Unit 4
Graphical Models and Solutions

Grade 10 Applied

Lesson Outline

BIG PICTURE

Students will:
- identify characteristics of quadratic relations;
- graphically solve systems of linear relations;
- solve problems by interpreting graphs of quadratic relations.

Day	Lesson Title	Math Learning Goals	Expectations
1	Going Around the Curve (Part 1)	• Collect data that can be modelled by a quadratic relation, using connecting cubes, and calculate first and second differences. • Draw the curve of best fit on chart paper. • Realize that the shape of the graphs are curves rather than lines.	QR2.01 CGE 5a, 7j
2	Going Around the Curve (Part 2)	• Complete the data collection from the previous lesson, if needed. • Communicate students' findings from the experiments to the entire class. • Define the shape of the curves of best fit as a parabola.	QR2.01 CGE 2c, 4f
3	Making a Difference	• Determine that if the table of values yields a constant second difference the curve is parabolic and vice versa. • Realize that there are other non-linear relationships that are not parabolic. • Develop a word wall of new vocabulary related to the quadratic.	QR2.02 CGE 5a, 5b
4	Features of Parabolic Graphs	• Identify the key features of parabolic graphs created on Days 1 and 2 (the equation of axis of symmetry, the coordinates of the vertex, the y-intercept, x-intercepts (zeros), and the max/min value). • Develop and use appropriate vocabulary related to parabolic curves.	QR2.03 CGE 4f
5	Canada's Baby Boom	• Collect data that can be represented as a quadratic relation from secondary sources. • Graph the data and draw a curve of the best fit (parabola). • Interpret data related to the context such as the maximum or minimum height. • Determine that quadratic relations are of the form $y = ax^2 + bx + c$ using technology.	QR3.01 CGE 4a, 7f
6	Data Collection Using Balls	• Use technology to collect data that will produce a quadratic relation. • Interpret data related to the context. • Make predictions based on the context of the relation.	QR3.02 CGE 3c, 4f
7	Take Me Out…	• Solve problems that arise from realistic situations that can be modelled by two linear relations. • Determine graphically the point of intersection of two linear equations. • Interpret the story the graph tells, including, point of intersection and what occurs before and after the intersection point.	ML3.01, LR3.03 CGE 3c

Day	Lesson Title	Math Learning Goals	Expectations
8	Solve the Problem Using a Graph	• Determine graphically the point of intersection of two linear relations. • Interpret the story told by a graph, i.e., point of intersection, what occurs before and after the intersection point. • Consider the possible number of points of intersection of two linear relations.	ML3.01, LR3.03 CGE 3c, 4f
9	Summative Assessment		

Unit 4: Day 1: Going Around the Curve (Part 1)

75 min

Math Learning Goals

- Collect data that can be modelled by a quadratic relation, using connecting cubes, and calculate first and second differences.
- Draw the curve of best fit on chart paper.
- Realize that the shape of the graphs are curves rather than lines.

Materials

- linking cubes
- chart paper
- grid chart paper
- BLM 4.1.1, 4.1.2, 4.1.3, 4.1.4, 4.1.5

Assessment Opportunities

Minds On…

Groups of 3 → Placemat

Students complete a placemat with the phrase "linear relationship" in the centre. They reflect on everything they recall about the characteristics of linear relations in their own section and share their results within their groups. They write the characteristics they agree upon in the centre. Repeat the process using the phrase "non-linear relationship."

Summarize characteristics on chart paper and post.

Recall that first difference implies a linear relationship.

Show students how to find second differences, using an example.

Curriculum Expectations/Demonstration/Observation/Checklist: Observe what characteristics students recall about linear and non-linear relations.

Action!

Groups of 3 → Experiments

Each group completes an assigned experiment (BLMs 4.1.1–4.1.5). They record their data in a table on chart paper, and plot the data on grid chart paper.

Groups who do not complete their experiment can be given some time during the next lesson.

Learning Skills/Teamwork/Observation/Checklist: Observe how well students work as a productive team to complete the task.

These experiments provide "clean data" from which a constant second difference can be determined.

Consolidate Debrief

Groups of 3 → Sharing

Groups who complete the activity post their graphs and tables of values and plan how they will present their work to the class.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete the practice questions.

Provide students with appropriate practice questions.

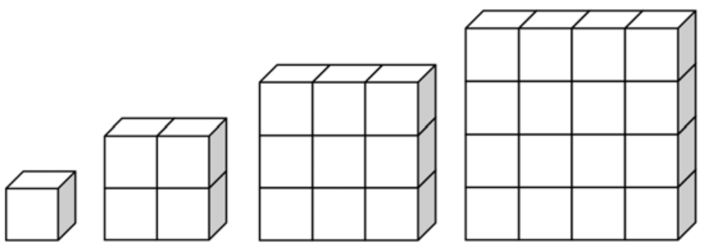
4.1.1: Going Around the Curve

Experiment A
A particular mould grows in the following way: If there is one “blob” of mould today, then there will be 4 tomorrow, 9 the next day, 16 the next day, and so on.
Model this relationship using linking cubes.

Purpose
Find the relationship between the side length and the number of cubes.

Hypothesis
What type of relationship do you think exists between the side length and the number of cubes?

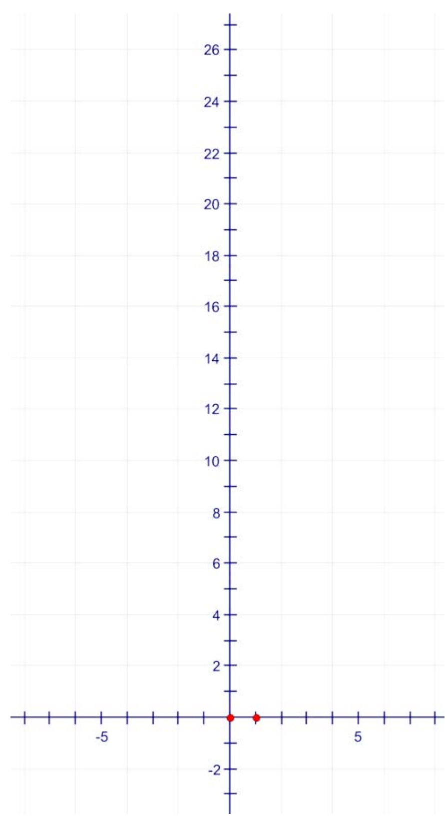
Procedure
1. Build the following sequence of models, using the cubes.



2. Build the next model in the sequence.

Mathematical Models
Complete the table, including first and second differences.
Make a scatter plot and a line of best fit.

Side Length	Number of Cubes	First Differences	Second Difference
0	0		



4.1.2: Going Around the Curve

Experiment B

Jenny wants to build a square pool for her pet iguana. She plans to buy tiles to place around the edge to make a full play area for her pet.
Model the relationship, comparing total play area (pool combined within the edging) to the side length of the pool, using linking cubes.

Purpose

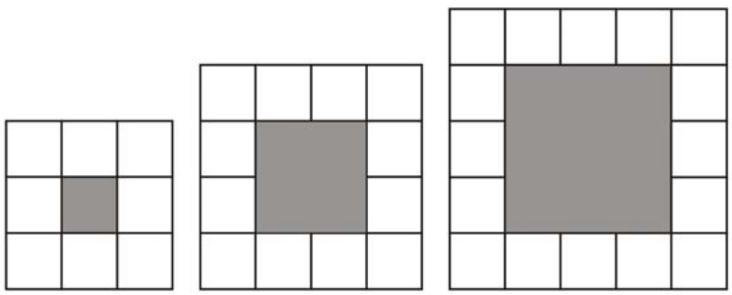
Find the relationship between the side length of the pool (shaded inside square) and the total play area.

