

# Unit 1

## Similar Triangles

Grade 10 Applied

### Lesson Outline

#### BIG PICTURE

Students will:

- investigate similar triangles using their prior knowledge of ratio and proportion;
- solve problems related to similarity, including those using imperial and metric measures;
- manipulate and solve algebraic equations, using prior skills and building new skills to solve equations involving fractions as needed to solve problems.

Day	Lesson Title	Math Learning Goals	Expectations
1	Similar Triangles: Perimeter and Area Relationship	<ul style="list-style-type: none"> <li>• Investigate the relationship between the perimeter and the area of similar triangles.</li> <li>• Use the Pythagorean relationship to find information about triangles.</li> </ul>	MT1.01, MT2.02 CGE 2c, 3c
2	What Is Similarity?	<ul style="list-style-type: none"> <li>• Investigate the properties of similar triangles using geoboards, e.g., corresponding angles are equal and corresponding sides are proportional.</li> </ul>	MT1.01 CGE 3b, 5a
3	Properties of Similar Triangles	<ul style="list-style-type: none"> <li>• Investigate the properties of similar triangles, i.e., corresponding angles are equal and corresponding sides are proportional, using concrete materials.</li> </ul>	MT1.01, MT1.02 CGE 3c, 4b
4	Let's Do Conversions <i>(lesson not included)</i>	<ul style="list-style-type: none"> <li>• Consolidate the results of their investigations and share with whole class.</li> <li>• Perform conversions between the metric and imperial systems.</li> <li>• Solve algebraic equations of the first degree with fractional coefficients to solve proportion problems.</li> </ul>	MT1.01, MT3.02, LR1.01 CGE 3b, 4b, 5e
5	Solving Those Proportions <i>(lesson not included)</i>	<ul style="list-style-type: none"> <li>• Consolidate the skill of solving algebraic equations of the first degree with fractional coefficients, using computer algebra systems and paper and pencil to solve proportion problems.</li> </ul>	LR1.01, MT1.02 CGE 4b, 5b
6	More Proportions and Conversions <i>(lesson not included)</i>	<ul style="list-style-type: none"> <li>• Apply new knowledge to determine the lengths of sides or angle values for similar triangles.</li> <li>• Practise solving proportional statements, as needed.</li> <li>• Use both the imperial and metric systems.</li> </ul>	MT1.02, MT3.01 CGE 4b, 5c
7	How Far? How High?	<ul style="list-style-type: none"> <li>• Solve problems involving similar triangles using measurement data.</li> <li>• Use both the imperial and metric systems.</li> <li>• Perform conversions between metric and imperial systems.</li> </ul>	LR1.01, MT1.03, MT3.01, MT3.02 CGE 5a, 5b
8	Using What We've Learned <i>(lesson not included)</i>	<ul style="list-style-type: none"> <li>• Solve problems involving similar triangles from given situations.</li> <li>• Perform conversions between metric and imperial systems.</li> </ul>	MT1.03, MT3.02 CGE 5g
9	Summative Assessment		



75 min

**Math Learning Goals**

- Investigate the relationship between the perimeter and the area of similar triangles.
- Use the Pythagorean relationship to find information about triangles.

**Materials**

- BLM 1.1.1

**Assessment Opportunities**

**Minds On... Whole Class → Guided Discussion**

Draw several different types of triangles on the board, e.g., right, isosceles, scalene. Ask students to identify them by name, and note that some triangles may have more than one name, e.g., right isosceles. Discuss what information is required to find the perimeter and the area of each triangle. Lead students to recognize that finding the height may require the use of the Pythagorean theorem. Review the Pythagorean theorem.

Orient triangles in various ways so that not all have horizontal bases.

**Groups of 3 → Making a Hypothesis**

Students discuss and make a hypothesis about the relationship between the area and the length of the perimeter of similar triangles, e.g., Given a triangle and a similar triangle whose perimeter is double, what is the effect on its area? Students include reasons for their hypothesis, e.g., their previous knowledge and understanding of area and perimeter, their conceptual understanding of the formulas, a guess resulting from a relevant sketch.

