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TIPS4RM

Leading
MATH
Success

TIPS4RM Targeted Implementation
and Planning Supports for
Revised Mathematics

Continuum and Connections

Patterning to Algebraic Modelling

Overview

Context Connections

- Positions patterning to algebraic modelling in a larger context and shows connections to everyday situations, careers, and tasks
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers
- Includes relevant specific expectations for each grade
- Summarizes prior and future learning

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied
- Includes advice on how to help students develop understanding

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (patterning to algebraic modelling) as an organizer

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question.*
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated
- Serves as a model for developing other questions relevant to the learning expected for the grade/course

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate
- Focuses on problem-solving strategies, involving multiple mathematical processes
- Provides an opportunity for students to engage with the problem at many levels
- Provides problems appropriate for students in Grades 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?




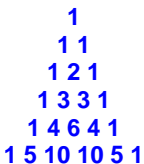
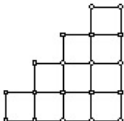
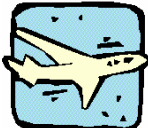




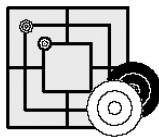

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical processes Reasoning and Proving, and Reflecting

Patterning to Algebraic Modelling

Context

- Patterning is an integral part of all strands in the elementary curriculum. It is the foundation for the study and application of relations, the cornerstone of higher-level mathematics.
- The mystery of algebra and the fear of “letters” are reduced when patterning is used to develop an understanding of variables.
- A math trail of patterns could include flowers, house numbers, and architectural designs – physical, numerical, and geometrical patterns.
- Curriculum expectations require students to identify, extend, create, analyse, discuss, and explain patterns to develop understanding that leads to algebraic modelling.

Context Connections

			
Architecture	Calendars	Geometric Designs	Pascal's Triangle
			
Staircases	Schedules	Music	Fibonacci Sequence
			
Cab Fares	Postage Rates	Puzzles and Games	Other Connections

Manipulatives

- pattern blocks
- colour tiles
- toothpicks
- cubes

Technology

- spreadsheets
- The Geometer's Sketchpad[®] 4
- calculators/graphing calculators