Hypothesis

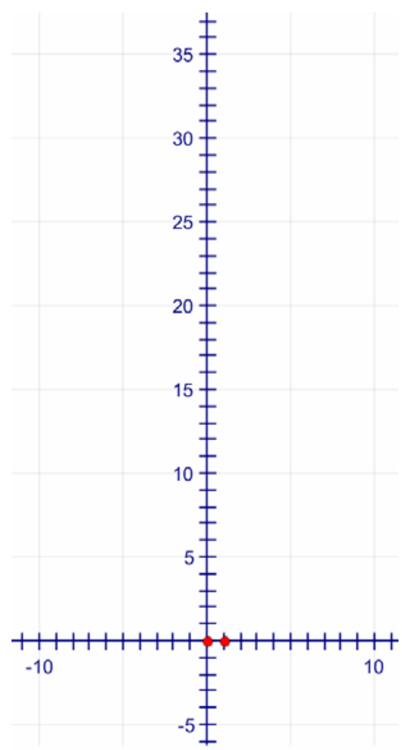
What type of relationship do you think exists between the side length and the play area?

Procedure

1. Build the following sequence of models using the cubes.
   Note: The pool is the shaded square, the tiles are white.



2. Build the next model in the sequence.



Mathematical Models

Complete the table, including first and second differences.
Make a scatter plot and a line of best fit.

Side Length	Total Play Area	First Differences	Second Difference
1			

4.1.3: Going Around the Curve

Experiment C

A particular mould grows in the following way: If there is one “blob” of mould today, then there will be 3 tomorrow, and 6 the next day.
Model this relationship using linking cubes.

Purpose

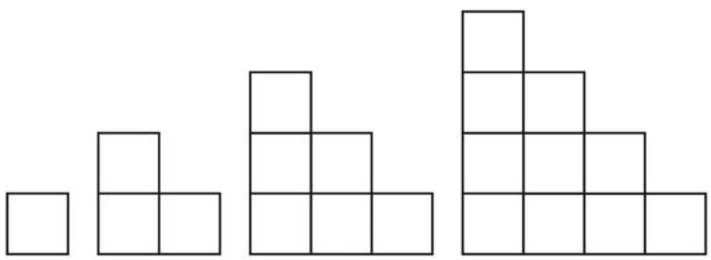
Find the relationship between the number of cubes in the bottom row and the total number of cubes.

Hypothesis

What type of relationship do you think exists between the number of cubes in the bottom row and the total number of cubes?

Procedure

1. Build the following sequence of models using the cubes.

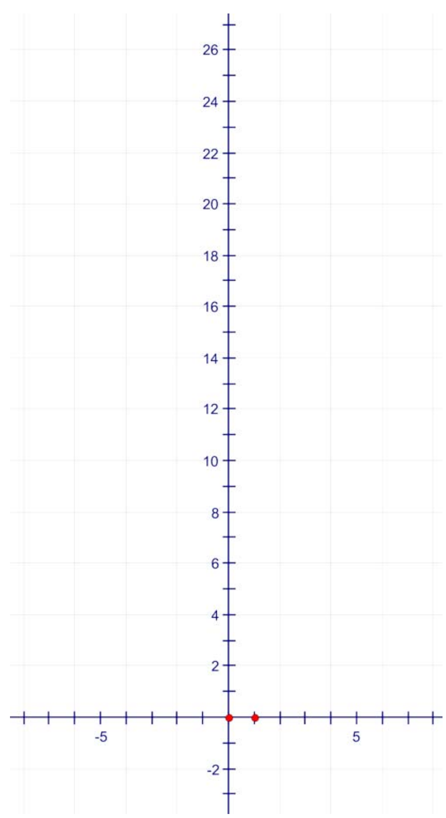


2. Build the next model in the sequence.

Mathematical Models

Complete the table, including first and second differences.
Make a scatter plot and a line of best fit.

Number of Cubes in the Bottom Row	Total Number of Cubes	First Differences	Second Difference



4.1.4: Going Around the Curve

Experiment D

Luisa is designing an apartment building in a pyramid design. Each apartment is a square. She wants to know how many apartments can be built in this design as the number of apartments on the ground floor increases.
Model this relationship, using linking cubes.

Purpose

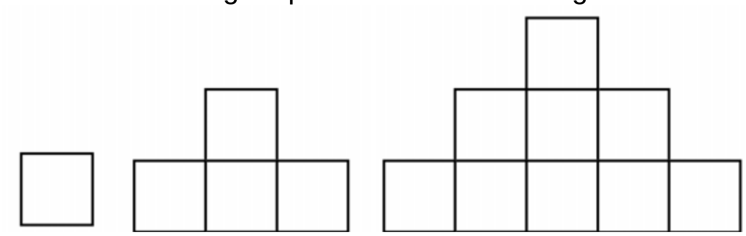
Find the relationship between the number of cubes in the bottom row and the total number of cubes.

Hypothesis

What type of relationship do you think exists between the number of cubes in the bottom row and the total number of cubes?

Procedure

1. Build the following sequence of models using the cubes.

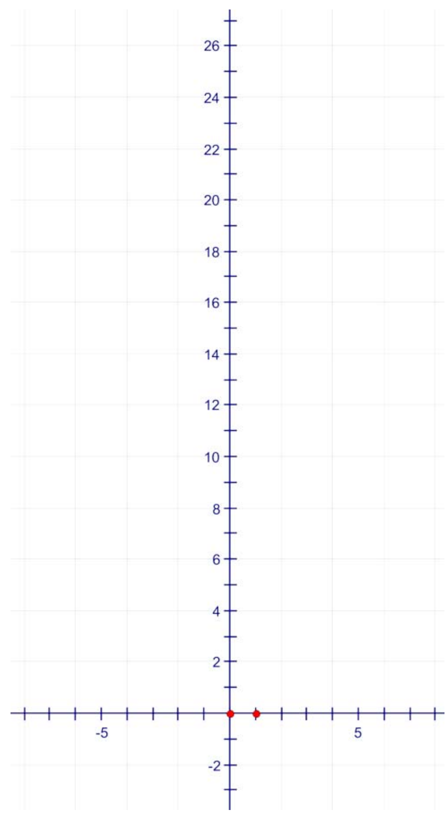


2. Build the next model in the sequence.

Mathematical Models

Complete the table, including first and second differences.
Make a scatter plot and a line of best fit.

Number of Cubes in the Bottom Row	Total Number of Cubes	First Differences	Second Difference
0	0		



4.1.5: Going Around the Curve

Experiment E

Liz has a beautiful pond in her yard and wants to build a tower beside it using rocks. She is unsure how big she will make it and how many rocks she will need. She is particularly concerned to have the nicest rocks showing.
Model the relationship comparing the length of the base to the number of visible rocks using linking cubes.

Purpose

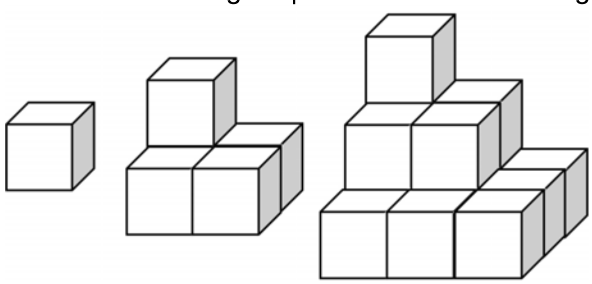
Find the relationship between the number of cubes on the side of the base and the total number of unhidden cubes.

Hypothesis

What type of relationship do you think exists between the length of the side of the base and the number of visible cubes?

