## Unit 7
### Quadratic Relations of the Form \( y = ax^2 + bx + c \)

#### Lesson Outline

**BIG PICTURE**

Students will:
- manipulate algebraic expressions, as needed to understand quadratic relations;
- identify characteristics of quadratic relations;
- solve problems by interpreting graphs of quadratic relations.

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title</th>
<th>Math Learning Goals</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The x Tiles</td>
<td>• Connect the algebraic representations to the graphical representations of quadratics of the forms ( y = x^2 + bx + c ) and ( y = (x - r)(x - s) ).&lt;br&gt;• Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using algebra tiles.</td>
<td>QR1.01, QR2.04, CGE 4b, 5a</td>
</tr>
<tr>
<td>2</td>
<td>Multiply a Binomial by a Binomial Using a Variety of Methods</td>
<td>• Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using the chart method, and the distributive property.</td>
<td>QR1.01, QR2.04, CGE 5a, 5b</td>
</tr>
<tr>
<td>3</td>
<td>Find the y-Intercept of Quadratic Equations</td>
<td>• Determine the connections between the ( y )-intercept in the graph of a quadratic relation and the ( c ) in the algebraic form ( y = x^2 + bx + c ), using technology.</td>
<td>QR1.01, QR2.04, CGE 3c, 5a</td>
</tr>
<tr>
<td>4</td>
<td>Finding ( x )-Intercepts and Introduction to Factoring Quadratic Equations</td>
<td>• Determine the connections between the factored form of a quadratic relation and the ( x )-intercepts of its graph.&lt;br&gt;• Factor simple trinomials of the form ( x^2 + bx + c ), using a variety of tools, e.g., algebra tiles, paper and pencil, and patterning strategies.</td>
<td>QR1.03, QR2.04, CGE 3c, 7b</td>
</tr>
<tr>
<td>5</td>
<td>Factoring Quadratic Relations of the Form ( y = ax^2 + bx + c ), Where ( a ) Is a Common Factor</td>
<td>• Determine the connections between the factored form and the ( x )-intercepts of a quadratic relation.&lt;br&gt;• Factor binomials and trinomials involving one variable up to degree two, by determining a common factor, using algebra tiles.</td>
<td>QR1.02, QR2.04, CGE 5b</td>
</tr>
<tr>
<td>6</td>
<td>Use Intercepts to Graph Quadratic Equations</td>
<td>• Consolidate factoring.&lt;br&gt;• Connect factors to the ( x )-intercepts of the graph of the quadratic relation.&lt;br&gt;• Graph quadratic relations, using intercepts.</td>
<td>QR1.02, QR1.03, QR2.04, CGE 3c</td>
</tr>
<tr>
<td>7</td>
<td>We Have a Lot in Common</td>
<td>• Determine the connections between the factored form and the ( x )-intercepts of the quadratic relation ( y = x^2 + bx ).</td>
<td>QR1.03, QR2.04, CGE 3c, 5b</td>
</tr>
<tr>
<td>8</td>
<td>What’s the Difference?</td>
<td>• Investigate the method of factoring a difference of two squares using patterning strategies and diagrams.&lt;br&gt;• Use the technique of factoring the difference of two squares to determine the ( x )-intercepts of a quadratic relation.</td>
<td>QR1.04, QR2.04, CGE 5b, 5e</td>
</tr>
<tr>
<td>9</td>
<td>Quick Sketches of Parabolas</td>
<td>• Use factoring techniques to find zeros and use understanding of the connection between the equation and the ( y )-intercept and symmetry to make “quick” sketches of the related parabola, given ( y = x^2 + bx + c ).</td>
<td>QR1.02, QR1.03, QR1.04, QR2.04, CGE 5g</td>
</tr>
<tr>
<td>Day</td>
<td>Lesson Title</td>
<td>Math Learning Goals</td>
<td>Expectations</td>
</tr>
<tr>
<td>------</td>
<td>---------------------------------------------------</td>
<td>------------------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>10,</td>
<td>Solve Problems Involving Quadratic Relations</td>
<td>• Solve problems involving quadratic relations by interpreting a given graph or a</td>
<td>QR3.01, QR3.02</td>
</tr>
<tr>
<td>11</td>
<td><em>(lessons not included)</em></td>
<td>graph generated with technology from its equation.</td>
<td>CGE 5g</td>
</tr>
<tr>
<td>12</td>
<td>Summative Assessment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Math Learning Goals
- Connect the algebraic representations to the graphical representations of quadratics of the forms $y = x^2 + bx + c$ and $y = (x - r)(x - s)$.
- Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using algebra tiles.

Materials
- graphing calculators
- algebra tiles
- BLM 7.1.1, 7.1.2, 7.1.3

Assessment Opportunities
- Minds On… Whole Class
  - Demonstration
  Toss a beach ball to a student. Ask the class to describe the path of the ball. Sketch the path of the ball on a graph with the independent axis representing horizontal distance from the teacher and the dependent axis representing height of the ball.
  Draw another graph that represents the height of the ball over time (horizontal axis represents time, vertical axis represents the distance from the ground).
  Lead a discussion that draws out the characteristics of the resulting parabola (symmetry, axis of symmetry, vertex, zeros, x-intercept, y-intercept).

- Whole Class → Gallery Tour
  Post students’ work: the two forms of the algebraic equation, a sketch of the graph, and a table of values. Students view the work of the other pairs.
  Debrief:
  - What did you notice about the graphs?
  - What did you notice about the tables of values?
  - What can you infer about the two forms of the equations?

- Whole Class → Modelling
  - How do we get from one form to the other?
  - What is the operation between the two sets of brackets?
  - Demonstrate connection to multiplication of $20 \times 12$, using base-ten materials.

  Review algebra tiles: “1” tile (unit tile), $x$ tile, and $x^2$ tile.

  Using overhead algebra tiles, model the method of multiplying the factors to generate an area which represents the expanded form of the expression, e.g., $x^2 + 7x + 10$, $x^2 + 6x + 8$, $x^2 + 6x + 5$, $x^2 + 5x + 4$, using algebra tiles. Show the connection to multiplication of $20 \times 12$, using base-ten materials.

- Mathematical Process/Using Tools/Observation/Mental Note: Observe students’ use of a graphing calculator to graph quadratic relations.

- Individual → Practice
  Using algebra tiles, students multiply and simplify three examples with positive terms only.
  $y = (x + 4)(x + 3)$  $y = (x + 2)(x + 5)$  $y = (x + 3)(x + 2)$
  They make a sketch of the algebra tile solution.
  Students check their answers on the graphing calculator by comparing the graphs of the two forms.