Other Resources

<http://standards.nctm.org/document/chapter6/alg.htm>

http://matti.usu.edu/nlvm/nav/frames_asid_169_g_1_t_2.html

<http://standards.nctm.org/document/eexamples/chap4/4.1/>

<http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#Rabbits>

Connections Across Grades

Selected results of word search using the *Ontario Curriculum Unit Planner*

Search Words: *pattern, variable, algebraic, model, differences, rule*

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
<ul style="list-style-type: none"> identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically; make tables of values for growing patterns given pattern rules (in words), then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a variety of tools; determine the term number of a given term in a growing pattern that is represented by a pattern rule in words, a table of values, or a graph; describe pattern rules (in words) that generate patterns by adding or subtracting a constant or multiplying or dividing by a constant to get the next term, then distinguish such pattern rules from pattern rules (given in words) that describe the general term by referring to the term number; determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term; extend and create repeating patterns that result from rotations, through investigation and using a variety of tools; demonstrate an understanding of different ways in which variables are used; 	<ul style="list-style-type: none"> represent linear growing patterns, using a variety of tools and strategies; make predictions about linear growing patterns, through investigation with concrete materials; develop and represent the general term of a linear growing pattern, using algebraic expressions involving one operation; compare pattern rules that generate a pattern by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term with pattern rules that use the term number to describe the general term; model everyday life relationships involving constant rates, where the initial condition starts at 0, through investigation and using tables of values and graphs; model everyday life relationships involving constant rates, using algebraic equations with variables to represent the changing quantities in the relationship; translate phrases describing simple mathematical relationships into algebraic expressions, using concrete materials; evaluate algebraic expressions by substituting natural numbers for the variables; make connections between evaluating algebraic expressions and determining the term in a pattern using the general term; 	<ul style="list-style-type: none"> represent, through investigation with concrete materials, the general term of a linear pattern, using one or more algebraic expressions; represent linear patterns graphically, using a variety of tools; determine a term, given its term number, in a linear pattern that is represented by a graph or an algebraic equation; describe different ways in which algebra can be used in everyday life situations; model linear relationships using tables of values, graphs, and equations, through investigation and using a variety of tools; translate statements describing mathematical relationships into algebraic expressions and equations; evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables; make connections between solving equations and determining the term number in a pattern, using the general term; solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a “balance” model. 	<p>Applied and Academic</p> <ul style="list-style-type: none"> describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three; substitute into and evaluate algebraic expressions involving exponents; pose problems, identify variables, and formulate hypotheses associated with relationships between two variables; relate the geometric representation of the Pythagorean theorem to the algebraic representation $a^2 + b^2 = c^2$; determine other representations of a linear relation arising from a realistic situation, given one representation; solve problems that can be modelled with first-degree equations, and compare the algebraic method to other solution methods. <p>Applied</p> <ul style="list-style-type: none"> solve for the unknown value in a proportion, using a variety of methods; multiply a polynomial by a monomial involving the same variable to give results up to degree three, using a variety of tools; carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology and techniques. <p>Academic</p> <ul style="list-style-type: none"> use patterning to derive the multiplication, division, and power exponent laws; multiply a polynomial by a monomial involving the same variable; design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology and techniques; identify $y = mx + b$ as a common form for the equation of a straight line; identify properties of the slopes of lines and line segments; 	<ul style="list-style-type: none"> verify properties of similar triangles; determine relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle and define the sine, cosine, and tangent ratios; determine the relationship for calculating the surface area of a pyramid; determine the value of a variable in the first degree, using a formula; identify $y = mx + b$ as a common form for the equation of a straight line; identify properties of the slopes of lines and line segments; determine the equation of a line, given its graph, the slope and y-intercept, the slope and a point on the line, or two points on the line; determine graphically the point of intersection of two linear relations; solve problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables; expand and simplify second-degree polynomial expressions; factor binomials, simple trinomials, and the difference of squares; collect data that can be represented as a quadratic equation; determine that a quadratic relation of the form $y = ax^2 + bx + c$ can be graphically represented as a parabola, and determine that the table of values yields a constant second difference;

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
<ul style="list-style-type: none"> ▪ solve problems that use two or three symbols or letters as variables to represent different unknown quantities; ▪ determine the solution to a simple equation with one variable, through investigation and using a variety of tools and strategies; ▪ compare, through investigation, different graphical representations of the same data; ▪ extend and create repeating patterns that result from rotations, through investigation and using a variety of tools. 	<ul style="list-style-type: none"> ▪ solve linear equations of the form $ax = c$ or $c = ax$ and $ax + b = c$, or variations such as $b + ax = c$ and $c = bx + a$, by modelling with concrete materials, by inspection, or by guess and check, with and without the aid of a calculator. 		<ul style="list-style-type: none"> ▪ determine the equation of a line, given its graph, the slope and y-intercept, the slope and a point on the line, or two points on the line; ▪ determine graphically the point of intersection of two linear relations. 	<ul style="list-style-type: none"> ▪ compare the graphical representations of a quadratic relation in the form of $y = x^2 + bx + c$ and the same relation in the factored form $y = (x - r)(x - s)$ and describe the connections between each algebraic representation and the graph; ▪ solve problems involving a quadratic relation by interpreting a given graph or a graph generated with technology from its equation.

Summary of Prior Learning

In earlier years, students:

- describe, represent, extend, and create growing and shrinking patterns using words, tables of values, and graphs;
- represent patterns numerically and geometrically;
- create and extend repeating patterns from reflections, with and without concrete materials;
- understand that patterns are repeated operations, actions or repeated change to an attribute.

In Grade 7, students:

- continue to identify, create, describe, and extend linear growth patterns using a variety of strategies;
- use patterns to relate multiplication of fractions by a whole number to repeated addition of fractions;
- develop formulas for area of trapezoids, volume of right prisms, and surface area of right prisms;
- use patterns to sort and classify triangles and quadrilaterals, to describe congruent shapes;
- continue to use patterns to solve simple linear equations;
- describe trends and make inferences from data.

In Grade 8, students:

- continue to identify, create, describe, and extend linear growth patterns;
- generalize patterns with equations;
- develop proficiency in multiplying and dividing fractions and integers;
- develop formulas for circumference and area of circles, volume of right prisms, and surface area of cylinders;
- sort and classify geometric figures and determine geometric relationships;
- connect patterns to equations;
- continue to describe trends and make inferences from data.