Procedure

1. Build the following sequence of models using the cubes.

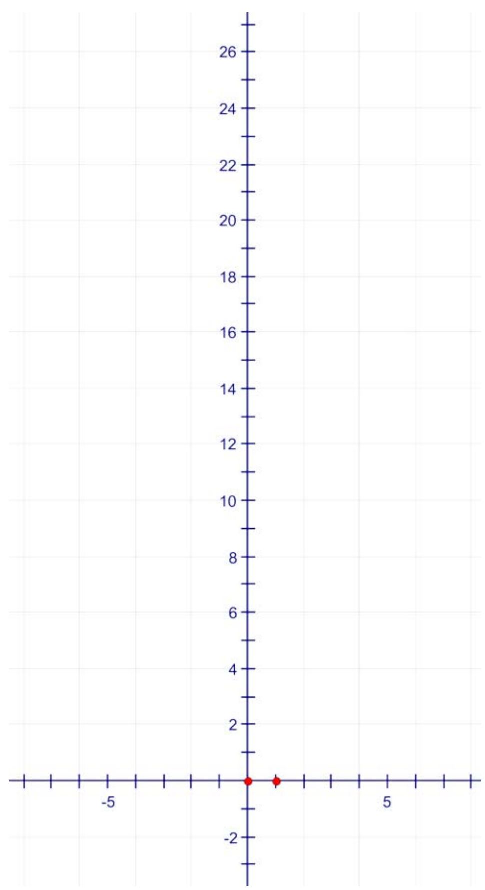


2. Build the next model in the sequence.

Mathematical Models

Complete the table, including first and second differences.
Make a scatter plot and a line of best fit.

Length of Side of Base	Total Number of Unhidden Cubes	First Differences	Second Difference



75 min

Math Learning Goals
- Complete the data collection from the previous lesson, if needed.
- Communicate students' findings from the experiments to the entire class.
- Define the shape of the curves of best fit as a parabola.

Materials
- materials from Day 1
- large chart (graph) paper

Assessment Opportunities

Minds On…

Whole Class → Review
Review the context of expectations for Day 1 experiments.

Tell students that the graphs obtained are only half of the full graph and direct them to extend them using symmetry. Demonstrate, as needed.

Pairs → Peer Coaching
Pair each student with someone who was not in their group on the previous day. Pairs discuss each experiment, checking the graph and the first and second differences.

Word Wall
- parabola
- quadratic equation/relation
- vertex
- axis of symmetry
- *x*-intercepts
- *y*-intercept
- zeros
- maximum/minimum value
- symmetry

Action!

Groups of 3 → Presentation
The original groups make any required changes, draw a large graph of their results, and present their findings to the class. Presentations should include the context provided in their problem; the models they constructed; the data they collected; an explanation of the pattern that they observed; the graphs they constructed; and the curve of best fit.

Curriculum Expectation/Presentation/Checkbric: Assess how students collect and represent data, and draw the curve of best fit. Observe results of experiments, ask oral questions, and clear up misconceptions.

Consolidate Debrief

Whole Class → Discussion
Focus discussion on the shapes of the entire graph, including negative *x* values. Explain that this particular curve is called a parabola. Students take particular note of the constant second difference in the table of values, connecting to their understanding that linear relations have common first differences.

Explain that the common second difference identifies the resulting curve as a parabola. Students predict what their graph would look like if it was extended to negative values of the independent variable.

Students should see the full parabolic shape rather than just half.

Reflection

Home Activity or Further Classroom Consolidation
- Complete a journal entry using the following writing prompt:
  Parabolic curves exist in the world in the following ways/places/actions…

OR

- Find pictures you think are parabolic and bring them to class.

75 min

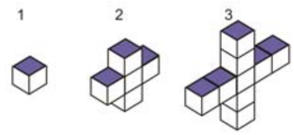
Math Learning Goals
- Determine that if the table of values yields a constant second difference the curve is parabolic and vice versa.
- Realize that there are other non-linear relationships that are not parabolic.
- Develop a word wall of new vocabulary related to the quadratic.

Materials
- connecting cubes
- graph paper

Assessment Opportunities

Minds On…

Whole Class → Investigation
Demonstrate a linear relationship, using the following construction.



Students individually complete a table of values with the headings Model Number, Number of Cubes, and First Difference and create a graph of the information. They connect the first difference with the slope of the line and reflect on the value of difference to determine if the relationship is linear or non-linear, and provide reasons for their conclusions.

Action!

Pairs → Summarizing
Students refer to data from any two experiments completed on Day 1. Engage student thinking by providing prompts: Prove that the data is non-linear.
Further prompting: We know all the graphs are parabolas; we call these quadratic relations.
Ask:
- How could we know that they are parabolas just from the table of values?
- What pattern do you see in the work you've done on the tables?

Before continuing, students should have the understanding that the first differences are not the same and therefore the data is non-linear.

Students complete the statement: The graph of a table of values will be a parabola if....

Curriculum Expectation/Demonstration/Checklist: Assess students' understanding that a constant second difference in a table of values determines that the relation is quadratic.

Students can confirm their predictions by selecting other sets of data from the experiments.

Groups of 4 → Investigation
Students build cubes of sides 1, 2, and 3, and record on a table of values the side length and volume. They calculate the volume for side lengths 4, 5, and 6, and put the data on a graph.

Ask:
- Is this linear or non-linear?
- Is the curve a parabola?
- How can you verify this?

Consolidate Debrief

Whole Class → Discussion
Discuss and summarize facts:
- The table of values associated with parabolas always has a common second difference.
- There are other curves that are not parabolas. These curves do not have common second differences.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete one task about parabolas:
- Determine if the situations are linear, quadratic, or neither, and provide reasons for your answers.

OR

- Place the parabolic picture that you brought to class on a grid and determine some of its points. Verify that it is or is not a parabola.

Provide a variety of situations and/or data that are linear, quadratic, or neither.

Unit 4: Day 4: Features of Parabolic Graphs

Grade 10 Applied

75 min

Math Learning Goals

- Identify the key features of parabolic graphs created on Days 1 and 2 (the equation of axis of symmetry, the coordinates of the vertex, the *y*-intercept, *x*-intercepts (zeros), and the max/min value).
- Develop and use appropriate vocabulary related to parabolic curves.

Materials

- BLM 4.4.1, 4.4.2, 4.4.3
- scissors
- glue sticks

Assessment Opportunities

Minds On…

Pairs → Timed Retell

Provide partners A and B with a different graph of a quadratic relation (BLM 4.4.1). They should not see each other's graph.

Student A describes the key features of the graph to student B. Student B sketches the graph from student A's oral prompts. Students compare how close the sketch was to the given graph. The partners switch roles.

Whole Class → Discussion

Discuss the terms they used to describe their graphs to their partners. Emphasize the key features that were focused on in the descriptions.

Ask: Did you have difficulties identifying or describing any of the key features? If so, what was the difficulty?

Word Wall
- axis of symmetry
- vertex
- the *y*-intercept
- zeros
- max/min value

The "Zero" is the value of *x* when y = 0

Action!

Whole Group → Guided Instruction

Guide students in defining the terms that describe the features of a parabola and label the graphs, using appropriate terminology (BLM 4.4.2).

Connect the terminology to parabolic graphs created from real data.

Consolidate Debrief

Groups of 3 → Discussion

Students rejoin groups of 3 and revisit the graphs they created on Days 1 and 2.

They cut out the key terminology (BLM 4.4.3) and glue them in the appropriate location on their graphs providing a rationale for their choice.

Curriculum Expectation/Demonstration/Mental Note: Observe students' interpretation of the key features of a parabola as they identify these features of their graph.