- Home Activity or Further Classroom Consolidation
  Complete the practice questions. Use worksheet 7.1.3 to show your solutions.
## 7.1.1: Quadratic Equations (Teacher)

Cut out each pair of quadratic equations and distribute to the students.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 + 3x + 2 )</td>
<td>( y = (x + 1)(x + 2) )</td>
</tr>
<tr>
<td>( y = x^2 + 2x - 3 )</td>
<td>( y = (x - 1)(x + 3) )</td>
</tr>
<tr>
<td>( y = x^2 - 1x - 6 )</td>
<td>( y = (x + 2)(x - 3) )</td>
</tr>
<tr>
<td>( y = x^2 + x - 6 )</td>
<td>( y = (x - 2)(x + 3) )</td>
</tr>
<tr>
<td>( y = x^2 + 5x + 4 )</td>
<td>( y = (x + 1)(x + 4) )</td>
</tr>
<tr>
<td>( y = x^2 + 2x - 3 )</td>
<td>( y = (x + 3)(x - 1) )</td>
</tr>
<tr>
<td>( y = x^2 - x - 6 )</td>
<td>( y = (x - 3)(x + 2) )</td>
</tr>
<tr>
<td>( y = x^2 + 6x + 5 )</td>
<td>( y = (x + 1)(x + 5) )</td>
</tr>
<tr>
<td>( y = x^2 - 4x - 5 )</td>
<td>( y = (x + 1)(x - 5) )</td>
</tr>
<tr>
<td>( y = x^2 - 4x - 12 )</td>
<td>( y = (x + 2)(x - 6) )</td>
</tr>
<tr>
<td>( y = x^2 + 6x + 8 )</td>
<td>( y = (x + 2)(x + 4) )</td>
</tr>
<tr>
<td>( y = x^2 - 5x + 6 )</td>
<td>( y = (x - 3)(x - 2) )</td>
</tr>
<tr>
<td>( y = x^2 - 2x - 3 )</td>
<td>( y = (x - 3)(x + 1) )</td>
</tr>
<tr>
<td>( y = x^2 + 7x + 12 )</td>
<td>( y = (x + 3)(x + 4) )</td>
</tr>
<tr>
<td>( y = x^2 + 7x + 10 )</td>
<td>( y = (x + 2)(x + 5) )</td>
</tr>
<tr>
<td>( y = x^2 - 7x + 10 )</td>
<td>( y = (x - 2)(x - 5) )</td>
</tr>
</tbody>
</table>
7.1.2: Graphing Quadratic Equations

Name:

1. Obtain a pair of equations from your teacher.
2. Press the **Zoom** button and press 6 (for ZStandard) to set the window to make the max and min on both axes go from –10 to 10.
3. Press the $y=$ button and key in your two equations into $Y_1$ and $Y_2$.
4. To change the graph of $Y_2$ to “animation”: Move the cursor to the left of $Y_2$. Press **Enter** four times to toggle through different graph styles available. You should see _____

5. Press **Graph**. First the $Y_1$ quadratic will appear, then the $Y_2$ quadratic will appear and be traced by an open circle.

6. Complete the three columns of the table below.

<table>
<thead>
<tr>
<th>Our Two Equations</th>
<th>What They Look Like</th>
<th>What We Think It Means</th>
</tr>
</thead>
</table>

7. Press 2$^{nd}$ **Graph** so that you can look at the tables of values for the two curves. Discuss what you see and complete the table.

<table>
<thead>
<tr>
<th>What We Noticed About the Table of Values</th>
<th>What We Think It Means</th>
</tr>
</thead>
</table>
7.1.3: Algebra Tile Template

1. \( y = (__) (__) = \) ______________ 

2. \( y = (__) (__) = \) ______________ 

3. \( y = (__) (__) = \) ______________ 

4. \( y = (__) (__) = \) ______________ 

5. \( y = (__) (__) = \) ______________ 

6. \( y = (__) (__) = \) ______________ 

TIPS4RM: Grade 10 Applied: Unit 7 – Quadratic Relations of the Form \( y = ax^2 + bx + c \)
Unit 7: Day 2: Multiply a Binomial by a Binomial
Using a Variety of Methods

Math Learning Goals
- Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using the chart method, and the distributive property.

Materials
- BLM 7.2.1, 7.2.2
- algebra tiles
- graphing calculators

Assessment Opportunities
- Representation with algebra tiles is best for expressions with positive terms only.
- This representation can be used for binomials with positive and negative terms.

Whole Class → Discussion
Students expand \((x + 1)(x + 1)\) with algebra tiles.
Discuss the meaning of repeated multiplication, i.e., \((x + 1)(x + 1) = (x + 1)^2\).

Individual → Practice
Students practise multiplication of a binomial with positives only, using BLM 7.2.1 Part A and algebra tiles.

Whole Class → Guided Instruction
Model the different combinations of multiplication, i.e., monomial × monomial and monomial × binomial with positive and negative terms, using the chart method.
Connect the use of algebra tiles to the “chart method.”
Model the chart method for multiplication of a binomial × binomial. Students practise this method, BLM 7.2.1 Part B.
Use algebra tiles to show each of these and recall the distributive property.
\[
\begin{align*}
   x(x + 3) \\
   2(x + 3)
\end{align*}
\]
Multiply \((x+2)(x+3)\) using tiles, and show “double distributive,” (the algebraic model).
Model the steps involved in the algebraic manipulation of multiplying a binomial by a binomial. Connect to distributive property by calling it “double distributive.”

Using the Distributive Property
Lead students to understand the connection between the chart method and the algebraic method of multiplying binomials.
\[
(\begin{array}{cc} x & +4 \\ +3 & x \end{array}) \quad (\begin{array}{cc} x & x^2 \\ 2x & 6 \end{array})
\]
Students practise this method, using BLM 7.2.1 Part C.

Learning Skills/Work Habits/Observation/Checklist: Assess how well students stay on task and complete assigned questions.

Consolidate Debrief

Individual → Journal
In your journal, write a note to a friend who missed today’s class. Summarize the three methods of multiplying binomials that you worked with. Use words, diagrams, and symbols in your explanation.

Home Activity or Further Classroom Consolidation
Solve the problems by multiplying the binomials.


