

# Unit 2 Trigonometry

## Lesson Outline

### BIG PICTURE

Students will:

- investigate the relationships involved in right-angled triangles to the primary trigonometric ratios, connecting the ratios to constants of proportionality between similar triangles developed in Unit 1;
- solve problems involving right-angled triangles, using the primary trigonometric ratios and the Pythagorean theorem, including problems that require using imperial and metric measurements.

| Day | Lesson Title                                                                   | Math Learning Goals                                                                                                                                                                                                                                                                                | Expectations                     |
|-----|--------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|
| 1   | What's My Ratio?                                                               | <ul style="list-style-type: none"> <li>Determine sine, cosine, and tangent ratios for right-angled triangles using concrete materials.</li> </ul>                                                                                                                                                  | MT2.01<br>CGE 3b                 |
| 2   | What's My Ratio? The Sequel                                                    | <ul style="list-style-type: none"> <li>Investigate the three primary trigonometric ratios for right-angled triangles.</li> <li>Summarize investigations.</li> </ul>                                                                                                                                | MT2.01<br>CGE 5a, 5e             |
| 3   | What's My Angle?                                                               | <ul style="list-style-type: none"> <li>Consolidate investigations for sine, cosine, and tangent ratios of right-angled triangles.</li> <li>Determine angles given trigonometric ratios, using scientific calculators.</li> <li>Research a career that requires the use of trigonometry.</li> </ul> | MT2.01, MT2.04<br>CGE 3e, 7b, 7f |
| 4   | Figure Out the Triangle                                                        | <ul style="list-style-type: none"> <li>Determine the measures of the sides and angles of right-angled triangles using the primary trigonometric ratios and the Pythagorean theorem.</li> </ul>                                                                                                     | MT2.01, MT2.04<br>CGE 4f         |
| 5   | Solving Right-Angled Triangles (Part 1)<br><i>(lesson not included)</i>        | <ul style="list-style-type: none"> <li>Determine the measures of the sides and angles of right-angled triangles, using the primary trigonometric ratios and the Pythagorean theorem.</li> </ul>                                                                                                    | MT2.02<br>CGE 5a                 |
| 6   | Solving Right-Angled Triangles (Part 2)<br><i>(lesson not included)</i>        | <ul style="list-style-type: none"> <li>Determine the measures of the sides and angles in right-angled triangles, using the primary trigonometric ratios and the Pythagorean theorem.</li> </ul>                                                                                                    | MT2.02, MT2.03<br>CGE 3f         |
| 7   | Solving Right-Angled Triangles (Part 3)<br><i>(lesson not included)</i>        | <ul style="list-style-type: none"> <li>Students make and use clinometers to measure angles.</li> <li>Determine the measures of inaccessible objects around the school using the primary trigonometric ratios and clinometers.</li> </ul>                                                           | MT2.03<br>CGE 3c                 |
| 8   | Trigonometric Applications<br><i>(lesson not included)</i>                     | <ul style="list-style-type: none"> <li>Solve problems involving the measures of sides and angles in surveying and navigation problems.</li> <li>Solve trigonometric problems by performing conversions between and within the imperial and metric systems.</li> </ul>                              | MT2.03, MT3.02<br>CGE 5b, 7b     |
| 9   | Who Uses Trigonometry? (Project Presentations)<br><i>(lesson not included)</i> | <ul style="list-style-type: none"> <li>Make a presentation to the class on careers that involve trigonometry.</li> </ul>                                                                                                                                                                           | MT2.04<br>CGE 2a, 2d             |
| 10  | Summative Assessment                                                           |                                                                                                                                                                                                                                                                                                    |                                  |



75 min

**Math Learning Goals**

- Determine sine, cosine, and tangent ratios for right-angled triangles using concrete materials.