Application
Concept Practice

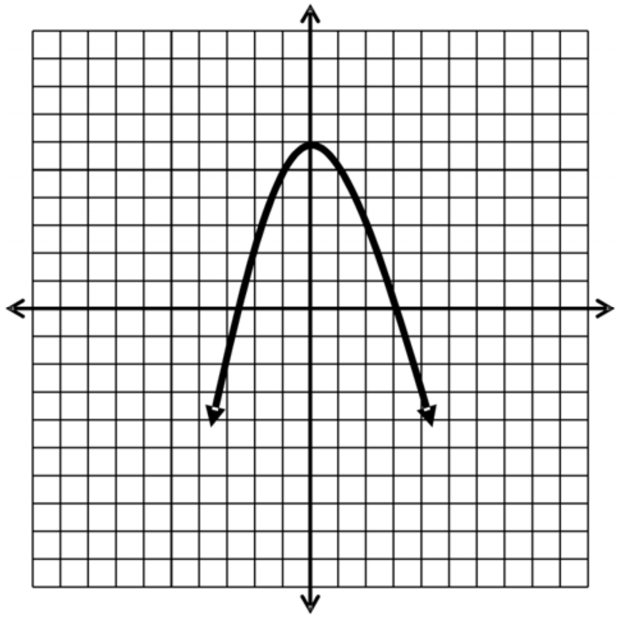
Home Activity or Further Classroom Consolidation

Complete the practice questions.

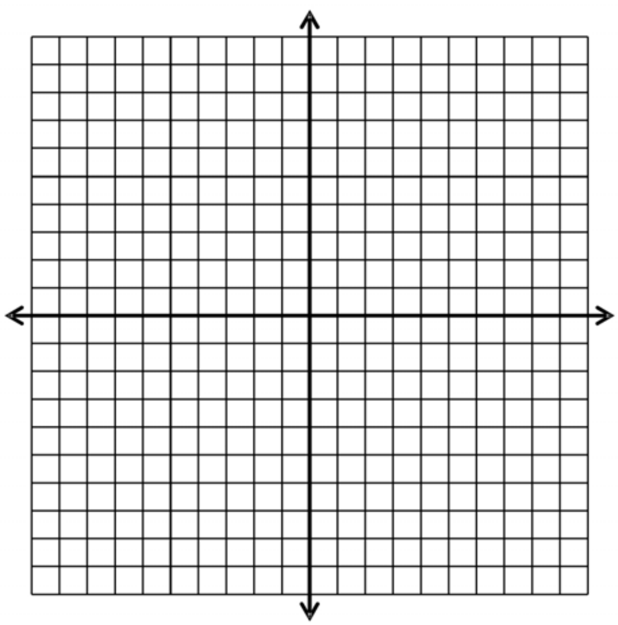
Provide students with questions that include parabolas that represent real data.

4.4.1: Key Features of Parabolic Graphs

Student A has 30 seconds to describe the following graph to student B.

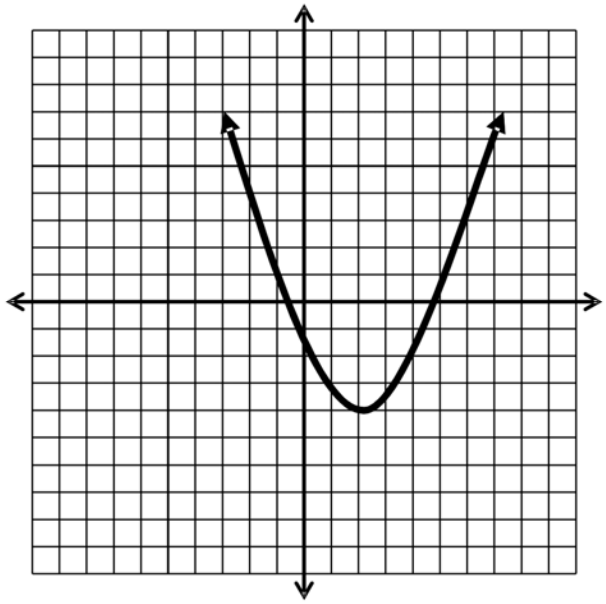


Using the grid below, student B sketches the graph described by Student A.

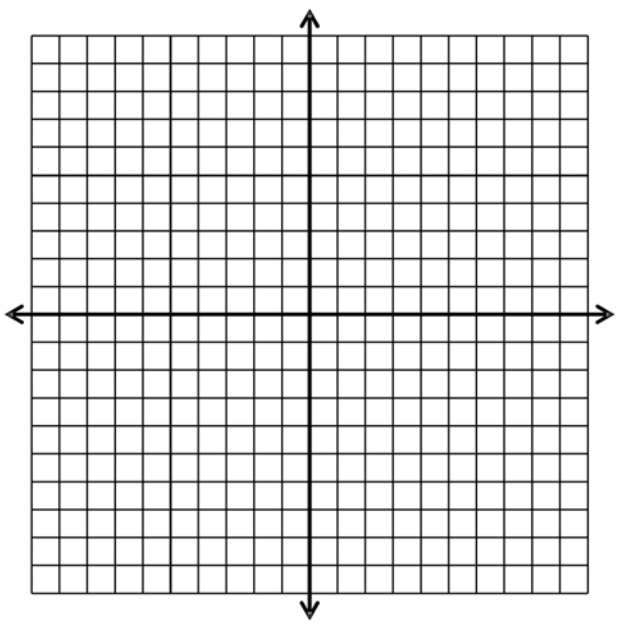


4.4.1: Key Features of Parabolic Graphs (continued)

Student B has 30 seconds to describe the following graph to student A.



Using the grid below, student A sketches the graph described by student B.

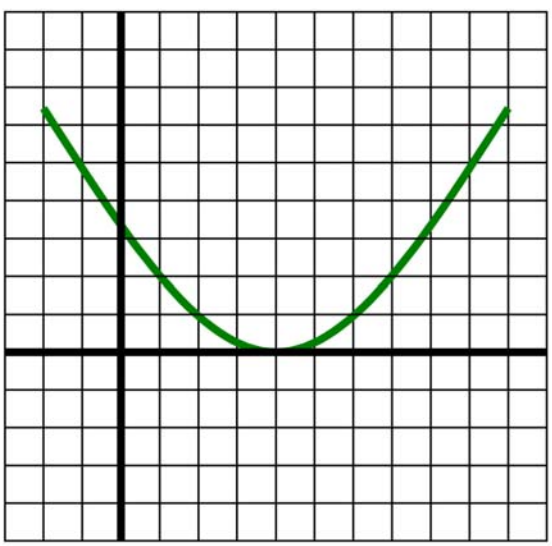


4.4.2: Key Features of Quadratic Relations

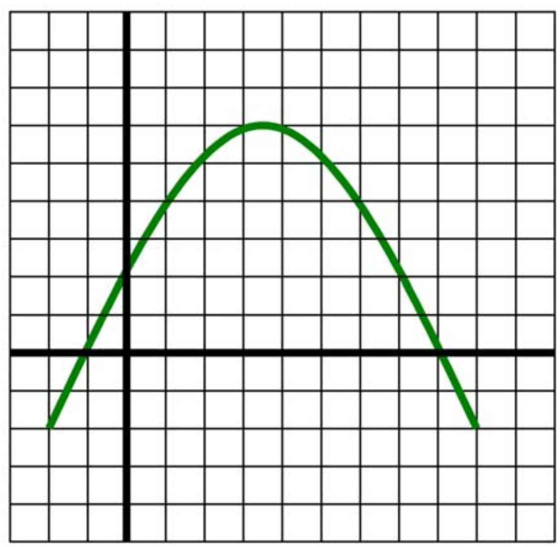
Terminology	Definition	How Do I Label It?	Graph A	Graph B
Vertex	The maximum or minimum point on the graph. It is the point where the graph changes direction.	(*x*,*y*)		
Minimum/ maximum value				
Axis of symmetry				
y-intercept				
x-intercepts				
Zeros				

Label the graphs using the correct terminology.