7.2.1: Multiply a Binomial by a Binomial

Name:

Part A
Use algebra tiles to multiply binomials and simplify the following:
1. \( y = (x + 1)(x + 3) = \) _________________
2. \( y = (x + 2)(x + 3) = \) _________________

Part B
Use the chart method to multiply and simplify the following:
1. \( y = (x + 1)(x + 3) = \) _________________
2. \( y = (x + 2)(x + 3) = \) _________________
3. \( y = (x + 2)(x - 1) = \) _________________
4. \( y = (x - 2)(x + 3) = \) _________________
5. \( y = (x - 1)(x - 1) = \) _________________
6. \( y = (x - 1)(x - 2) = \) _________________
### 7.2.1: Multiply a Binomial by a Binomial (continued)

#### Part C
Multiply and simplify the two binomials, using the chart method and the distributive property.

1. \((x + 4)(x - 3)\)
   
<table>
<thead>
<tr>
<th></th>
<th>+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

2. \((x - 3)(x - 3)\)
   
<table>
<thead>
<tr>
<th></th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

3. \((x + 2)^2\)
   
<table>
<thead>
<tr>
<th></th>
<th>+2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>+2</td>
<td></td>
</tr>
</tbody>
</table>

4. \((x + 2)(x - 1)\)
   
<table>
<thead>
<tr>
<th></th>
<th>+2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

5. \((x - 2)(x + 1)\)
   
<table>
<thead>
<tr>
<th></th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>+1</td>
<td></td>
</tr>
</tbody>
</table>

6. \((x - 1)^2\)
   
<table>
<thead>
<tr>
<th></th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

7. \((x - 1)(x - 2)\)
   
<table>
<thead>
<tr>
<th></th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

8. \((x - 3)(x - 4)\)
   
<table>
<thead>
<tr>
<th></th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>
7.2.1: Multiply a Binomial by a Binomial (continued)

Answers to Part B

1. \( y = x^2 + 4x + 3 \)
   \[
   \begin{array}{ccc}
   \hline
   x & 1 \\
   \hline
   x & x^2 & x \\
   3 & 3x & 3 \\
   \hline
   \end{array}
   \]

2. \( y = x^2 + 5x + 6 \)
   \[
   \begin{array}{ccc}
   \hline
   x & 2 \\
   \hline
   x & x^2 & 2x \\
   3 & 3x & 6 \\
   \hline
   \end{array}
   \]

3. \( y = x^2 + x - 2 \)
   \[
   \begin{array}{ccc}
   \hline
   x & 2 \\
   \hline
   x & x^2 & 2x \\
   -1 & -x & -2 \\
   \hline
   \end{array}
   \]

4. \( y = x^2 + x - 6 \)
   \[
   \begin{array}{ccc}
   \hline
   x & -2 \\
   \hline
   x & x^2 & -2x \\
   3 & 3x & -6 \\
   \hline
   \end{array}
   \]

5. \( y = x^2 - 2x + 1 \)
   \[
   \begin{array}{ccc}
   \hline
   x & -1 \\
   \hline
   x & x^2 & -x \\
   -1 & -x & 1 \\
   \hline
   \end{array}
   \]

6. \( y = x^2 - 3x + 2 \)
   \[
   \begin{array}{ccc}
   \hline
   x & -1 \\
   \hline
   x & x^2 & -x \\
   -2 & -2x & 2 \\
   \hline
   \end{array}
   \]

Answers to Part C

1. \( x^2 + x - 12 \)

2. \( x^2 - 6x + 9 \)

3. \( x^2 + 4x + 4 \)

4. \( x^2 + x - 2 \)

5. \( x^2 - x - 2 \)

6. \( x^2 - 2x + 1 \)

7. \( x^2 - 3x + 2 \)

8. \( x^2 - 7x + 12 \)
7.2.2: Chart Template for Distributive Property

1. \( y = (   )(   ) = \) ____________  
2. \( y = (   )(   ) = \) ____________

3. \( y = (   )(   ) = \) ____________  
4. \( y = (   )(   ) = \) ____________

5. \( y = (   )(   ) = \) ____________  
6. \( y = (   )(   ) = \) ____________

7. \( y = (   )(   ) = \) ____________  
8. \( y = (   )(   ) = \) ____________
**Math Learning Goals**
- Determine the connections between the y-intercept in the graph of a quadratic relation and the \(c\) in the algebraic form \(y = ax^2 + bx + c\), using technology.

**Materials**
- graphing calculators
- BLM 7.3.1, 7.3.2

**75 min**

**Minds On… Whole Class ➔ Modelling**
Pair students, and assign a person A and person B. Ask: What is meant by the y-intercept of a graph? A answers B. Ask: When we studied linear equations we looked at \(y = 3x + 2\). What did the 2 represent? B answers A.

Explain the value of knowing the y-intercept for graphing.

*Mathematical Processes/Communicating/Observation/Mental Note:*
Circulate as students work in pairs to note their use of mathematical language.

**Action! Pairs ➔ Guided Investigation**
Pairs complete BLM 7.3.1 to identify the y-intercept of quadratic relations given in standard form and factored form.

**Consolidate Debrief**

**Whole Class ➔ Summarizing**
Students summarize how the y-intercept of the graph relates to the equation of linear relations and quadratic relations, including why it is valuable to know the y-intercept for graphing.

Connect to the understanding that the y-intercept occurs when \(x = 0\) for any relation, referring to algebraic and graphical forms during the discussions.

**Home Activity or Further Classroom Consolidation**
Complete worksheet 7.3.2.

*Practice*

Provide algebra tiles template 7.1.3 or chart template 7.2.2, as needed.
7.3.1: Finding the \( y \)-Intercept of a Quadratic Equation

Name:

1. Use the graphing calculator to find the \( y \)-intercept for each of the equations:
   Note any patterns you see.

   \[
   \begin{array}{|c|c|}
   \hline
   \text{Equation} & \text{\( y \)-intercept} \\
   \hline
   y = x^2 - x - 2 & \\
   y = x^2 + 2x - 8 & \\
   y = x^2 - x + 6 & \\
   y = (x - 1)(x - 2) & \\
   y = (x + 4)(x + 3) & \\
   y = (x + 3)^2 & \\
   \hline
   \end{array}
   \]