**Action!**

**Groups of 3 → Guided Investigation**

Groups work through BLM 1.1.1. Encourage students to show their work and present their solution in an organized manner. Different groups may come up with different solutions. Have these solutions placed on chart paper for sharing. If groups finish early, ask them if they can come up with an alternative way to solve the problem.

Some students may choose to use GSP®4.

**Mathematical Processes/Problem Solving/Checklist:** Assess how students state a hypothesis, apply problem-solving strategies, and adjust their hypothesis based on new information.

**Consolidate Debrief Whole Class → Guided Discussion**

Consider the results of the investigation. Share different solutions. Facilitate a discussion by asking leading questions such as:

- Considering the formula for the area of a triangle, why do you think the area will be 4 times the original area when the perimeter is doubled?
- Does this logic hold true for halving the perimeter? Explain.
- What do you think will happen if the perimeter is tripled?
- How could you check this?
- What other tools could you use to solve this problem?

There is an opportunity to discuss Pythagorean triples.

**Home Activity or Further Classroom Consolidation**

Investigate if your conclusion to today's problem will be true if the original shape is a rectangle.

Application

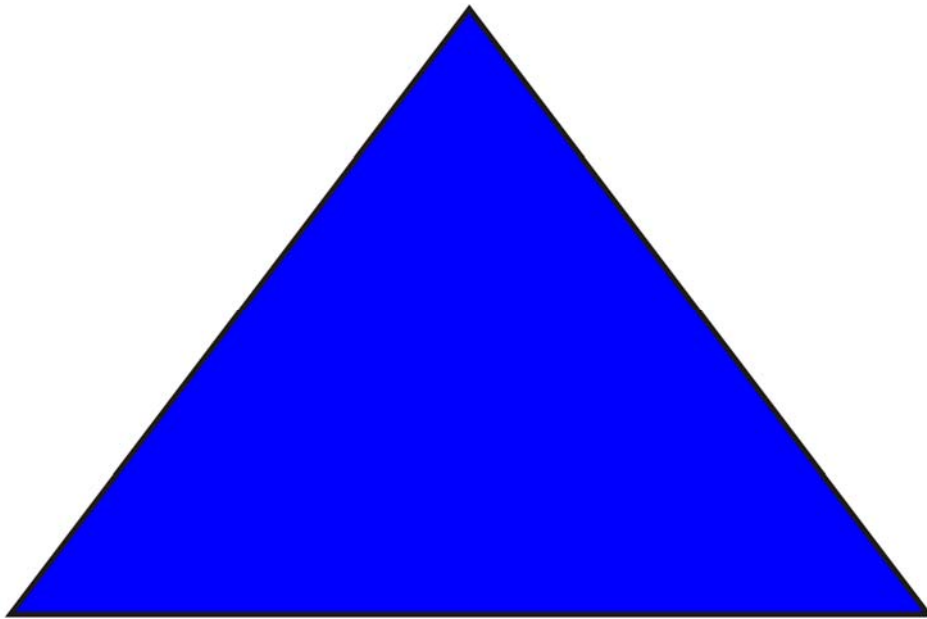
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## 1.1.1: Growing and Shrinking Triangles

### Investigation

Find area and perimeter of the triangle.



If another triangle of the same shape has a perimeter that is double, what is the effect on the area? If another triangle of the same shape has a perimeter that is half, what is the effect on the area?

### Hypothesis

#### Complete the investigation.

Show your work and explain your reasoning. Generalize by stating the relationship between the perimeter and the area of similar triangles. Adjust your hypothesis, as needed.



75 min

**Math Learning Goals**

- Investigate the properties of similar triangles using geoboards, e.g., corresponding angles are equal and corresponding sides are proportional.

**Materials**

- BLM 1.2.1, 1.2.2, 1.2.3
- 11-pin transparent geoboards
- geobands

**Assessment Opportunities**

**Minds On... Pairs → Guided Discussion**

One member of the pair is A and the other is B.