Graph A



Graph B



4.4.3: Key Terminology

Axis of symmetry

vertex

y-intercept

zero

zero

Maximum
value

Minimum
value

Unit 4: Day 5: Canada's Baby Boom

Grade 10 Applied

75 min

Math Learning Goals

- Collect data that can be represented as a quadratic relation from secondary sources.
- Graph the data and draw a curve of the best fit (parabola).
- Interpret data related to the context such as the maximum or minimum height.
- Determine that quadratic relations are of the form $y = ax^2 + bx + c$ using technology.

Materials

- graphing calculators
- BLM 4.5.1, 4.5.2, 4.5.3

Assessment Opportunities

Minds On…

Individual → Summarizing

Students complete a Frayer model for one feature from the previous day's lesson (BLM 4.5.1).

Provide small graphs of parabolas to include on their model.

Action!

Individual → Exploration Using Technology

Students complete BLM 4.5.2 as an example of quadratic regression. They consider what kind of equation will model the data.

Students use quadratic regression to find equations for data collected on Day 1.

Make the connection between $y = ax^2 + bx + c$ as the equation for a quadratic relation and the parabola.

Fathom™ can be used as an alternative to handheld technology.

Note: The correlation coefficient will be needed to discuss the quality of the fit for each model. This authentic data contrasts the "clean data" collected and analysed Day 1 and 2 of the unit.

Consolidate Debrief

Whole Class → Discussion

Students share their responses to the following questions with respect to the graph of "Canada's Baby Boom."

- What is the vertex of this parabola?
- In what year did the number of births reach a maximum?
- Why do you think this year had the greatest number of births?
- What was the maximum number of births?
- Could you use a parabola to represent birth rates for other periods of time? Explain.
- Is the parabola useful for predicting births into the future?

Curriculum Expectation/Application/Checklist: Assess students' use of the graph to answer related questions.

Application
Reflection

Home Activity or Further Classroom Consolidation

Complete worksheet 4.5.3.

4.5.1: Key Features of a Parabola

Write the feature of a parabola that you were given in the centre of the graphic. Complete the chart. Include sketches and graphs with your work.

Definition	**Facts/Characteristics**
Examples	**Non-examples**

4.5.2: Quadratic Power – Modelling Canada's Baby Boom

Your Task

The Baby Boom occurred right after World War II. Determine if a parabola can be a useful model for the number of births per year for this post-war Baby Boom period.

Procedure

To access Canada's Baby Boom data

1. Open the E-STAT website at http://estat.statcan.ca.
2. Select your language of choice. Click **Accept and Enter** at the bottom of the screen.
3. Click **Search CANSIM** on the left side bar and then click **Search by Table Number**.
4. In the blank box, type **053-0001** to retrieve Table 053-0001 – Vital statistics, births, deaths, and marriages, quarterly.
5. On the subset selection page choose as follows:
   - Under Geography, select **Canada**
   - Under Estimates, select **Births**
   - Under From, select **Jan. 1950**
   - Under To, select **Dec. 1967**
6. Click the **Retrieve as Individual Time Series** button.
7. In the Output specification screen under output format selection, click the **down arrow** and select **Plain Text Table, Time as Rows**.
8. From "The frequency of the output data will be" pull-down menu, select **Converted to Annual (Sum)**.
9. Press the **Go** button.
10. Record the data on a sheet of paper using the following headings: Year and Births.

Year	Births
1950	372009
1951	381092
1952	403559
1953	417884
1954	436198
1955	442937
1956	450739
1957	469093
1958	470118
1959	479275
1960	478551
1961	475700
1962	469693
1963	465767
1964	452915
1965	418595
1966	387710
1967	370894

Graphing Calculator Analysis

1. Create a Births vs. Year scatter plot on your graphing calculator.
2. Perform a Linear Regression on your graphing calculator. Write the equation of the line.
3. How well does the Linear Regression fit the data?
4. Perform a Quadratic Regression on your graphing calculator. Write the equation of the line.
5. How well does the Quadratic Regression fit the data?
6. Which regression best describes the data?

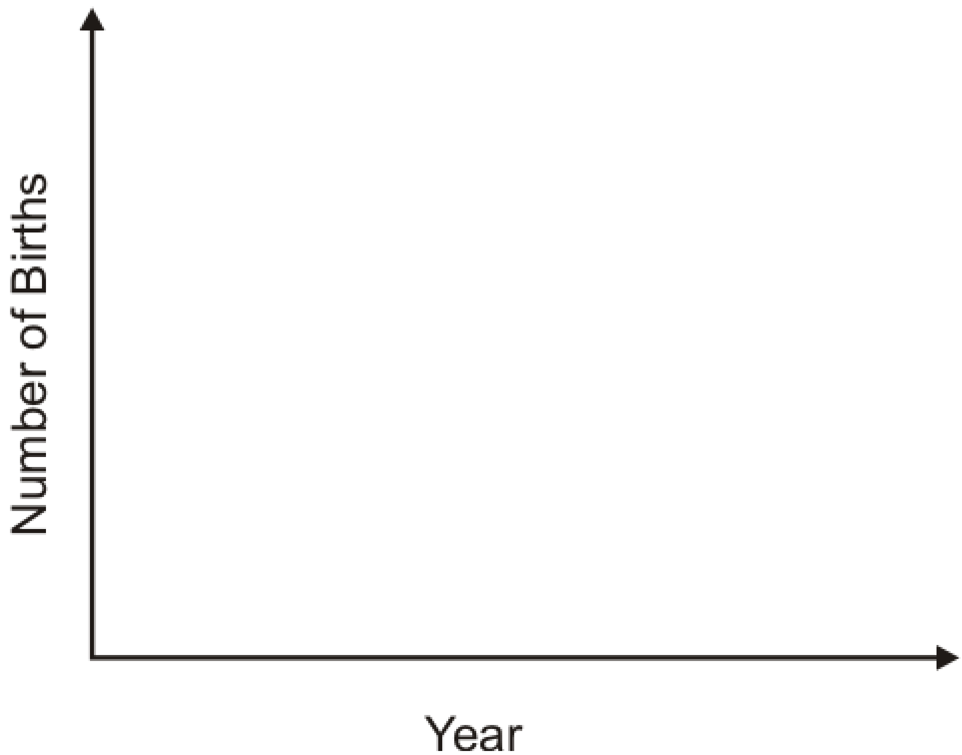
Source:
http://www.keypress.com/fathom/pages/community_exchange/activities_and_documents/activities.php

4.5.3: Modelling Canada's Baby Boom

You examined Baby Boom births in Canada, between 1950–1967.
Sketch what you think the graph of the number of births in Canada between 1950–2005 might look like.

Explain your reasoning and any assumptions you made.

Births in Canada 1950–2005



Unit 4: Day 6: Data Collection Using Balls

Grade 10 Applied

75 min

Math Learning Goals

- Use technology to collect data that will produce a quadratic relation.
- Interpret data related to the context.
- Make predictions based on the context of the relation.

Materials

- BLM 4.6.1, 4.6.2
- graphing calculators
- CBR/CBL
- 3 different types of balls

Assessment Opportunities

Minds On…

Groups of 4 → Form a Hypothesis

Demonstrate that three different types of balls bounce to different heights.

Students make two hypotheses about the bouncing balls, guided by the questions:

- What will be the shape of the graph that represents the height of the ball over time?

Students describe the shape of the graph in words and make a sketch, and include the reasons for their hypothesis.

- Will the time in the air between the first bounce and the second bounce vary depending on which ball is used?

If yes, students hypothesize the order, from the longest time in the air, to the shortest.

They include reasons for their hypotheses (BLM 4.6.2).

Other similar experiments can be used that model quadratic relations using technology, e.g., ramp activities.

The balls chosen should bounce to different heights, e.g., beach, soccer, basket.

Action!

Groups of 4 → Experiment

Groups complete the ball bounce experiment (BLM 4.6.1), using technology to gather data that can be modelled with a quadratic relation, and analyse the graph. Prompt students in the analysis of their graphs by asking relevant questions.