2. How can you determine the \( y \)-intercept by looking at a quadratic equation?

3. Which form of the quadratic equation is easiest to use to determine the \( y \)-intercept?
   Explain your choice.

4. Using your conclusion from question 2, state the \( y \)-intercept of each and check using a graphing calculator.

   \[
   \begin{array}{|c|c|c|}
   \hline
   \text{Equation} & \text{\( y \)-intercept} & \text{Does it check?} \\
   & & \text{Yes} \ | \ \text{No} \\
   \hline
   y = x^2 - 2x - 8 & & \\
   y = x^2 - x - 6 & & \\
   y = x^2 + 3x + 2 & & \\
   y = (x - 4)(x - 1) & & \\
   y = (x - 2)(x + 5) & & \\
   \hline
   \end{array}
   \]

5. Explain the connection between the \( y \)-intercept and the value of \( y \) when \( x = 0 \).
7.3.2: Quadratic Equations

Name:

1. Find the $y$-intercept for each of the following quadratic equations given in factored form. Write the equations in standard form. Show your work.

   a) $y = (x - 5)(x + 2)$

      standard form:

      $= \\

      y$-intercept:

   b) $y = (x + 4)(x - 3)$

      standard form:

      $= \\

      y$-intercept:

   c) $y = (x - 4)^2$

      standard form:

      $= \\

      y$-intercept:

   d) $y = (x + 5)^2$

      standard form:

      $= \\

      y$-intercept:

2. Find the $y$-intercept for each of the following quadratic equations:

   a) $y = (x + 4)(x + 2)$

      y$-intercept:

   b) $y = (x - 6)^2$

      y$-intercept:
Math Learning Goals
- Determine the connections between the factored form of a quadratic relation and the x-intercepts of its graph.
- Factor simple trinomials of the form \( x^2 + bx + c \), using a variety of tools, e.g., algebra tiles, paper and pencil, and patterning strategies.

Materials
- graphing calculators
- algebra tiles
- BLM 7.4.1, 7.4.2, 7.4.3

Assessment Opportunities
- Remind students of the symmetrical nature of a parabola.

Minds On… Whole Class ➔ Demonstration
Review how to find the y-intercept for a quadratic equation given the standard form or factored form.
Introduce x-intercepts for quadratic equations. Model how to make a sketch of the parabola given its y-intercept and two x-intercepts. Place a dot on the y-intercept of 4 and x-intercept of 4 and 1. Ask a student to sketch the graph. Repeat for y-intercept of +6 and x-intercept of –3 and –2.
Each student makes a sketch of the graph y-intercept –6 and x-intercept of –2 and 3. Check the answer.
Lead students to the understanding that knowing the y-intercept and both x-intercepts can greatly assist in graphing some parabolas of the form \( y = ax^2 + bx + c \).

Action! Pairs ➔ Investigation
Students use graphing calculators to investigate the x-intercepts (BLM 7.4.1).
Lead students to realize that equations in factored form help to find the x-intercepts without a calculator. Establish the need to change equations in standard form to factored form. Tell students that the process is called factoring.

Whole Class ➔ Demonstration
Model the use of algebra tiles and the rectangular area model to factor trinomials, using expression \( x^2 + 4x + 3 \).
Ask: What are the length and width of the rectangle?
Point out that there are no other ways to make rectangles with this set of tiles.
Students should note that the dimensions of the rectangle are \((x + 1)\) and \((x + 3)\). These are called the factors. Identify the x-intercepts of the corresponding relation \( y = (x + 1)(x + 3) \).
Students use algebra tiles to factor quadratic expressions (BLM 7.4.2). Remind them that, in building the rectangle, the “lines” or “cracks” must line up.

Individual Practice
Students complete the chart and the questions on BLM 7.4.2. Take up as a class. Students complete BLM 7.4.3 Part A.

Curriculum Expectations/Oral Question/Anecdotal Note: Assess how students make connections between the graph and the quadratic equation.

Consolidate Debrief
Whole Class ➔ Summary
Complete BLM 7.4.3 Part B question 1, clarifying each aspect of the question.

Home Activity or Further Classroom Consolidation
Concept Practice
Complete worksheet 7.4.3 Part B.
7.4.1: Finding the \( x \)-Intercepts of a Quadratic Equation

Name:

To find the \( x \)-intercepts:
1. Enter the equation in \( Y_1 \). \( y = x^2 - x - 6 \)
2. Press ZOOM and 6 (Zstandard) to set the scale for your graph. The calculator will then show the parabola.
3. Press 2nd TRACE 1 to view the Calculate screen.

4. Select 2: ZERO. Your screen should be similar to the following screen.

5. You will be asked to enter a left bound. You can move the cursor to the left of one \( x \)-intercept (or just enter an \( x \) value that is to the left of the \( x \)-intercept). Press ENTER.

6. Repeat for the right bound, being sure that you are to the right of the same \( x \)-intercept.

7. The next screen will say guess. You can guess if you want but it is not necessary. Press ENTER. You will get one \( x \)-intercept.

8. Repeat steps 3 through 7 to get the other \( x \)-intercept.
7.4.1: Finding the x-Intercepts of a Quadratic Equation (continued)

1. Use the graphing calculator to find the x-intercepts for each of the following:

<table>
<thead>
<tr>
<th>Equation</th>
<th>First x-intercept</th>
<th>Second x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 - 4x - 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x^2 + 2x - 8 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x^2 - x + 6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = (x - 1)(x - 2) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = (x + 4)(x + 3) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = (x - 3)(x + 5) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Can you determine the x-intercepts by looking at a quadratic equation? Explain.

3. Which form of the quadratic equation did you find the easiest to use when determining the x-intercepts? Explain the connection between the factors and the x-intercepts.
7.4.2: Area with Algebra Tiles

Name:

Using algebra tiles create the rectangles for the following areas. Complete the following chart.

<table>
<thead>
<tr>
<th>Area of Rectangle</th>
<th>Number of $x^2$ Tiles</th>
<th>Number of $x$ Tiles</th>
<th>Number of Unit Tiles</th>
<th>Sketch of Rectangle</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 4x + 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 5x + 6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 6x + 8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 7x + 12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Find a relationship between the number of $x$ tiles and the numbers in the expressions for the length and width.

2. Find a relationship between the number of unit tiles and the numbers in the expressions for the length and width.

3. If the area of a rectangle is given by $x^2 + 8x + 15$, what expression will represent the length and the width?
7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic

Name:

**Part A**

For each of the following, shade in the appropriate rectangular area. Then shade in the tiles that represent the length and width for each of those areas. Use the length and width to represent and state the factors. State the $x$-intercepts. Check using a graphing calculator.