Ask: What does it mean to say that two shapes are similar?

A responds to B, then B responds.

If two triangles are similar, what might be true of their angles and their sides?

B responds to A, then A responds.

**Individual → Activating Prior Knowledge**

Option 1

Students complete the Before column of the Anticipation Guide (BLM 1.2.1).

Option 2

Students complete the What I Know and What I Want to Know columns (BLM 1.2.1).

Select one of the two options on BLM 1.2.1 to activate prior knowledge.

**Action!**

**Pairs → Investigation**

**Learning Skills/Teamwork/Observation/Anecdotal Note:** Observe pairs of students for cooperative learning, sharing of responsibilities, on-task behaviour.

Students complete questions 1–4 on BLM 1.2.2.

Guide students through question 5 to establish properties of similar triangles before completing the remaining questions. Include how to write a similarity equation for the corresponding sides of similar triangles. For question 6, students represent each triangle on a separate geoboard to determine the corresponding angle measurements by translating, rotating, or reflecting.



Provide only the number of bands needed.

Establish that one unit is the horizontal or vertical length between two pegs on the geoboard.

**Consolidate Debrief Pairs → Reflecting**

Students complete the After column or the What I Learned column on BLM 1.2.1.

**Home Activity or Further Classroom Consolidation**

Complete worksheet 1.2.3.

Application

## 1.2.1: What Is Similarity?

### Anticipation Guide

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		In a triangle, I can calculate the length of the third side if I know the length of the other two sides.		
		All triangles are similar.		
		All squares are similar.		
		When I enlarge a geometric shape, the number of degrees in each angle will become larger.		

Notes:

### K-W-L Chart

Statement	What I Know	What I Want to Know	What I Learned
Pythagorean relationship			
If two triangles are similar, then...			

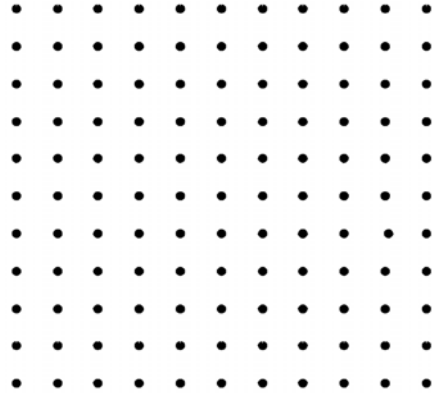
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## 1.2.2: What Is Similarity?

- On your geoboard create a right-angled triangle with the two perpendicular sides having lengths 1 and 2 units.
  - Create two more triangles on your geoboard that are enlargements of the triangle created in a).

- Draw the three triangles using different colours on the grid and label the vertices, as indicated:
  - triangle one (label vertices ABC)
  - triangle two (label vertices DEF)
  - triangle three (label vertices GHJ)



- Determine the lengths of the hypotenuse of each of the triangles.

$\triangle ABC$	$\triangle DEF$	$\triangle GHJ$

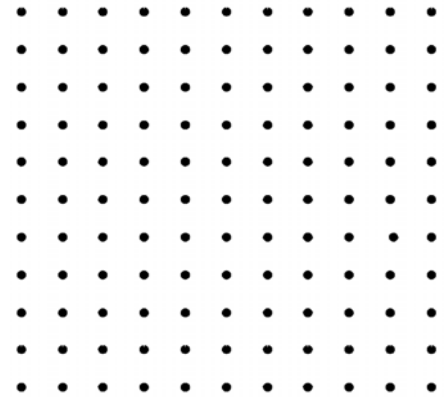
- Indicate the length of each side of each triangle on the diagram.

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## 1.2.2: What Is Similarity? (continued)

4. a) Place  $\triangle ABC$ ,  $\triangle DEF$ , and  $\triangle GHJ$  on the geoboard so that the one vertex of each triangle is on the same peg and two of the sides are overlapping.