**Materials**

- BLM 2.1.1, 2.1.2, 2.13
- string
- protractors
- ribbon
- clothespins

**Assessment Opportunities**

**Minds On... Groups of 4 → Review**

Distribute a right-angled triangle cut-out to each student (BLM 2.1.1). Students find the other three students with similar right-angled triangles to form their working group for the day.

Students use protractors and rulers to confirm if the corresponding angles are equal or sides are equal/proportional.

Reinforce that congruent figures are similar but similar figures are not necessarily congruent. Identify the presence of complementary angles in each of their triangles.

**Action!**

**Whole Class → Demonstration**

Identify the opposite, adjacent sides, and hypotenuse of a right-angled triangle with respect to a given angle. Connect to previous learning where students examined corresponding side measurements of two similar triangles. Explain that they are to explore the ratios within one right-angled triangle and the information that investigation might yield.

Form right-angled triangles using string for sides and three students as vertices of each right-angled triangle. Demonstrate using signs and clothespins, how the opposite and adjacent sides interchange depending on which angle is referenced. Work from inside the triangle. Students holding the sign stand at the reference angle and then walk to the opposite or adjacent sides and the hypotenuse.

Working in groups of four, students measure the lengths of the sides of their triangle and fill in column 1 on BLM 2.1.2. Then they fill in the next four columns of their chart.

Individually, students complete BLM 2.1.3 and share their answers.

**Math Process/Reflecting/Oral Question:** Assess how students reflect on the results of the activity by asking an appropriate question.

**Whole Class → Instruction**

Introduce terminology of *sine*, *cosine*, and *tangent* to define each ratio and as a convenient way of referencing the ratios. Students place these terms above the appropriate columns in BLM 2.1.2.

**Consolidate Debrief Whole Class → Demonstration**

Repeat the string demonstration, asking which sides would be needed to represent the sine, cosine, and tangent ratios as a way of checking students' responses to BLM 2.1.2.

**Home Activity or Further Classroom Consolidation**

Concept Practice

Construct a right-angled triangle that has angle measures different from your group of similar triangles. Measure the lengths of the sides of the triangle in both imperial and metric units. Determine the sine, cosine, and tangent ratios for your triangle.

There is one congruent pair in each set of four similar triangles.

Record the letter names of triangles that go together so groups can be verified before activity.