1. \( y = x^2 + 3x + 2 = (\quad)(\quad) \)

2. \( y = x^2 + 5x + 4 = (\quad)(\quad) \)

   x-intercepts _____, ______
   
   Check with calculator.

3. \( y = x^2 + 6x + 5 = (\quad)(\quad) \)

4. \( y = x^2 + 4x + 4 = (\quad)(\quad) \)

   x-intercepts _____, ______
   
   Check with calculator.
7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic (continued)

**Part B**
Using the diagrams in Part A, find the x- and y-intercepts for each quadratic relation. Use the information to make the sketch on the grid provided.

1. **standard form:**
   \[ y = x^2 + 3x + 2 \]

   **factored form:**

   **y-intercept:**

   **first x-intercept:**

   **second x-intercept:**

2. **standard form:**
   \[ y = x^2 + 5x + 4 \]

   **factored form:**

   **y-intercept:**

   **first x-intercept:**

   **second x-intercept:**
7.4.3: Factoring Using Algebra Tiles and Making Connections to the Graphic (continued)

3. standard form:  
\[ y = x^2 + 6x + 5 \]

factored form:

y-intercept:

first x-intercept:

second x-intercept:

4. standard form:  
\[ y = x^2 + 4x + 4 \]

factored form:

y-intercept:

first x-intercept:

second x-intercept:

5. In what way is the last example different from the others?
Math Learning Goals
- Determine the connections between the factored form and the x-intercepts of a quadratic relation.
- Factor binomials and trinomials involving one variable up to degree two, by determining a common factor using algebra tiles.

Materials
- algebra tiles
- graphing calculators
- BLM 7.5.1, 7.5.2, 7.5.3
- glue stick

Assessment Opportunities

Minds On… Four Corners
Review the forms of equations and representations of quadratics and label four stations: factored form, standard form, graphical form, and chart or algebra tiles representation. Randomly hand out the cards (BLM 7.5.1). Students move to the corner for the form represented on their card and check that everyone has the same form. They can help one another express their representation in the other forms and jot them on the back of their cards. They then “find” the other three people holding the related cards. Groups justify they all have different forms of representations of the same quadratic. Paste these on a single page and post them for a gallery walk.

Curriculum Expectations/Demonstration/Marking Scheme: Collect worksheets and assess students’ understanding of the concepts needed to complete the Home Activity from Day 4.

Action! Investigation
Students complete BLM 7.5.2.
Discuss the ideas investigated and extend to relations that include negative coefficients for b and c, e.g., \( y = x^2 - x - 6 \). Students should understand that \((r) \times (s) = c \) and \( r + s = b \), yielding factors \((x + r)(x + s)\) and x-intercepts of \(-r\) and \(-s\).

Students complete BLM 7.5.3 and share their solutions on the overhead.

Whole Class → Guided Instruction
Introduce common factoring by displaying overhead tiles for 3x + 3.
Demonstrate that this is 3 groups of \((x + 1)\), thus this can be expressed as 3(x + 1). The “common factor” is 3. Repeat for 2x + 4 and 5x – 10.
Display the overhead tiles needed to factor 2x^2 + 2x + 2.
Ask:
- Can these be placed into groups where each type of tile is equally represented in all groups? Why or why not?
- How many groups did you get? What are the contents of each group?
Show how the factored answer would be expressed.
Complete several more examples, such as: 3x^2 + 6x + 12, 2x^2 + 6x – 8, 2x^2 – 8x + 10, 4x^2 + 8x + 8.
For 4x^2 + 8x + 8, some students may think that 2 groups of \((2x^2 + 4x + 4)\) would be appropriate when 4 groups of \((x^2 + 2x + 2)\) is the most appropriate answer. Use this example to discuss the greatest common factor.

Consolidate Debrief

Individual → Journal
Students summarize how to factor a trinomial, including conditions that make it possible for a trinomial to be factored and how to determine a common factor.

Home Activity or Further Classroom Consolidation
Complete the practice questions.

Concept Practice
7.5.1: Four Corners

\[ y = x^2 + 4x + 3 \]

\[ y = (x + 1)(x + 3) \]

\[ y = x^2 + 3x + 2 \]
7.5.1: Four Corners (continued)

\[ y = (x + 2)(x + 1) \]

\[ y = x^2 + 5x + 6 \]

\[ y = (x + 3)(x + 2) \]
7.5.1: Four Corners (continued)

\[ y = x^2 + 6x + 5 \]

\[ y = (x + 5)(x + 1) \]

\[ y = x^2 + 5x + 4 \]
7.5.1: Four Corners (continued)

\[ y = (x + 1)(x + 4) \]

\[ y = x^2 + 1x - 6 \]

\[
\begin{array}{ccc}
  x & x^2 & +3x \\
-2 & -2x & -6 \\
\end{array}
\]

\[ y = (x + 3)(x - 2) \]
### 7.5.1: Four Corners (continued)

\[
y = (x - 1)(x - 3)
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(x^2)</td>
</tr>
<tr>
<td>-3</td>
<td>-3x</td>
</tr>
</tbody>
</table>

\[
y = (x - 3)(x - 4)
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(x^2)</td>
</tr>
<tr>
<td>-4</td>
<td>-4x</td>
</tr>
</tbody>
</table>

\[
y = x^2 - 4x + 3
\]
7.5.1: Four Corners (continued)

\[ y = x^2 - 7x + 12 \]
7.5.2: Factored Form and $x$-Intercepts

Name:

Use algebra tiles to find the length and width for each given area. Use the graphing calculator to find the $x$-intercepts of the corresponding quadratic relation. Graph both the area model and factored form of the quadratic relation to check that these are the same before finding the $x$-intercepts.

<table>
<thead>
<tr>
<th>Area</th>
<th>Length</th>
<th>Width</th>
<th>Factored Form</th>
<th>First $x$-intercept of corresponding relation</th>
<th>Second $x$-intercept of corresponding relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 4x + 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 5x + 6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 6x + 8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 7x + 12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What do you notice about the constant term in the length and width expressions and the coefficient of the $x$ term in the area expressions?

2. What do you notice about the constant term in the length and width expressions and the constant term in the area expressions?

3. If an area is expressed as $x^2 + 10x + 21$, what must be true of the constant terms in the length and width expressions?

4. If the standard form of a quadratic relation is $y = x^2 + bx + c$, and it has $x$-intercepts of $r$ and $s$, then the same relation would then be $y = (x - r)(x - s)$. How would you find the value of $r$ and $s$?
### BLM 7.5.3: Match It!