- b) Copy your model on the grid.

5. a) What do you notice about the corresponding angles of  $\triangle ABC$ ,  $\triangle DEF$ , and  $\triangle GHJ$ ?

- b) What do you notice about the corresponding sides of  $\triangle ABC$ ,  $\triangle DEF$ , and  $\triangle GHJ$ ?

### Summary

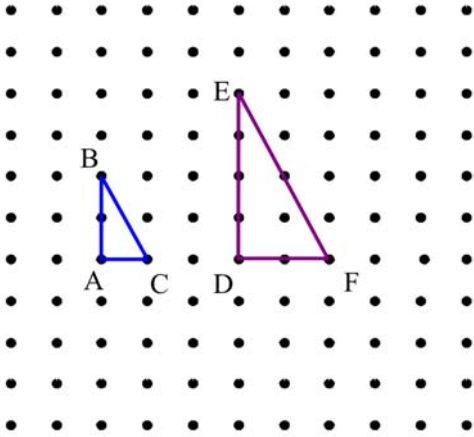
I know the following about similar triangles:

## 1.2.2: What Is Similarity? (continued)

6. Use the geoboards to explore whether the following triangles are similar.

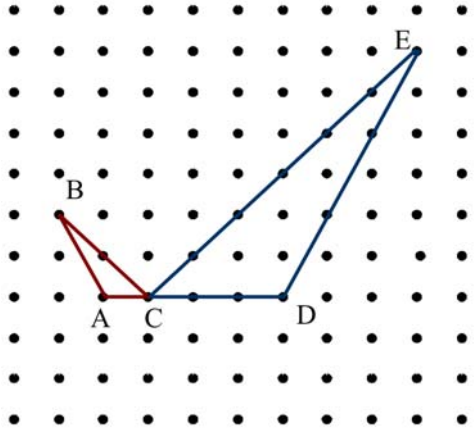
a)

Explain your reasoning.



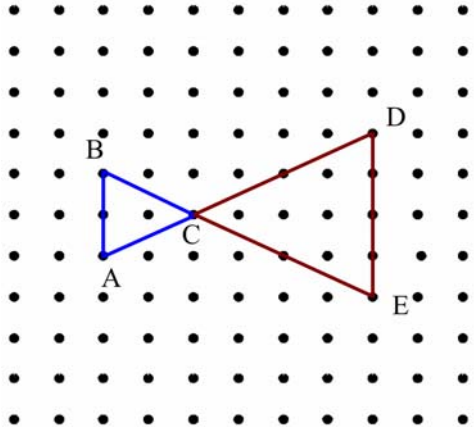
b)

Explain your reasoning.



c)