In Grade 9 Applied, students:

- connect the algebraic and geometric representations of a single-variable term up to degree three;
- determine relationships of optimal values of rectangles;
- connect the geometric and algebraic representations of the Pythagorean theorem;
- develop formulas for volume;
- continue to determine geometric relationships;
- continue to describe trends and make inferences from data.

In Grade 10 Applied, students:

- develop the trigonometric ratios of sine, cosine, and tangent;
- develop the formula for surface area of pyramids;
- continue to use patterns to distinguish between linear and non-linear relations;
- identify the properties of slope and properties of linear and quadratic relationships;
- use patterns to connect the various representations and connect the graphical representations $y = x^2 + bx + c$ and $y = (x - r)(x - s)$ of quadratic relations.

In later years

- Students' choice of courses will determine the degree to which they apply their understanding of concepts related to patterning and algebraic modelling.

Instruction Connections

Suggested Instructional Strategies	Helping to Develop Understanding															
<p>Grade 7</p> <ul style="list-style-type: none"> Provide a variety of experiences with concrete materials (e.g., toothpicks, pattern blocks, interlocking cubes) for students as they explore patterns individually and in groups. Most patterns will be linear (add a constant to get successive terms) to lead to the study of linear relations in Grades 9 and 10. Use a table of values to organize information and develop understanding that connects the term number to the value of the term, e.g., the term number to the actual term. See Developing Proficiency Questions, Grade 7. The Understanding column for the following chart is a necessary step in building understanding of the connections between concrete and algebraic models. Example: <table border="1" data-bbox="219 737 581 894"> <thead> <tr> <th>Stage</th> <th>Perimeter</th> <th>Understanding</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>6</td> <td>6</td> </tr> <tr> <td>2</td> <td>8</td> <td>$6 - 1 + 3$</td> </tr> <tr> <td>3</td> <td>10</td> <td>$6 - 1 + 2 + 3$</td> </tr> <tr> <td>4</td> <td>12</td> <td>$6 - 1 + 2 + 2 + 3$</td> </tr> </tbody> </table> <p><i>Explanation: To determine the perimeter of stage 4 start with 6 (hexagon sides) then subtract, 1 since one side is not included once a square is added. Add 2 for each middle square, then add 3 for the final square.</i></p> <ul style="list-style-type: none"> Ask students to make predictions from concrete materials, tables, and graphs. Have students express the word rule stated above algebraically as $6 - 1 + 2(n - 2) + 3$ where n is the stage number (n can also be called the “term number”). Most patterns can be interpreted in more than one way (see Grade 8 example below). Encourage students to look at multiple pattern rules for the same pattern. The Understanding column of the chart could vary – encourage different representations. Have students create pattern rule(s) in word statements. Grade 7 sample response: The rule for this pattern is: 6 – 1 added to two times the number of middle squares added to 3. The number of middle squares is two less than the stage number. (The last sentence must be included for a complete description.) Expect students to explain their rules to each other and to justify their rules by showing how their rule(s) work for given stages and later stages/terms. Play games such as What’s My Pattern? in which the teacher or a student thinks of a one-operation expression (e.g., $n + 3$). Students give “input” value (e.g., 4), teacher gives “output” value (7). This is repeated until students determine correct expression. This plays well in pairs. Frequently use everyday relationships that show linear patterns, e.g., wages, speed, house numbers. Give students a number pattern or graph and have them create a situation that could be modelled by the pattern. Have students represent patterns concretely, numerically, and graphically. 	Stage	Perimeter	Understanding	1	6	6	2	8	$6 - 1 + 3$	3	10	$6 - 1 + 2 + 3$	4	12	$6 - 1 + 2 + 2 + 3$	<ul style="list-style-type: none"> Use the “fill-down” feature of a spreadsheet when students are comfortable with explaining a pattern rule based on the previous term, e.g., using recursion Help students understand the benefits of using a pattern rule based on the term number, e.g., by asking for the one-hundredths term. Encourage students to analyse charts by looking across a row as well as down a column. Help students connect new patterns to prior experiences – look for common elements in building the patterns, e.g., using the first term, using repeated terms to create a multiplication, using constants. Use a concrete model to establish a pattern. Encourage students to add several rows to a chart to see a pattern unfold. Help students see the connections between number patterns, algebraic expressions, tables, and graphs. If a student is uncomfortable with one representation, e.g., an algebraic model based on the term number, this will help them to see the connections between a model with which they are comfortable and a new one they are striving to understand. Help students develop proficiency at converting simple word statements into algebraic expressions when translating word rules into algebraic expressions, e.g., two less than the term number translates into $n - 2$, where n is the term number. Have peers and small groups check oral translations and provide supportive feedback while practising examples. Use of calculators makes the study of patterns more accessible. Guide students to see the connection between constant rates of change, constant first differences, and graphic representations of linear relations. Use of data collection devices is incorporated in the expectations related to quadratics. Use of computer algebra systems makes the study of the patterns more accessible. Students need to see the connection between the constant second difference, the algebraic, and the graphical representation of a quadratic relation. Students need to connect the x-intercepts of a quadratic relation to the factored form.
Stage	Perimeter	Understanding														
1	6	6														
2	8	$6 - 1 + 3$														
3	10	$6 - 1 + 2 + 3$														
4	12	$6 - 1 + 2 + 2 + 3$														