**Name:**

Match each pair of numbers on the left with the correct product and sum on the right.

<table>
<thead>
<tr>
<th></th>
<th>Pair of Numbers</th>
<th>Product and Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( r = -1, s = -5 )</td>
<td>a. ( r \times s = 5 ) ( r + s = 6 )</td>
</tr>
<tr>
<td>2.</td>
<td>( r = 1, s = 6 )</td>
<td>b. ( r \times s = -5 ) ( r + s = -4 )</td>
</tr>
<tr>
<td>3.</td>
<td>( r = 2, s = -3 )</td>
<td>c. ( r \times s = -5 ) ( r + s = 4 )</td>
</tr>
<tr>
<td>4.</td>
<td>( r = 1, s = -5 )</td>
<td>d. ( r \times s = 5 ) ( r + s = -6 )</td>
</tr>
<tr>
<td>5.</td>
<td>( r = -3, s = 4 )</td>
<td>e. ( r \times s = 6 ) ( r + s = 7 )</td>
</tr>
<tr>
<td>6.</td>
<td>( r = -1, s = 5 )</td>
<td>f. ( r \times s = 6 ) ( r + s = -7 )</td>
</tr>
<tr>
<td>7.</td>
<td>( r = -6, s = 2 )</td>
<td>g. ( r \times s = -6 ) ( r + s = -1 )</td>
</tr>
<tr>
<td>8.</td>
<td>( r = 1, s = 5 )</td>
<td>h. ( r \times s = 12 ) ( r + s = 13 )</td>
</tr>
<tr>
<td>9.</td>
<td>( r = -1, s = -6 )</td>
<td>i. ( r \times s = -12 ) ( r + s = -4 )</td>
</tr>
<tr>
<td>10.</td>
<td>( r = 1, s = 12 )</td>
<td>j. ( r \times s = -12 ) ( r + s = 1 )</td>
</tr>
</tbody>
</table>
Unit 7: Day 6: Use Intercepts to Graph Quadratic Relations

Math Learning Goals
- Consolidate factoring.
- Connect factors to the x-intercepts of the graph of the quadratic relation.
- Graph quadratic relations, using intercepts.

Materials
- BLM 7.6.1
- large chart (grid) paper

Assessment Opportunities
- Include some examples where the parabola is opening downward, and where there is a double root.
- Note the use of the word sketch. The graph is not intended to be exact, but to show the symmetry, the intercepts, and the general movement of the curve.
- Students can visualize and draw their solution in the air also.
- Some students may still need algebra tiles to factor.

Minds On...

Pairs ➔ Review
Students discuss their journal entry from Day 5 and refer to their answers on BLM 7.5.1 and 7.5.2.

Pairs factor the following quadratic expressions, discuss what is the same and different about each, and how the results would correspond to the x-intercepts of the corresponding relation:
\[ x^2 + 11x + 30, \quad x^2 - 11x + 30, \quad x^2 - x - 30, \quad x^2 + x - 30. \]

Curriculum Expectation/Observation/Checklist: Listen to students’ conversations to identify level of understanding of factoring trinomials and the connection between the factors and the x-intercepts.

Action!

Whole Class ➔ Demonstration
Using large chart (grid) paper, place a dot on the x-axis at –4 and 2, and a dot on the y-axis at –8. Point out the –4 and 2 are the x-intercepts and –8 is the y-intercept of a parabola.

Ask volunteers to sketch the parabola that goes through these points. Point out that, at this time, they are focusing on the symmetry of the parabola, and that the parabola intersects correctly on the y-axis, and the x-axis. Repeat this several times using other values for the intercepts.

Discuss question A on BLM 7.6.1 and do question B together. Refer to question F to be sure students understand what is different in this question.

Individual ➔ Practice
Students complete BLM 7.6.1.

Students who finish early can assist students who are having difficulty, and/or put an answer on the overhead and make a sketch of the relation on large chart (grid) paper.

Whole Class ➔ Discussion
Students check their answers using the response on the overhead and on the chart paper grid.

Lead a discussion that compares graphing linear relations using intercepts and quadratic relations using intercepts. Discuss how factoring provides an efficient method for making a sketch of the graph, recognizing that it is a sketch only.

Home Activity or Further Classroom Consolidation
Complete the practice questions.

Differentiated Exploration Reflection

Provide students with appropriate practice questions.
7.6.1: Use Intercepts to Graph It!