Students revise their hypothesis and reflect on their original reasoning.

Math Process/Communication/Mental Note: Circulate among groups, listen to their language, check for correct use of terms for the parabola, e.g., *vertex*, *maximum*.

Groups transfer the data from BLM 4.6.2 to chart paper for each of the balls, colour coding the data for different balls. Groups post their results.

Students in each group can change responsibility at each station, e.g., ball dropper, recorder, technology person, and task coordinator.

Collecting the data can be expedited by setting up two areas in the room, each area having three stations (one station for each type of ball). Divide the class into two large groups and form groups of four within each to rotate through the three stations.

Consolidate Debrief

Whole Class → Sharing and Guided Discussion

Ask some groups to present their data, graphs, and analysis. They refer to maximum height, when it occurs, and time taken between first and second bounce. They share their original hypothesis and their reflections on their original reasoning, as well as new learning.

Lead a discussion about the different scales groups chose for recording their data and the relative merits of each. Determine the length of time in the air, on average, between the first and second bounce for each ball type, and the order from the most time in the air to the least. Refer to the horizontal intercepts, and what they mean in terms of the experiment.

Examine the shape of the graph (parabolic) and discuss the connection to the horizontal axis representing time. Identify key features of the graph and their meaning in terms of the context, e.g., maximum height, and when it occurs, axis of symmetry, to reinforce previous learning.

Ask questions that require students to analyse the graphs acknowledging any discrepancies in the results of different groups and discuss why they may have occurred. Discuss authentic situations where knowing the relationship between height and time may be important, e.g., space launch/firecrackers/movie stunts.

Encourage students to draw from their reasoning skills and their understanding of science and physics throughout the discussion.

Home Activity or Further Classroom Consolidation

Practice

Search for data that confirms or refutes your graphs from the previous Home Activity regarding Baby Boom data.

Complete the practice questions that require interpretation of quadratic graphs generated by real data.

Provide students with appropriate practice questions.

4.6.1: Ball Bouncing Instructions

Record your hypothesis on the Ball Bouncing Record Sheet.
Use the CBR and the graphing calculator to examine the relationship between the bounce heights of your ball versus time after it is dropped from 1 m.

Instructions to Use Technology

1. Press the APPS button on your calculator and select **CBR/CBL**.
2. Follow the instructions on the calculator screen and press **ENTER**.
3. Run the Ranger program on your calculator.
4. From the main menu of the Ranger Program, select **3:APPLICATIONS**.
5. Select **1:METERS** and then select **3:BALL BOUNCE**.
6. Follow the directions on the screen of your calculator. Release the ball so that the bottom of the ball is 1 m above the floor. The CBR should be at the same height as the bottom of the ball. Drop the ball and then press the trigger key on the CBR just before the ball strikes the ground. Try to keep the CBR steady as it collects the data. This may take a little practice.
7. Your graph should have a minimum of 3 bounces. If you are not satisfied with the results of your experiment, press **ENTER**, select **5:REPEAT SAMPLE**, and try again. Repeat it until you get a nice graph of the ball bouncing.

Data Collection

The goal is to capture the motion of the ball that represents the period from the first bounce to the second bounce.

1. Press **ENTER** to return to the PLOT MENU. Select **7:QUIT** to exit the Ranger Program. The data that you will work with will be in L1 and L2.
2. Use the built-in Select feature of the calculator to select the data you want. Follow the keystrokes below.
   a) Press **2nd [STAT]**, scroll over to OPS menu and then scroll down to #8:SELECT, (and press **ENTER**.
   b) After the bracket (, enter where you want to store the selected data. To use L3 and L4, press **2nd L3 [,] 2nd L4** and **ENTER**.
   c) To actually select a part of the graph you will use, use the arrow keys to move to the left end of the parabola that you want to keep. This should be the first bounce. Press **ENTER**. This sets the left bound. Use the arrow keys to move to the right end of the parabola that you want. This should be the second bounce. Press **ENTER**. The selected data will be placed in L3, L4 and then this data will be displayed.
   d) Sketch a graph of this single parabola using the instructions and grid provided on your Ball Bouncing Record Sheet.
3. Repeat the experiment for each of the different balls.

4.6.2: Ball Bouncing Record Sheet

Hypothesis

1. We hypothesise the shape of the graph that represents the height of the ball over time will be ______________________________

   Sketch the graph. Explain your reasoning.

2. We hypothesise the time in the air between the first and the second bounce will be ________________ (*the same/different*) for each ball.

   Explain your reasoning.

Data Collection

Follow the steps described on the Ball Bouncing Instructions sheet.
To make a graph of each ball bounce, use the **TRACE** function or the **LIST** found using the **STAT** key **EDIT** from the graphing calculator, to complete the table of values for 7 points, starting at the first bounce and ending with the second bounce. Include the maximum point. Choose an appropriate scale to make an accurate sketch of the graph.

1. Ball type: ____________________

Times (s)	Height (cm)
	0
	(max)
	0

4.6.2: Ball Bouncing Activity (continued)

2. Ball type: ____________________

Times (s)	Height (cm)
	0
	(max)
	0

3. Ball type: ____________________

Times (s)	Height (cm)
	0
	(max)
	0

75 min

Math Learning Goals

- Solve problems that arise from realistic situations that can be modelled by two linear relations.
- Determine graphically the point of intersection of two linear equations.
- Interpret the story the graph tells, including point of intersection and what occurs before and after the intersection point.

Materials

- BLM 4.7.1, 4.7.2, 4.7.3, 4.7.4

Assessment Opportunities

Minds On…

Whole Class → Read a News Article

A student volunteer reads the article to the class, or form small groups and ask a volunteer reader to read to the group (BLM 4.7.1). Set a context for discussion of the article. BLM 7.4.2 provides a basis for discussion, with suggested questions.

This **Minds On…** reinforces previous learning about quadratics and provides a segue to linear systems.

Action!

Whole Class → Demonstration

Represent graphically a situation that is modelled by a linear system. Students should understand the meaning of specific parts of the graph, e.g., point of intersection.

Demonstrate the graphical method of solving a linear system (BLM 7.4.3). Discuss the questions; emphasize the meaning of the point of intersection in terms of the problem.

Pairs → Context Building

Students think of two other situations that could be represented by a system of linear relations. They share their ideas with a partner and then with the larger group.

Pair students and identify person A and B, assigning questions to each pair. While A does the work on paper, B coaches A. Then they alternate (BLM 4.7.4).

Each pair selects one question from the practice questions and prepares to present their solution to the class.

Select appropriate linear system problems from the textbook if more are required.

Math Process/Communicating/Observation/Anecdotal Note: Circulate and observe (listen to) the math language and note the presentation of the graphs as students coach one another. Provide informal feedback, as required.

Some students may require assistance interpreting the words, if the equations are not provided.

Consolidate Debrief

Whole Class → Presentation

Pairs present their solution, identifying the significance of the point of intersection in the context of their problem.

Application
Reflection

Home Activity or Further Classroom Consolidation

Think of a situation that could be represented by a system of linear equations and draw an appropriate graphical representation. Write a paragraph that describes the events depicted on the graph, and the meaning of the point of intersection, and the points before and after the point of intersection.

4.7.1: Article

Students read this article in **Minds On….** This activity sets the context for the next activity.