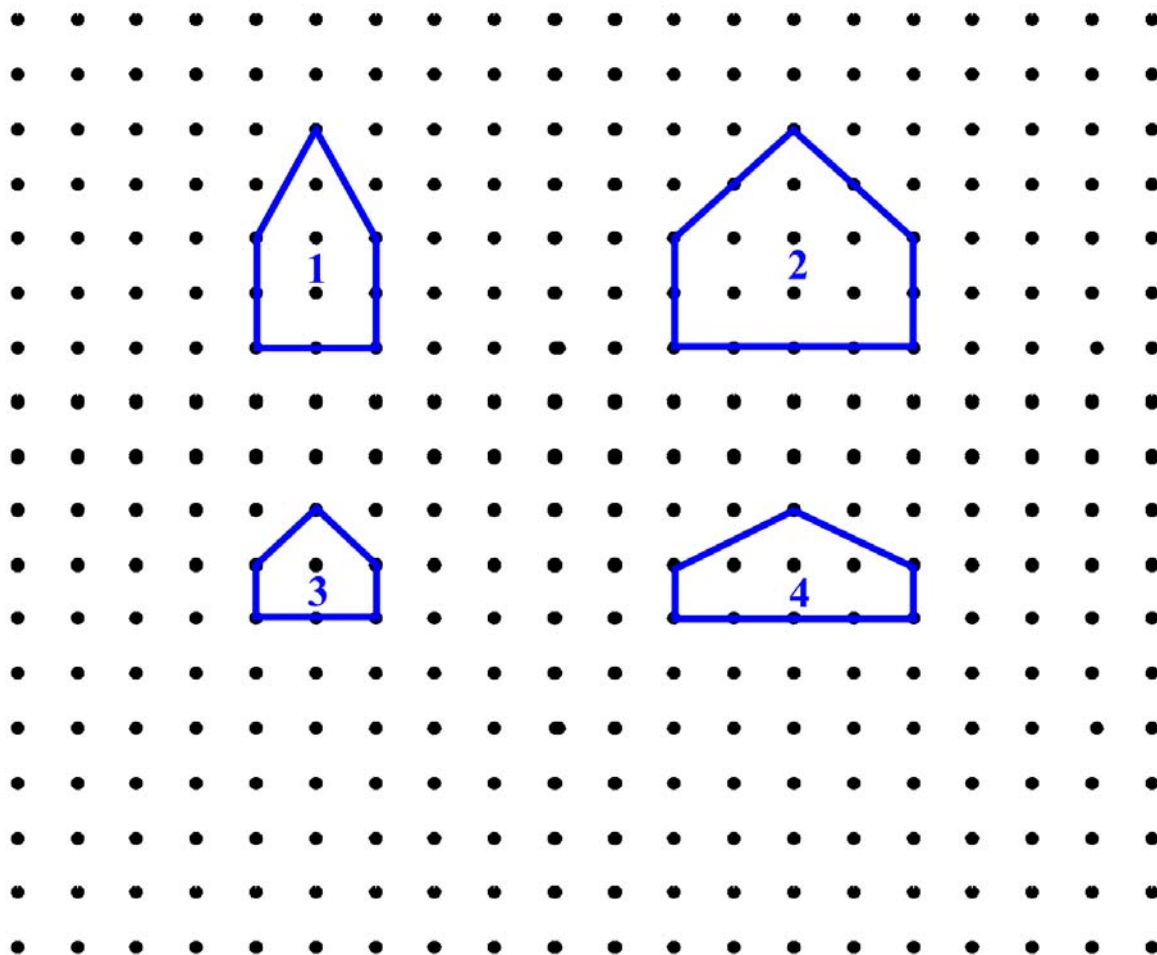
Explain your reasoning.





## 1.2.3: Exploring Similarity

1. Which of the following four houses are similar? Explain why.  
Label the diagrams.



2. On the grid, draw a house that is similar to one of the figures.

Complete the following statement:

The house I drew is similar to house # \_\_\_\_\_.

I know this because:



**Math Learning Goals**

- Investigate the properties of similar triangles, i.e., corresponding angles are equal and corresponding sides are proportional, using concrete materials.

**Materials**

- BLM 1.3.1, 1.3.2
- GSP<sup>®</sup>4 (optional)
- protractors
- rulers
- square, legal- and letter-sized paper

**Assessment Opportunities**

**Minds On... Small Groups → Discussion**

Students complete a Frayer model for similar triangles based on their learning from the previous day’s lesson (BLM 1.3.1). Students should keep this work for reference throughout the course.

Discuss briefly the differences and similarities between similar shapes and congruent shapes.

**Action! Whole Class → Instructions**

Discuss the purpose of the paper folding activity.

Students follow the fold instructions to create similar triangles (BLM 1.3.2).

**Groups of 3 → Exploration**

Each student in the group completes BLM 1.3.2 using a different-sized piece of paper ( $8\frac{1}{2} \times 11$ ,  $8\frac{1}{2} \times 8\frac{1}{2}$ ,  $8\frac{1}{2} \times 14$ ) and they compare their results.

**Reasoning and Proving/Oral Question/Anecdotal Note:** As students work, circulate, and ask questions so they can demonstrate they are using reasoning skills.

