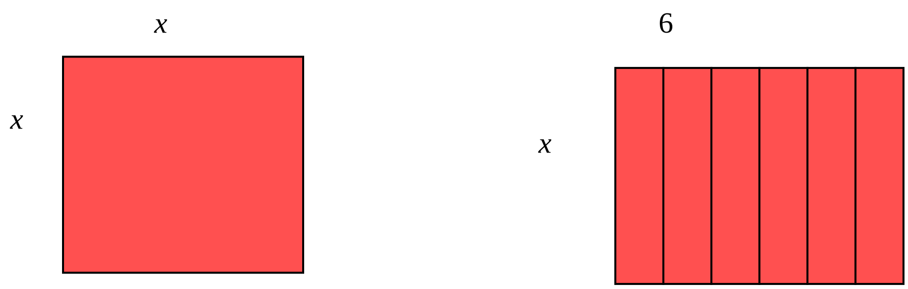


Explanation

This is the only rectangle that can be formed with algebra tiles. Factoring requires identifying the length and the width that results in this rectangle's area. In this case the dimensions are (x) and (x + 6).

Alternative Explanation

Separate the x^2 tile from the x tiles to show the length and width of each part of the expression.



In the square, the area is given by x times x. In the rectangle, the area is given by x times 6. The total area is given by $(x)(x) + (x)(6)$, we say that x is a common factor of each term.

Explain

x multiplies x, and x multiplies 6, thus x multiplies x and 6; that is, x multiplies $(x + 6)$, or $x\,(x + 6)$

Repeat the process with $x^2 + 3x$.

2. **Algebraic Model for Common Factoring**
   Use the principle of what is common to each term to factor the right-hand side of each equation. Lead the class through algebraic thinking in factoring the following examples:

$x^2 - 1x$	$x^2 + 5x$	$x^2 - 2x$
$= x\,(x - 1)$	$= x\,(x + 5)$	$= x\,(x - 2)$

Math Learning Goals

- Investigate the method of factoring a difference of two squares using patterning strategies and diagrams.
- Use the technique of factoring the difference of two squares to determine the x-intercepts of a quadratic relation.

Materials

- graphing calculators
- BLM 7.8.1

75 min

Assessment Opportunities

Minds On…

Individual → Reflection

Students reflect on the connection between $x^2 - 36$, $y = x^2 - 36$, and the corresponding graph. They fold a paper into thirds and write the headings "**I Think, I Wonder, I Know**" in the columns. Students complete the first and second columns, and share their reflection with a partner.

Math Process/Communicating/Observation/Anecdotal Note: Assess students' use of mathematical language related to quadratic relations.

Action!

Pairs → Investigation

Students use a graphing calculator to identify the intercepts of quadratic relations of the form $y = x^2 - a^2$ and connect the x-intercepts to the factors (BLM 7.8.1).

Circulate to clear any misconceptions and to guide pairs, as needed.

Whole Class → Guided Instruction

Activate students' prior knowledge by factoring the relation $y = x^2 + 7x + 12$.

$3 \times 4 = 12$, $3 + 4 = 7$. Therefore $y = x^2 + 7x + 12$ can be expressed in factored form as follows: $y = (x + 3)(x + 4)$

Ask how $y = x^2 - 49$ could be written as a trinomial. [Answer: $y = x^2 + 0x - 49$]

Model the process: $(+7)(-7) = -49$ and $(+7) + (-7) = 0$

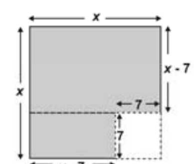
Therefore, $y = x^2 - 49$ can be expressed in factored form as $y = (x + 7)(x - 7)$.

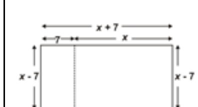
Reinforce the fact that the bx term is $0x$ and thus is not written in the expression. (Zero times x is zero.)

Explain why this type of quadratic is called a "difference of perfect squares," illustrating both algebraically and pictorially.

Students practise solving problems involving factoring a difference of squares.

The shaded area is $x^2 - 49$. You can see the two squares and the shaded area is the difference between their areas. When we factor, we want to find length and width of the rectangle having this area.





The length is $x + 7$ and width is $x - 7$.

Consolidate Debrief

Whole Class → Discussion

Students complete the third column in their "**I Think, I Wonder, I Know**" chart.

Review factoring a difference of squares and its connection to the graph, as needed.

Practice

Home Activity or Further Classroom Consolidation

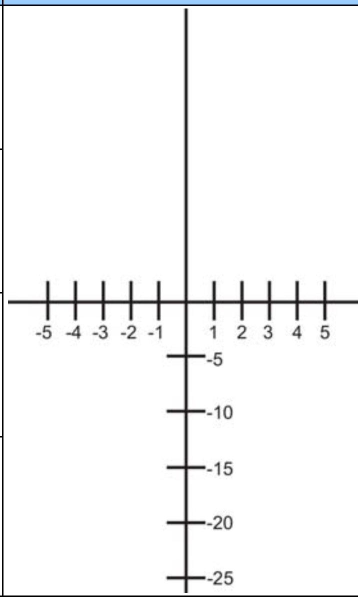
Practise factoring and connecting the factors to the graph.

7.8.1: Graphing Relationships of the Form $y = x^2 - a^2$

Name:

1. Working with a partner and one graphing calculator, set your Window: Xmin = –10; Xmax = 10; Xscl = 1; Ymin = –36; Ymax = 10; Yscl = 1; Xres = 1.

   Complete the following table.

	Relation in standard form	Sketch each graph. Label as A, B, C, D, or use different colours.	y-intercept	x-intercept (r)	x-intercept (s)	Relation in factored form $y = (x - r)(x - s)$
A	$y = x^2 - 4$					
B	$y = x^2 - 9$					
C	$y = x^2 - 16$					
D	$y = x^2 - 25$					

Consider your results from question 1 and answer the following questions.

2. What is the same about the relations?

3. What is the same about the graphs?

4. What is the same about the vertex of each graph?

5. What do you notice about the r and s values of each relation?

6. Solve this puzzle. How can you find the y-intercept and the x-intercepts of the graph of a quadratic relation of the form $y = x^2 - a^2$?