> While warming up at the bottom of the fifth inning in a game against the Toronto Blue Jays in August 1983, New York Yankees left fielder Dave Winfield threw a ball – and hit a sea gull which was waddling along near the Yankees dugout. His feat was met with a chorus of boos from Toronto fans.
>
> Despite protesting his innocence, Winfield was arrested, charged with “willfully causing unnecessary cruelty to animals,” and required to post a $500 bond before leaving Canada.
>
> Among his supporters was Yankees manager Billy Martin. “They say Winfield hit that bird on purpose,” he remarked. “They wouldn’t say that if they saw some of the throws he’s been making all year!”

http://anecdotage.com/index.php?aid=18302

4.7.2: Discussion Guide (Teacher)

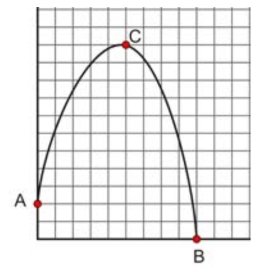
Guide students' thinking with the following questions:

1a. Do you think it would be easy to intentionally throw a ball and hit a sea gull sitting on the field?

1b. Would it be more difficult to intentionally hit a seagull if it was in flight? Explain.
Elicit these points:
- directional accuracy needed since the ball does not travel in a straight line,
- precise vertical and horizontal motion required,
- interference such as friction or wind effects accuracy,
- unpredictable motion of the sea gull.

2. What is the shape of the *path* of the ball? (A sponge ball could be carefully tossed to demonstrate the path of a thrown ball.)

   Draw this on a grid and ask the following questions to illicit context



   - How should the axes be labelled considering this graph represents the path of the ball? (horizontal and vertical distance from Winfield)
   - What does point A represent? (The vertical intercept, which in this case is the height from which the ball is thrown)
     Note: Specific answer depends on sketch and scale.
   - What does point B represent? (The horizontal intercept, which in this case is where the ball hits the sea gull. This is also the point of intersections between the parabola and the horizontal axis, which is where the sea gull was sitting. As well, it is the distance the sea gull was from where Winfield was standing.)
     Note: Specific answer depends on sketch and scale.
   - What does point C represent? (The maximum point, which in this case is where the ball stops going up, and starts coming down. The maximum height is _____ m and is _____m from where Winfield is standing.)
     Note: Specific answer depends on sketch and scale.
   - Describe how the graph would change if Winfield hit the sea gull when it was sitting on top of the dugout instead of on the ground. (Point B would be higher on the curve.)

3. What if the horizontal axis represents time in seconds? Describe, and sketch, how the graph would change. What different information would point A, B, and C provide in this case? (A: when the ball was thrown; B: time elapsed from the when the ball was thrown until the sea gull was hit; C: time elapsed from when the ball was thrown until the ball reached its maximum height). When would one type of graph be more useful than the other?

4. Compare these situations to the ball bounce activity done previously. What is the same and what is different?

Extension: Draw a sketch that would represent the ball hitting a sea gull while the sea gull was in flight. Be sure to label the axis, and show the flight of the bird. Include any assumptions you make.

4.7.3 A Cell Phone Problem

Two cellular phone companies charge a flat fee plus an added cost for each minute of long distance time used.

Call-a-Lot Plan	$C = 0.50t + 20$	Where C represents the total monthly cost in dollars and t represents the number of minutes
Talk-More Plan	$C = 0.25t + 25$	

Create a table of values showing the total charges for a month for up to 30 minutes.

Graph the relations on the same set of axes. Use an appropriate scale.

Call-a-Lot

t	*C*

Talk-More

t	*C*

1. Identify the slope and the C-intercept of the Call-a-Lot line. How do these relate to the total charges?

2. How could the slope and C-intercept been utilized to draw the graph?

3. For Talk-More, what does the ordered pair (8, 27) mean?

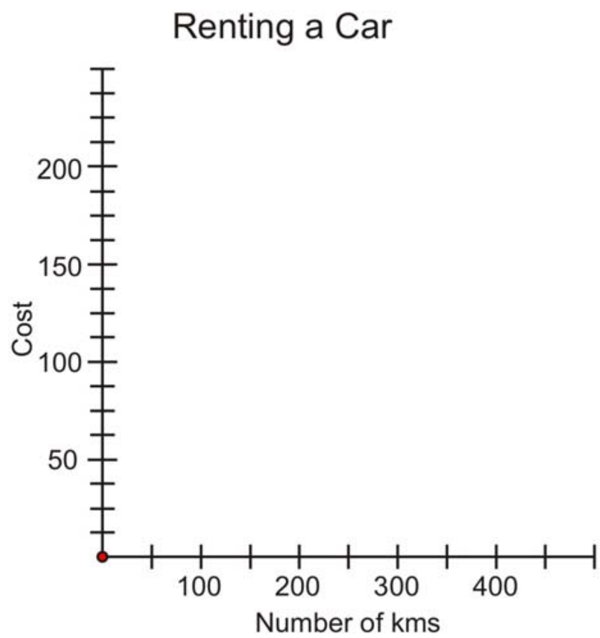
4. Leslie used 13 minutes on the Talk-More plan. How much will it cost?

5. Derek had a bill of $29 last month on the Call-a-Lot Plan. How many minutes did he use?

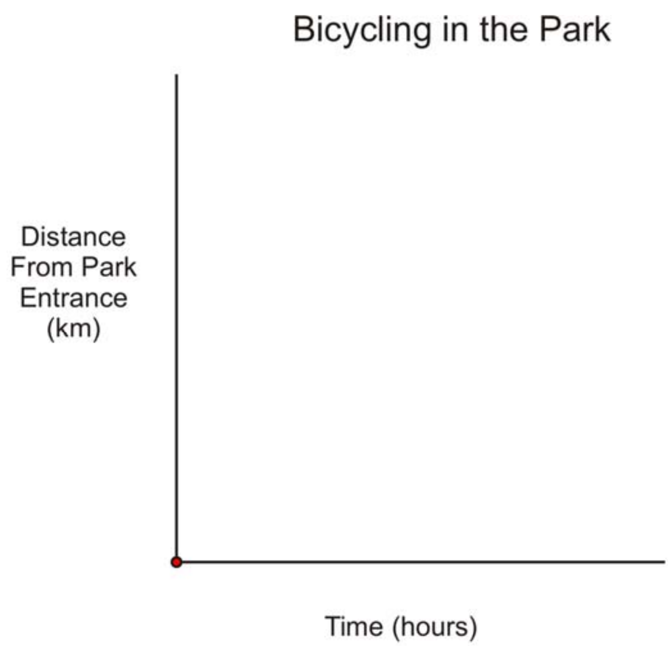
6. Under what conditions would Talk-More be a better plan than Call-a-Lot?

4.7.4: Meaning of the Point of Intersection

1. Your family wants to rent a car for a weekend trip. **Cars R Us** charges $60.00 per weekend for a midsize car plus $.20 per km. **Travel With Us** charges $.50 per km.
   a. Graph both options on the grid and determine the number of kilometres where both companies will cost the same amount.
   b. Explain what this means for your weekend trip.



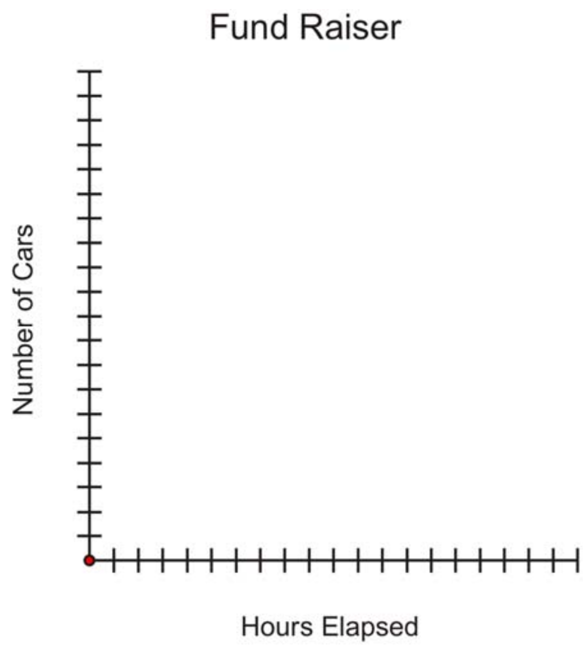
2. Anthony and Anne are bicycling at a Provincial Park. Anthony travels at the rate of 10 km/hr and begins 2 km from the park entrance. Anne begins a the park entrance and travels at 15 km/hr. They both travel at a constant rate towards the Outdoor Education Centre. Graph both routes on the grid and determine the meaning of the point of intersection.