**Consolidate Debrief Whole Class → Discussion**

Discuss answers from BLM 1.3.2, reinforcing that similar triangles have equal angles and sides that are proportional. Students should see this connection with the results of the exploration.

Groups share their summaries.

Consolidate how to determine a similarity equation for the corresponding sides of similar triangles, and how to solve the similarity equation using their knowledge of ratios and proportions.

**Home Activity or Further Classroom Consolidation**

Find the missing information for pairs of similar triangles.

*Concept Practice*

Using grid paper or GSP<sup>®</sup>4 facilitates this activity.

Preview the activity prior to assigning it to class.

See Mathematical Processes in LMS Library.

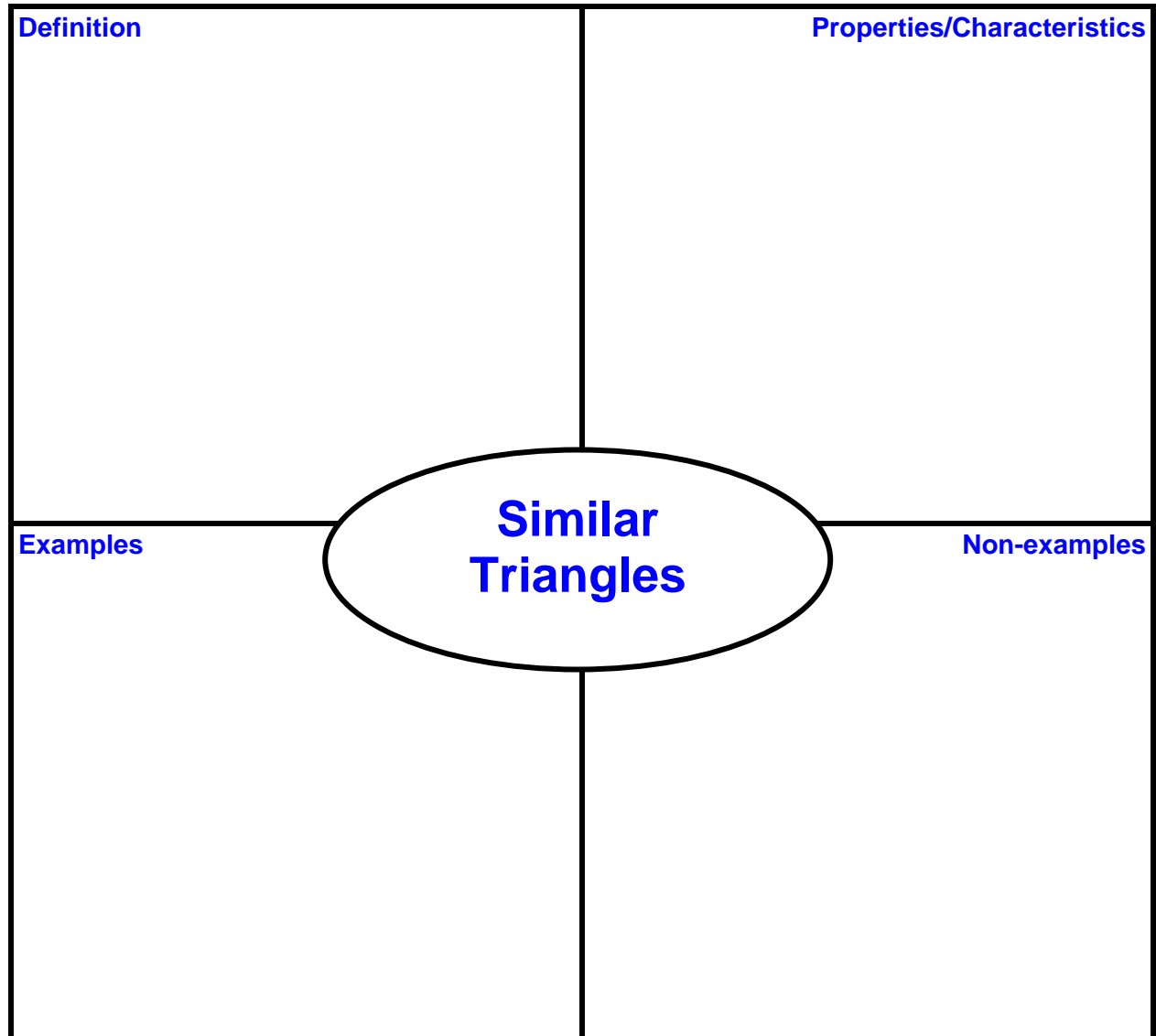


Provide students with several pairs of similar triangles, with some information given about each pair.

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## 1.3.1: Similar Triangles



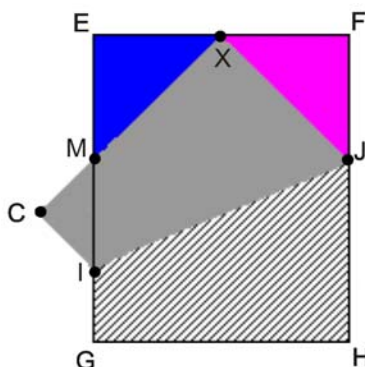
## 1.3.2: Paper Folding

You will each need:

- one sheet of blank paper ( $8\frac{1}{2} \times 11$  or  $8\frac{1}{2} \times 14$  or  $8\frac{1}{2} \times 8\frac{1}{2}$ )  
Each member of your group should have a different size.
- protractor
- ruler

### Part A: Paper Folding

1. Randomly choose a point along the top edge and fold the paper so that the bottom right-hand corner touches this point and label it as shown:



2. Identify any congruent shapes. Explain how you know these shapes are congruent.
  
3. Identify any pairs of triangles you think are similar. Use angles and side lengths to prove or disprove your conjecture.

Similar triangles	Prove/disprove using angles	Prove/disprove using sides

4. Compare your results with the other members of your group. What do you notice?
  
5. If you fold the paper to a different point X along EF, do you think you will still have the same pairs of similar triangles? Explain why or why not. Check with two other points, and compare results with the other members of your group.