Word Wall

- complementary angles
- opposite side
- adjacent side
- hypotenuse

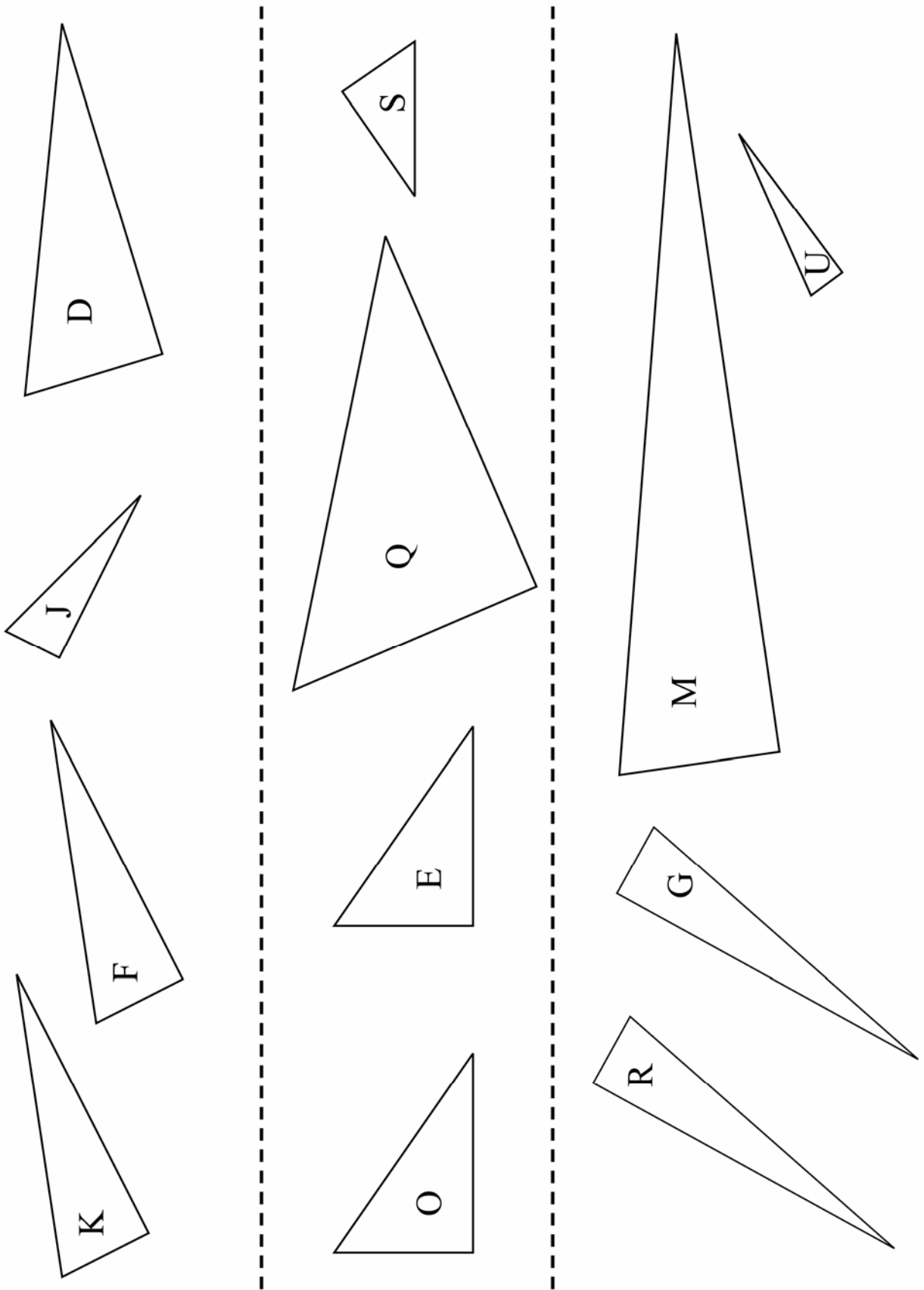
Use a ribbon to mark the student who represents the reference angle.