Suggested Instructional Strategies	Helping to Develop Understanding																
<p>Grade 8</p> <ul style="list-style-type: none"> Use concrete models, word descriptions, graphs, and algebraic expressions to describe patterns. The simplified expression for the Grade 7 pattern shown above is $(4 + 2n)$. Students in Grade 8 should be able to verify through substitution that different correct algebraic models will yield the same result for any value of n. Integrate words and math symbols. Further develop the concept of variable by encouraging students to discover, justify, and explain alternative algebraic representations based on different interpretations of the pattern. <p>Example: Count the number of sides for all shapes, then subtract the interior sides (hexagon sides) $+ 4 \times$ (number of squares) $- 2 \times$ (number of squares). For the 4th stage this would be: $6 + 4(3) - 2(3)$, which equals 12. For the n^{th} stage this would be: $6 + 4(n - 1) - 2(n - 1)$.</p> <p>If $n = 10$ then:</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">$6 - 1 + 2(n - 2) + 3$</td> <td style="width: 50%; border: none;">$6 + 2(n - 1)$</td> </tr> <tr> <td style="border: none;">$= 6 - 1 + 2(10 - 2) + 3$</td> <td style="border: none;">$= 6 + 2(10 - 1)$</td> </tr> <tr> <td style="border: none;">$= 6 - 1 + 16 + 3$</td> <td style="border: none;">$= 6 + 18$</td> </tr> <tr> <td style="border: none;">$= 24$</td> <td style="border: none;">$= 24$</td> </tr> </table> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">$6 + 4(n - 1) - 2(n - 1)$</td> <td style="width: 50%; border: none;">$2n + 4$</td> </tr> <tr> <td style="border: none;">$= 6 + 4(10 - 1) - 2(10 - 1)$</td> <td style="border: none;">$= 2(10) + 4$</td> </tr> <tr> <td style="border: none;">$= 6 + 36 - 18$</td> <td style="border: none;">$= 24$</td> </tr> <tr> <td style="border: none;">$= 24$</td> <td></td> </tr> </table> <ul style="list-style-type: none"> Justify an algebraic pattern rule. Example: Jenna correctly determined that the n^{th} term of the sequence $\{3, 5, 7, \dots\}$ is $3 + 2(n - 1)$. Justify Jenna's answer. Note that it would be insufficient to show that the expression "works" for a few cases. Students explain that the sequence starts with 3 and then 2 is added $(n - 1)$ times. So the n^{th} term is $3 + (2 + 2 + 2 + \dots + 2)$ where the number of 2s is $(n - 1)$. Continue to use a variety of everyday relationships to show linear patterns. <p>Grade 9 Applied</p> <ul style="list-style-type: none"> Use concrete materials and algebraic methods to show that two algebraic models are equivalent. Example: Each expression above simplifies to become $(2n + 4)$. Recall familiar patterns from Grade 8 – calculate first differences and look for patterns in the algebraic models of linear relations to make connections between tables of values, equations, and graphs. Connect the making of predictions using an algebraic model with making predictions by extrapolating or interpolating graphs. Use contextual situations to apply algebraic modelling to solve problems, e.g., given information about rental companies, students find which one offers the best deal. Connect geometric and algebraic models, e.g., the Pythagorean theorem. Connect concrete, graphical, numeric, and algebraic representations. 	$6 - 1 + 2(n - 2) + 3$	$6 + 2(n - 1)$	$= 6 - 1 + 2(10 - 2) + 3$	$= 6 + 2(10 - 1)$	$= 6 - 1 + 16 + 3$	$= 6 + 18$	$= 24$	$= 24$	$6 + 4(n - 1) - 2(n - 1)$	$2n + 4$	$= 6 + 4(10 - 1) - 2(10 - 1)$	$= 2(10) + 4$	$= 6 + 36 - 18$	$= 24$	$= 24$		
$6 - 1 + 2(n - 2) + 3$	$6 + 2(n - 1)$																
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$= 6 + 4(10 - 1) - 2(10 - 1)$	$= 2(10) + 4$																
$= 6 + 36 - 18$	$= 24$																
$= 24$																	