4.7.4: Meaning of the Point of Intersection (continued)

3. For a car wash fundraiser **Team A** washes 2 cars per hour starting a 7:00 a.m. **Team B** begins washing cars at 9:00 a.m. and washes 3 cars per hour.

   Graph the car washing progress of each team on the grid and determine the meaning of the point of intersection, as well as the meaning of the points before and after the point of intersection.



Unit 4: Day 8: Solve the Problem Using a Graph

Grade 10 Applied

75 min

Math Learning Goals

- Solve problems that arise from realistic situations that can be modelled by two linear relations.
- Determine graphically the point of intersection of two linear equations.
- Interpret the story told by a graph, i.e., point of intersection, what occurs before and after the intersection point.

Materials

- BLM 4.8.1

Assessment Opportunities

Minds On…

Whole Class → Presentation

Two or three students share their graphs and paragraphs from the Day 7 Home Activity.

Action!

Whole Class → Demonstration

Discuss methods of graphing linear relations and merit of each.

Demonstrate graphing lines using slope and y-intercepts, using $y = \frac{2}{5}x + 1$ and using intercept/intercept using $3x - 4y - 12 = 0$.

Individual → Practice

Students practise graphing systems of linear equations, finding the point of intersection, and explaining the meaning of what they found (BLM 4.8.1).

Curriculum Expectation/Observation/Checklist: Observe students' accuracy in graphing straight lines and identifying the point of intersection.

Select appropriate linear system problems from the textbook.

Consolidate Debrief

Pairs → discussion

Students compare their results with a partner (BLM 4.8.1).

As a class, discuss the methods of the graphing the meaning of the point of intersection (BLM 4.8.1). Note special cases 5 and 6. Summarize the possibilities for points of intersection of two straight lines (1 point, i.e., different lines $\not\,$; no points, i.e., parallel lines $||$; infinite points, i.e., same line).

Application
Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

Complete the practice questions.

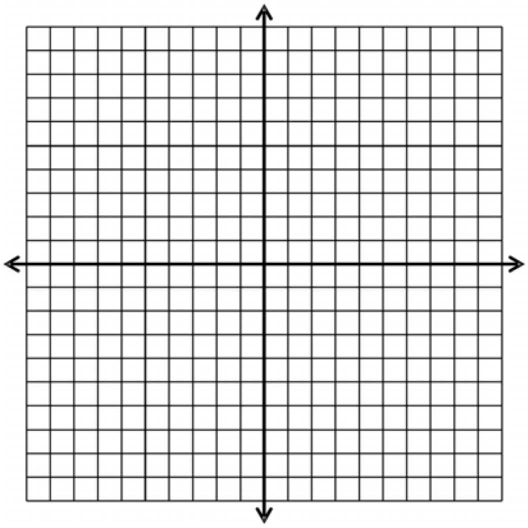
Provide students with appropriate practice questions to prepare for unit assessment.

4.8.1: Pairs of Equations

Graph the following pairs of equations and find the point of intersection. Label your graphs. State what the point of intersection means for each question.

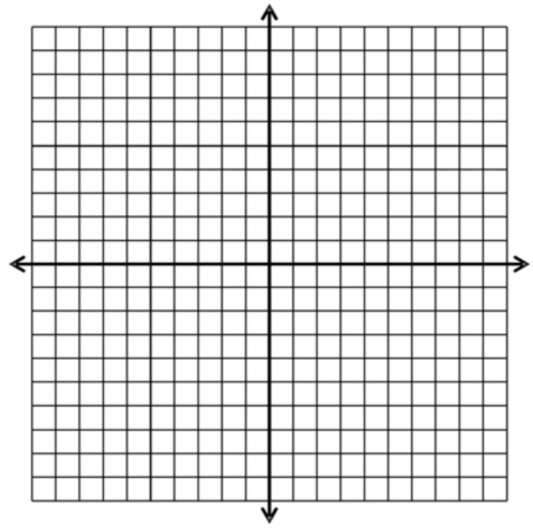
1. $y = -x - 7$

   $y = x - 3$



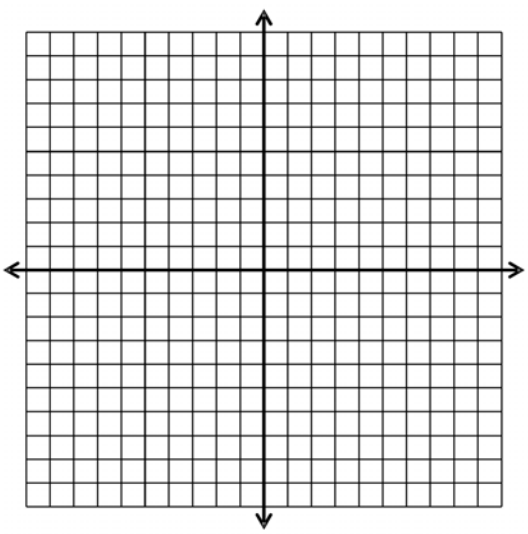
2. $y = 2x - 10$

   $y = -x - 4$



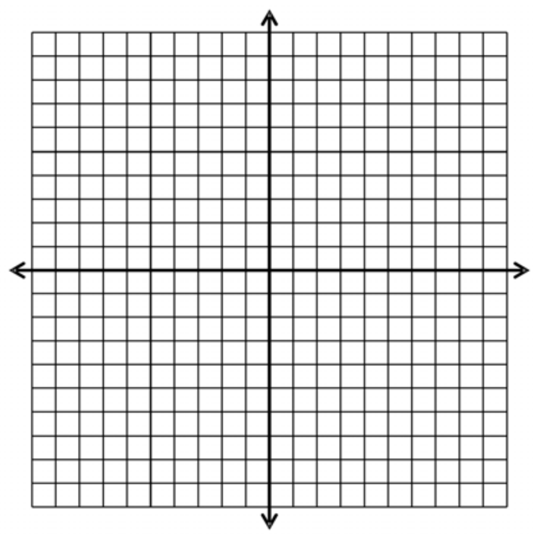
3. $2x - 3y - 6 = 0$

   $x + 3y - 3 = 0$



4. $y = \frac{1}{3}x - 5$

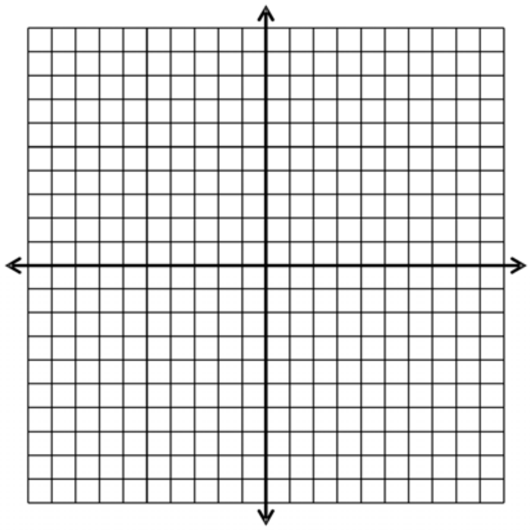
   $y = -\frac{2}{3}x - 2$



4.8.1: Pairs of Equations (continued)

5. $y = 3x + 2$

$2y - 6x - 12 = 0$



6. $y = -\frac{1}{2}x + 3$

$5x + 10y = 30$