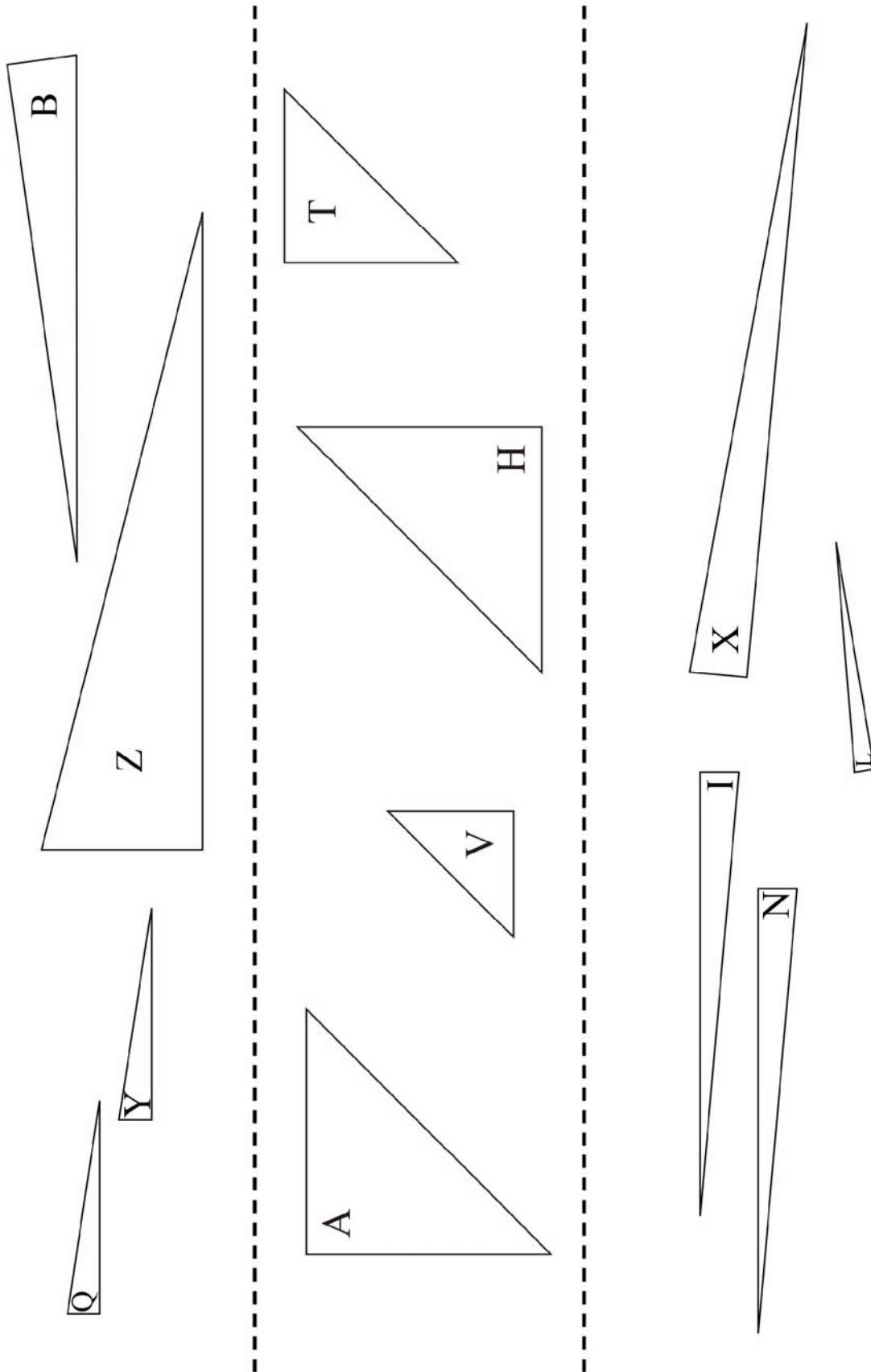
See *Mathematical Processes in Leading Math Success* library

Assign triangles with different angles, e.g., 42°, 48°, 90°.

## 2.1.1: Similar Triangles Template (Teacher)



## 2.1.1: Similar Triangles Template (Teacher) (continued)



## 2.1.2: What's My Ratio? Group Activity

Fill in each of the columns with information for your triangle.

| $\Delta$<br>(state letter) | Angle                                | Length of<br>hypotenuse | Length<br>opposite side | Length of<br>adjacent side | Determine the ratio to two decimal places   |                                             |                                           |
|----------------------------|--------------------------------------|-------------------------|-------------------------|----------------------------|---------------------------------------------|---------------------------------------------|-------------------------------------------|
|                            |                                      |                         |                         |                            | $\frac{\text{opposite}}{\text{hypotenuse}}$ | $\frac{\text{adjacent}}{\text{hypotenuse}}$ | $\frac{\text{opposite}}{\text{adjacent}}$ |
| —                          | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
|                            | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
| —                          | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
|                            | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
| —                          | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
|                            | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
| —                          | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |
|                            | $\sphericalangle$ _____ <sup>o</sup> |                         |                         |                            |                                             |                                             |                                           |

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### 2.1.3: What's My Ratio? Individual Reflection

1. If you have a fifth triangle that is similar to your four triangles, what would your hypothesis be about the following ratios? Explain.