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## 1.3.2: Paper Folding (continued)

### Part B: Further Folding

1. Find a point  $X$  along  $EF$  so that  $XFJ$  is an isosceles triangle. Explain how you can verify that this is an isosceles triangle.
2. If  $XFJ$  is an isosceles triangle, how do you know that  $XEM$  and  $MCI$  must also be isosceles? Verify that they are.
3. Find a point  $X$  along  $EF$  such that the fold will result in congruent triangles. Describe or draw a picture of the placement of  $X$ . Name the congruent triangles and verify that they are congruent.
4. Compare where you placed point  $X$  with where the others in your group placed point  $X$ .

### Part C: As a Team of Three

1. Compare your results for the questions in parts A and B. Summarize the information and be prepared to share your response.
2. Investigate other single folds of the paper that will produce two or three similar shapes. Make or draw a model and explain your thinking.



**Math Learning Goals**

- Solve problems involving similar triangles using measurement data.
- Use both the imperial and metric systems.
- Perform conversions between metric and imperial systems.

**Materials**

- BLM 1.7.1, 1.7.2
- measuring tape
- metre stick
- yard stick
- mirrors

**Assessment Opportunities**

**Minds On... Whole Class → Investigation**

Explain the activities How Far? (BLM 1.7.1) and How High? (BLM 1.7.2).  
 Ask: Did you know that your arm is about ten times longer than the distance between your eyes? Verify by measuring.  
 Use the classroom clock or a parked car you can see through the classroom window as an example of the object whose distance you want to determine.

Verifying the ratio of arm length to distance between eyes can lead to a discussion on accuracy as well as specifying the endpoints used to measure the distance between eyes.

**Action! Groups of 4 → Activity**

Students complete the activities How Far? (BLM 1.7.1) and How High? (BLM 1.7.2).  
 Assign each group a different unit of measure (yards, feet, inches, metres, centimetres).  
 Each student writes a complete solution.