Given the standard form of the quadratic relation, identify the value of the sum and product needed to factor. Express the relation in factored form, identify the $x$-intercepts and $y$-intercept, and use these results to make a sketch of each parabola.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Product and Sum</th>
<th>Pair of Numbers</th>
<th>Factored Form</th>
<th>$x$-intercepts</th>
<th>$y$-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $y = x^2 + 6x + 5$</td>
<td>$r \times s = 5$  \ $r + s = 6$</td>
<td>$r = 1, s = 5$</td>
<td>$y = (x+1)(x+5)$</td>
<td>$-1$ and $-5$</td>
<td>$5$</td>
</tr>
<tr>
<td>B $y = x^2 - 4x - 5$</td>
<td>$r \times s = -5$ \ $r + s = -4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C $y = x^2 + 4x - 5$</td>
<td>$r \times s = -5$ \ $r + s = 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D $y = x^2 - 6x + 5$</td>
<td>$r \times s = 5$ \ $r + s = -6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E $y = x^2 + 7x + 6$</td>
<td>$r \times s = 6$ \ $r + s = 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F $y = x^2 - 6x + 9$</td>
<td>$r \times s = $ \ $r + s = $</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G $y = x^2 - x - 6$</td>
<td>$r \times s = $ \ $r + s = $</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H $y = x^2 + 13x + 12$</td>
<td>$r \times s = $ \ $r + s = $</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I $y = x^2 - 4x - 12$</td>
<td>$r \times s = $ \ $r + s = $</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J $y = x^2 + x - 12$</td>
<td>$r \times s = $ \ $r + s = $</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sketch of the relation
### 7.6.1: Use Intercepts to Graph It! (Answers)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Product and Sum</th>
<th>Pair of Numbers</th>
<th>Factored Form</th>
<th>x-intercepts</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $y = x^2 + 6x + 5$</td>
<td>$r \times s = 5$</td>
<td>$r = 1$ $s = 5$</td>
<td>$(x+1)(x+5)$</td>
<td>$-1$ and $-5$</td>
<td>$5$</td>
</tr>
<tr>
<td>B $y = x^2 - 4x - 5$</td>
<td>$r \times s = -5$</td>
<td>$r = 1$ $s = -5$</td>
<td>$(x+1)(x-5)$</td>
<td>$-1$ and $5$</td>
<td>$-5$</td>
</tr>
<tr>
<td>C $y = x^2 + 4x - 5$</td>
<td>$r \times s = -5$</td>
<td>$r = -1$ $s = 5$</td>
<td>$(x-1)(x+5)$</td>
<td>$1$ and $-5$</td>
<td>$-5$</td>
</tr>
<tr>
<td>D $y = x^2 - 6x + 5$</td>
<td>$r \times s = 5$</td>
<td>$r = -1$ $s = -5$</td>
<td>$(x-1)(x-5)$</td>
<td>$1$ and $5$</td>
<td>$5$</td>
</tr>
<tr>
<td>E $y = x^2 + 7x + 6$</td>
<td>$r \times s = 6$</td>
<td>$r = 1$ $s = 6$</td>
<td>$(x+1)(x+6)$</td>
<td>$-1$ and $-6$</td>
<td>$6$</td>
</tr>
<tr>
<td>F $y = x^2 - 6x + 9$</td>
<td>$r \times s = 9$</td>
<td>$r = -3$ $s = -3$</td>
<td>$(x-3)(x-3)$</td>
<td>$3$</td>
<td>$9$</td>
</tr>
<tr>
<td>G $y = x^2 - x - 6$</td>
<td>$r \times s = -6$</td>
<td>$r = 2$ $s = -3$</td>
<td>$(x+2)(x-3)$</td>
<td>$-2$ and $3$</td>
<td>$-6$</td>
</tr>
<tr>
<td>H $y = x^2 + 13x + 12$</td>
<td>$r \times s = 12$</td>
<td>$r = 1$ $s = 12$</td>
<td>$(x+1)(x+12)$</td>
<td>$-1$ and $-12$</td>
<td>$12$</td>
</tr>
<tr>
<td>I $y = x^2 - 4x - 12$</td>
<td>$r \times s = -12$</td>
<td>$r = -6$ $s = 2$</td>
<td>$(x-6)(x+2)$</td>
<td>$6$ and $-2$</td>
<td>$-12$</td>
</tr>
<tr>
<td>J $y = x^2 + x - 12$</td>
<td>$r \times s = -12$</td>
<td>$r = -3$ $s = 4$</td>
<td>$(x-3)(x+4)$</td>
<td>$3$ and $-4$</td>
<td>$-12$</td>
</tr>
</tbody>
</table>
Unit 7: Day 7: We Have a Lot in Common

75 min

**Math Learning Goals**
- Determine the connections between the factored form and the $x$-intercepts of the quadratic relations $y = ax^2 + bx + c$.

**Materials**
- graphing calculators
- algebra tiles
- overhead algebra tiles
- BLM 7.7.1, 7.7.2, 7.7.3

**Assessment Opportunities**

**Whole Class ➔ Discussion**
Use the context of kicking a soccer ball. Depending on the kick, the ball may travel different heights and distances. Lead a discussion as to how to draw a height vs. horizontal distance graph for three scenarios in which there are different height and distance measures.

Ask:
- If you were to represent this scenario graphically:
  - what are appropriate labels for the axes? [height above ground in metres; horizontal distance from the kicker’s toe in metres]
  - where would the point be that represents the kick?
  - what would one of the $x$-intercepts of the graph be?
  - what might the general equation of the graph look like?

**Pairs ➔ Group Investigation**
Students work through the graphing calculator investigation (BLM 7.7.1).

Provide each pair with one of the following equations: $y = x^2 + 4x$, $y = x^2 + 6x$, $y = x^2 - 2x$, $y = x^2 - 5x$. Pairs of students complete questions 1–3 on BLM 7.7.1.

Form groups of four where each student in the group has a different equation. Students complete questions 4–9 (BLM 7.7.1). They compare what is the same about their graphs and what is different and comment on the $x$-intercept, $y$-intercept, and the general shape. Lead a discussion about the algebraic representations, e.g., $y = x(x + 4)$, $y = (x - 0)(x + 4)$ and $y = (x + 4)(x - 0)$ being equivalent ways of expressing the equation $y = x^2 + 4x$ in factored form.

Use algebra tiles to demonstrate common factoring of $y = x^2 + 6x$, then do Example 1 and 2 of BLM 7.7.2. Students complete BLM 7.7.2. (See Teacher BLM 7.7.3.)

**Mathematical Process/Using Tools/Observation/Anecdotal Note:** Assess students’ use of graphing calculators and algebra tiles to identify the factors.

**Whole Class ➔ Demonstration**
Relate the algebra tile method of factoring to the algebraic method. Demonstrate the algebraic method using several examples, including some with negatives.

**Consolidate Debrief**

**Individual ➔ Journal**
Students describe the three methods of common factoring that were investigated in the lesson, and describe the connections of the factors to the $x$-intercepts of the corresponding relation.

**Home Activity or Further Classroom Consolidation**

**Concept Practice**
Complete the practice questions.
7.7.1: Investigate Relations of the Form \( y = ax^2 + b \)

Name:

1. Obtain a graphing calculator and equation from your teacher.
2. Type in the equation (using the \( Y = \) button on your calculator). Key in \textbf{zoom 6} to get the max and min from –10 to 10 on your window.
3. Fill in the table.

<table>
<thead>
<tr>
<th>Your equation</th>
<th>A sketch of your quadratic equation (including the ( x )- and ( y )-intercepts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = )</td>
<td><img src="image.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Coordinates of \( x \)-intercepts \((_____, _____)\) and \((_____, _____)\) and the coordinate of the \( y \)-intercept \((_____, _____)\)

4. In your group, sketch all four graphs.

5. Identify what is the same and what is different in these four graphs.
6. Fill in the following table.

<table>
<thead>
<tr>
<th>Standard form</th>
<th>From the graph, identify the x-intercepts r and s</th>
<th>Write each equation in factored form $y = (x - r)(x - s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 + 4x$</td>
<td>$r = ____$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s = ____$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 + 6x$</td>
<td>$r = ____$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s = ____$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 - 2x$</td>
<td>$r = ____$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s = ____$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 - 5x$</td>
<td>$r = ____$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s = ____$</td>
<td></td>
</tr>
</tbody>
</table>

7. Clear the $y =$ screen on one of the four calculators, enter the factored form of the four equations, and graph. Are these graphs the same as the ones in your sketch? If yes, continue to question 8. If no, revise and check. Ask your teacher for assistance, if needed.