| $\frac{\textit{opposite}}{\textit{hypotenuse}} =$ | $\frac{\textit{adjacent}}{\textit{hypotenuse}} =$ | $\frac{\textit{opposite}}{\textit{adjacent}} =$ |
|---------------------------------------------------|---------------------------------------------------|-------------------------------------------------|
| Explanation:                                      | Explanation:                                      | Explanation:                                    |

2. Identify a relationship between the ratios in the chart for:

$$\frac{\textit{opposite}}{\textit{hypotenuse}} \text{ and } \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

3. Identify a relationship if you divide the ratio for  $\frac{\textit{opposite}}{\textit{hypotenuse}}$  by  $\frac{\textit{adjacent}}{\textit{hypotenuse}}$  for one of the angles.



75 min

**Math Learning Goals**

- Investigate the three primary trigonometric ratios for right-angled triangles.
- Summarize investigations.

**Materials**

- large chart (grid) paper
- 3 colours of stickers

**Assessment Opportunities**

**Minds On... Groups of 4 → Activity**

Students transfer information from the final three columns of BLM 2.1.2 onto the appropriate large grid paper, labelled Sine, Cosine, and Tangent. They use stickers of three different colours, one for each ratio.

**Note:** Developing the graph is used to represent the data collected and provides an example of a non-linear and non-quadratic relation.

Set up one large grid for each of the three ratios to graph results of the whole class.

Horizontal axis – degrees from 0 to 90

Vertical axis – ratios from 0 to 1 in increments of 0.1

**Action!**

**Whole Class → Discussion**

Ask: What do you think the ratio for the sine of 42 degrees is?

Students write individual responses using the graph.

Demonstrate how to obtain the sine, cosine, and tangent ratio using the scientific calculator. Practise with various ratios and angles.

**Individual → Activity**

Students check answers from the Day 1 Home Activity using a calculator. Using stickers, students add more data points to the three large grids.

**Math Process/Using Tools/Observation/Mental Note:** Observe how students use their calculator to find sine, cosine, and tangent of angles.



**Consolidate Debrief Whole Class → Discussion**

Use the three large classroom grids that were created to discuss:

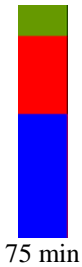
- type of relationship – linear vs. non-linear;
- type of variable – discrete vs. continuous.

**Home Activity or Further Classroom Consolidation**

Use guess and check and your calculator to determine the angle pertaining to the given ratios. Don't use the inverse key.

Exploration

Give students a value that represents a sine ratio, cosine ratio, and a tangent ratio.



**Math Learning Goals**

- Consolidate investigations for sine, cosine, and tangent ratios of right-angled triangles.
- Determine angles given trigonometric ratios, using scientific calculators.
- Research a career that requires the use of trigonometry.

**Materials**

- large grids of sine, cosine, and tangent graphs
- overhead calculator
- BLM 2.3.1

**Assessment Opportunities**

**Minds On... Whole Class → Exploration**

Pose the following: Given an angle in a right-angled triangle we can determine the trigonometric ratios. Can we now determine the angle if we are given the value of the trigonometric ratio?

Response: If we look at the chart on the vertical axis (ratio) we can determine the angle on the horizontal axis. When we think we know it, we can check using technology.

Have the sine, cosine, and tangent graphs on display.

**Action! Whole Class → Guided Exploration**

Introduce the inverse trigonometric ratio key on a scientific calculator. Students check how close their answers are from the Day 2 Home Activity.

Using an overhead calculator demonstrate how to find the size of an angle given the ratio for a specific trigonometric relation. Compare the result with the graph.

**Pairs → Practice**

Each member of the pair creates six questions, each of which is the value of the ratio of sine or cosine. The other student has to determine the angle using the large grid. They check each other's answers using the calculator.

Connect the calculator inverse trigonometric button with the graphical representation of the sine and cosine graphs.

Locate the three grids around the room to allow students to circulate. This will reinforce that the ratios for sine and cosine must be between 0 and 1.