Students should have had a lesson on conversion prior to this lesson.

**Curriculum Expectation/Demonstration/Checklist:** Assess how students apply the properties of similar triangles to solve problems.



**Consolidate Debrief Whole Class → Report**

Each group reports on its findings, not giving the units used. Using estimation skills, the class determines the unit of measure the group used.

**Home Activity or Further Classroom Consolidation**

*Application*

Find the height or length of an inaccessible object, using similar triangles, e.g., the height of a tree or streetlight, using either metric or imperial units. Write a short report which includes a mathematical solution, a labelled diagram, and a conversion to imperial or metric.

## 1.7.1: How Far?

Your arm is about ten times longer than the distance between your eyes. Verify.

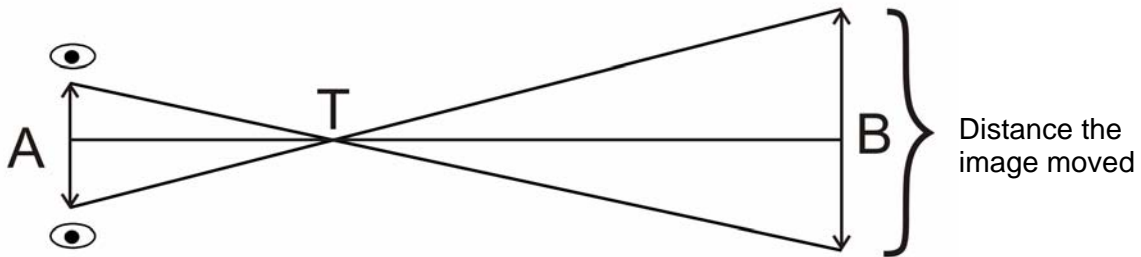
Arm Length \_\_\_\_\_

Distance Between Eyes \_\_\_\_\_

Ratio of Arm Length to Distance Between Eyes \_\_\_\_\_

### Task

1. Select an object from which you want to determine the distance. \_\_\_\_\_ (object)
2. Estimate the width of the object. \_\_\_\_\_
3. Hold one arm straight out in front of you, elbow straight, thumb pointing up. Close one eye, and align one side of your thumb with a particular spot on the front of the object. Without moving your head or arm, sight with the other eye. Your thumb will appear to jump sideways.
  - a) Approximate the number of widths of the object your thumb appeared to move. \_\_\_\_\_
  - b) What is the distance the image moved? \_\_\_\_\_
- 4.



In the diagram:

T is the position of your thumb.

AT represents the length of your arm.

TB represents the distance from your thumb to the object.

- a) Indicate all known measurements on the diagram. Include units.
- b) Identify which triangles are similar. Label the triangle vertices.  
Write the proportion needed to find the distance the object is from you.
- c) Determine the distance the object is from you, using two different methods.

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## 1.7.2: How High?

### Part 1

1. Select an object whose base is at right angles to the ground and whose height you cannot measure.
2. Measure the length of the shadow of the object. (Indicate units.)
3. Hold a metre/yard stick at right angles to the ground, and measure the length of its shadow. (Use the same units as in question 2.)
4. Draw similar triangles representing this situation in the space below. Label the diagram and indicate all known measurements with units.

5. Write the proportion needed to find the desired height.

6. Calculate the height of the object. *Show your work.*



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## 1.7.2: How High? (continued)

### Part 2

1. Select an object whose height you cannot measure. \_\_\_\_\_
2. Lay a small mirror horizontally on the ground exactly 1 metre/yard in front of the object.
3. Slowly walk backwards until you can just see the top of the object in the mirror. Determine your distance from the mirror.
4. Measure the distance from the ground to your eye level.
5. Draw similar triangles representing this situation in the space below. Label the diagram and indicate all known measurements with units.
6. Write the proportion needed to find the desired height.
7. Calculate the height of the object. *Show your work.*