8. Can the equations in the third column of your table be simplified? Explain.

9. Record the simplified versions of your relation in factored form.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 + 4x$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 + 6x$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 - 2x$</td>
<td></td>
</tr>
<tr>
<td>$y = x^2 - 5x$</td>
<td></td>
</tr>
</tbody>
</table>
7.7.2: Factoring $x^2 + bx$

Consider the outer portion of the algebra tile representation as the length and width of a room. The rectangle is the carpet. Colour in as many cells as required for each example to form a rectangle. To factor, form a rectangle using the tiles, then determine the length and width of the room.

**Example 1:** $x^2 + 2x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + 2x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)

**Example 2:** $x^2 + 3x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + 3x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)

Use your algebra tiles to factor the following:

1. $x^2 + 4x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + 4x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)

2. $x^2 + 1x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + 1x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)

3. $x^2 + 5x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + 5x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)

4. $x^2 + x$

factored form (______) (______)

the coordinates of the $x$-intercepts of $y = x^2 + x$

(______, ______) and (______, ______)

the coordinate of the $y$-intercept (______, ______)
7.7.3: Two Methods for Factoring $x^2 + bx$ (Teacher)

1. **Algebra Tile Model for Common Factoring**
   Use overhead tiles to demonstrate the concept of common factoring for the example $x^2 + 6x$.

   ![Algebra Tile Model](image)

   **Explanation**
   This is the only rectangle that can be formed with algebra tiles. Factoring requires identifying the length and the width that results in this rectangle’s area. In this case the dimensions are $(x)$ and $(x + 6)$.

   **Alternative Explanation**
   Separate the $x^2$ tile from the $x$ tiles to show the length and width of each part of the expression.

   ![Alternative Explanation](image)

   In the square, the area is given by $x$ times $x$. In the rectangle, the area is given by $x$ times 6. The total area is given by $(x)(x) + (x)(6)$; we say that $x$ is a common factor of each term.

   **Explain**
   $x$ multiplies $x$, and $x$ multiplies 6, thus $x$ multiplies $x$ and 6; that is, $x$ multiplies $(x + 6)$, or $x$ $(x + 6)$

   Repeat the process with $x^2 + 3x$.

2. **Algebraic Model for Common Factoring**
   Use the principle of what is common to each term to factor the right-hand side of each equation. Lead the class through algebraic thinking in factoring the following examples:

   \[
   \begin{align*}
   x^2 - 1x & = x (x - 1) \\
   x^2 + 5x & = x (x + 5) \\
   x^2 - 2x & = x (x - 2)
   \end{align*}
   \]
**Math Learning Goals**
- Investigate the method of factoring a difference of two squares using patterning strategies and diagrams.
- Use the technique of factoring the difference of two squares to determine the x-intercepts of a quadratic relation.

**Materials**
- graphing calculators
- BLM 7.8.1

**Assessment Opportunities**

**Minds On… Reflection**
Students reflect on the connection between $x^2 - 36$, $y = x^2 - 36$, and the corresponding graph. They fold a paper into thirds and write the headings “I Think, I Wonder, I Know” in the columns. Students complete the first and second columns, and share their reflection with a partner.

**Math Process/Communicating/Observation/Anecdotal Note:** Assess students’ use of mathematical language related to quadratic relations.

**Action! Pairs → Investigation**
Students use a graphing calculator to identify the intercepts of quadratic relations of the form $y = x^2 - a^2$ and connect the x-intercepts to the factors (BLM 7.8.1).

**Whole Class → Guided Instruction**
Activate students’ prior knowledge by factoring the relation $y = x^2 + 7x + 12$.

$3 \times 4 = 12, 3 + 4 = 7$. Therefore $y = x^2 + 7x + 12$ can be expressed in factored form as follows: $y = (x + 3)(x + 4)$

Ask how $y = x^2 - 49$ could be written as a trinomial. [Answer: $y = x^2 + 0x - 49$]

Model the process: $(+7)(-7) = -49$ and $(+7) + (-7) = 0$

Therefore, $y = x^2 - 49$ can be expressed in factored form as $y = (x + 7)(x - 7)$.

Reinforce the fact that the bx term is 0x and thus is not written in the expression. (Zero times x is zero.)

Explain why this type of quadratic is called a “difference of perfect squares,” illustrating both algebraically and pictorially.

Students practise solving problems involving factoring a difference of squares.

**Consolidate Debrief**

**Whole Class → Discussion**
Students complete the third column in their “I Think, I Wonder, I Know” chart. Review factoring a difference of squares and its connection to the graph, as needed.

**Home Activity or Further Classroom Consolidation**
Practise factoring and connecting the factors to the graph.
7.8.1: Graphing Relationships of the Form \( y = x^2 - a^2 \)

Name:

1. Working with a partner and one graphing calculator, set your Window: \( \text{Xmin} = -10; \) \( \text{Xmax} = 10; \) \( \text{Xscl} = 1; \) \( \text{Ymin} = -36; \) \( \text{Ymax} = 10; \) \( \text{Yscl} = 1; \) \( \text{Xres} = 1. \)

   Complete the following table.

<table>
<thead>
<tr>
<th>Relation in standard form</th>
<th>Sketch each graph. Label as A, B, C, D, or use different colours.</th>
<th>( y )-intercept</th>
<th>( x )-intercept ((r))</th>
<th>( x )-intercept ((s))</th>
<th>Relation in factored form ( y = (x - r)(x - s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> ( y = x^2 - 4 )</td>
<td><img src="image" alt="Graph A" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>B</strong> ( y = x^2 - 9 )</td>
<td><img src="image" alt="Graph B" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C</strong> ( y = x^2 - 16 )</td>
<td><img src="image" alt="Graph C" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>D</strong> ( y = x^2 - 25 )</td>
<td><img src="image" alt="Graph D" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Consider your results from question 1 and answer the following questions.

2. What is the same about the relations?

3. What is the same about the graphs?

4. What is the same about the vertex of each graph?

5. What do you notice about the \( r \) and \( s \) values of each relation?

6. Solve this puzzle. How can you find the \( y \)-intercept and the \( x \)-intercepts of the graph of a quadratic relation of the form \( y = x^2 - a^2 \)?