*Think Literacy: Mathematics, Grades 7–9, p. 102*

**Connecting/Observation/Mental Note:** Observe students' facility with using calculators and reading graphs to find the information needed.

**Consolidate Debrief Groups of 4 → Placemat**

On a placemat, students brainstorm what careers might require the use of trigonometry and where they would be able find out more information about that career.

Prompting may be required for this placemat activity.

**Whole Class → Research**

Explain the research project and how students' work will be evaluated (BLM 2.3.1). Students plan their project and complete the form for approval.

**Home Activity or Further Classroom Consolidation**

Begin your approved research project.

*Exploration*



## 2.3.1: Who Uses Trigonometry Project

**Content:** Choose a career of interest that uses trigonometry.

Suggestions:

|             |              |               |
|-------------|--------------|---------------|
| Aerospace   | Archaeology  | Astronomy     |
| Building    | Carpentry    | Chemistry     |
| Engineering | Geography    | Manufacturing |
| Navigation  | Architecture | Optics        |
| Physics     | Sports       | Surveying     |

**Process:** Decide how you will learn more about the use of trigonometry in your chosen career.

Suggestions:

|                   |               |           |
|-------------------|---------------|-----------|
| Internet research | text research | interview |
| job shadow        | job fair      |           |

**Product:** Select the way you will share what you learn.

Suggestions:

|                         |                 |                     |        |
|-------------------------|-----------------|---------------------|--------|
| skit                    | newspaper story | brochure            | poster |
| electronic presentation | photo essay     | verbal presentation | report |

### Personal Selection Chart

| Your name:                         |                                           |         |
|------------------------------------|-------------------------------------------|---------|
| Due date:                          |                                           |         |
| Content                            | Process<br>(you may choose more than one) | Product |
|                                    |                                           |         |
| Teacher's comments and suggestions |                                           |         |

- Your final submission must include the following:
  - the career/activity investigated
  - a brief description of your process
  - description of the career/activity, including how trigonometry plays a role
  - list of sources used
- Your final submission can include some of the following:
  - for a career
    - type of education/training required
    - potential average salary
    - employability
    - example of job posting (newspaper, Internet, etc.)
  - for a topic or activity
    - historical background
    - related issues



**Math Learning Goals**

- Determine the measures of the sides and angles of right-angled triangles using the primary trigonometric ratios and the Pythagorean relationship.

**Materials**

- BLM 2.4.1
- cardboard signs for sine, cosine, tangent, and Pythagorean relationship

**Assessment Opportunities**

**Minds On... Whole Class → Review**

Discuss any issues regarding the research assignment.

Review the conventions for labelling triangles (opposite, adjacent, hypotenuse).

Review the ratios sine, cosine, and tangent, using the terms *opposite*, *adjacent*, and *hypotenuse*.

**Pairs → Investigation**

Draw a right-angled triangle on the board or overhead and provide the degrees of one of the acute angles and the length of one side.

Students investigate how they might use what they have learned previously to find one of the missing sides.

Circulate and ask leading questions, and listen to their dialogue to identify any misconceptions.

Ask: How did you know to use that particular ratio?

Pairs share their strategy for solving the problem with the rest of the class.

Provide further examples and demonstration, as required.

**Curriculum Expectation/Oral Question/Anecdotal Note:** Observe how students label the triangle and identify the ratio to determine the missing sides.

Word Wall

- ratio
- sine
- cosine
- tangent
- hypotenuse

**Action!**

**Whole Class → Guided Instruction**

Using questions 1–4, guide students to determine whether they would use sine, cosine, tangent ratios, or the Pythagorean theorem to solve for the unknown side or indicated angle (BLM 2.4.1). Start at the reference angle on the diagram and draw arrows to the two other pieces of information stated in the problem. One of the pieces will be unknown. Label the sides as opposite, adjacent, or hypotenuse and decide which is the appropriate ratio needed to solve the problem.

As students complete questions 1–4, summarize the correct solution(s). Students then complete questions 5–8 individually.

**Note:** Some of the questions can use more than one method.

Students could use a mnemonic device or make up a sentence to help them to remember the primary trigonometric ratios, e.g., SOHCAHTOA

**Consolidate Debrief**

**Pairs → Share Solutions**

Pairs share their solutions for questions 5–8; identify incongruent solutions; and make corrections, as required.

**Home Activity or Further Classroom Consolidation**

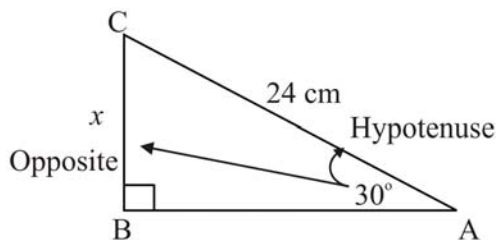
Concept Practice

Complete the practice questions.

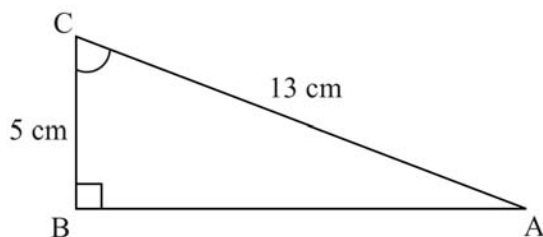
Provide students with appropriate practice questions.

## 2.4.1: What's My Triangle?

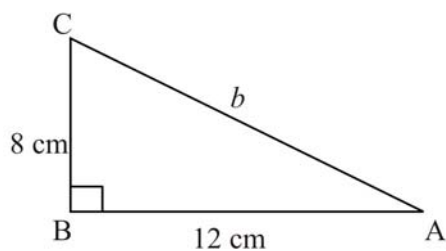
1. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $x$ .  
Solve for  $x$ .



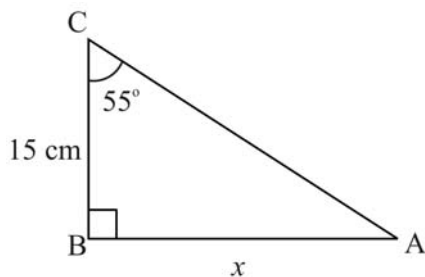
2. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $\angle C$ .  
Solve for  $\angle C$ .



3. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $b$ .  
Solve for  $b$ .

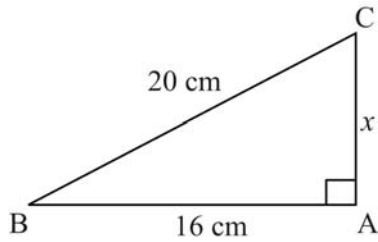


4. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $x$ .  
Solve for  $x$ .

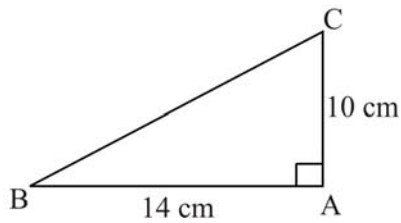


## 2.4.1: What's My Triangle? (continued)

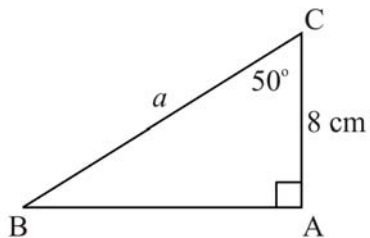
5. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $x$ . Solve for  $x$ .



6. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $\angle B$ . Solve for  $\angle B$ .



7. Decide whether to use sine, cosine, tangent, or the Pythagorean relationship to find  $a$ . Solve for  $a$ .



8. Decide whether to use sine, cosine, tangent, or Pythagorean relationship to find  $\angle C$ . Solve for  $\angle C$ .