Suggested Instructional Strategies	Helping to Develop Understanding
<p>Grade 10 Applied</p> <ul style="list-style-type: none"> • Use algebraic modelling to describe trigonometric relationships. • Use patterning skills to determine relationships within metric and imperial systems as well as between the systems. • Use geometric contexts to introduce quadratic relationships, e.g., determine the relationship between the surface area and the side length of a cube. • Use technology to generate equations that model the quadratic relationships. • Use patterning skills to determine the relationship between the coefficients in a trinomial and their binomial factors. 	

Connections Across Strands

Note

Summary or synthesis of curriculum expectations is in plain font.

Verbatim curriculum expectations are in italics.

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
<ul style="list-style-type: none"> relate multiplication of fractions by a whole number to repeated addition of fractions <i>determine, through investigation, the relationships among fractions, decimals, percents, and ratios</i> 	<ul style="list-style-type: none"> develop formulas for the area of trapezoids, volume of right prisms, and surface area of right prisms 	<ul style="list-style-type: none"> recognize patterns to help sort and classify triangles and quadrilaterals use patterns to describe congruent shapes <i>determine, through investigation using a variety of tools, polygons or combinations of polygons that tile a plane, and describe the transformation(s) involved</i> 	<ul style="list-style-type: none"> solve linear equations of the form $c = ax$, $ax = c$, $ax + b = c$, $c = ax + b$, using a variety of methods, e.g., inspection, guess and check see Connections Across the Grades, p. 3 	<ul style="list-style-type: none"> use patterns to identify, describe, and interpret trends, and make inferences and conclusions from data presented in tables, graphs, and charts make predictions about a population when given a probability

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
<ul style="list-style-type: none"> <i>represent the multiplication and division of fractions, using a variety of tools and strategies</i> <i>represent the multiplication and division of integers, using a variety of tools</i> 	<ul style="list-style-type: none"> develop formulas for circumference of a circle, area of a circle, volume for right prisms, and surface area of a cylinder 	<ul style="list-style-type: none"> <i>sort and classify quadrilaterals by geometric properties, including those based on diagonals, through investigation and using a variety of tools</i> <i>determine the Pythagorean relationship, through investigation and using a variety of tools</i> <i>determine through investigation using concrete materials the relationship between the numbers of faces, edges, and vertices of a polyhedron</i> 	<ul style="list-style-type: none"> make connections between patterns and equations see Connections Across the Grades, p. 3 	<ul style="list-style-type: none"> use patterns to identify, describe, interpret trends, and make inferences and conclusions from data (including rate of change of data) presented in tables, graphs, and charts

Grade 9 Academic

Number Sense and Algebra	Measurement and Geometry	Analytic Geometry	Linear Relations
<ul style="list-style-type: none"> see Connections Across the Grades, p. 3 	<ul style="list-style-type: none"> determine the relationship of optimal values of rectangles, e.g., maximum area given perimeter; minimum perimeter given area determine the relationships of optimal values by varying the dimensions of square-based prisms and cylinders relate the geometric representation of the Pythagorean theorem to the algebraic representation $a^2 + b^2 = c^2$ develop formulas for volumes of pyramids, cones, and spheres, and surface area of pyramids determine, through investigation using a variety of tools, geometric relationships, e.g., angles of triangles, quadrilaterals, polygons, properties of polygons 	<ul style="list-style-type: none"> determine the properties of linear and non-linear relationships using graphs and equations determine the properties of slopes of lines and line segments connect various forms of slopes (rise/run, change in y/change in x, $\Delta y/\Delta x$, $(y_2 - y_1)/(x_2 - x_1)$ and lines $y = mx + b$, $ax + by + c = 0$, $x = a$, $y = b$) through investigations, explain the geometric significance of m and b in $y = mx + b$ determine, through investigation, connections among the representations of a constant rate of change of a linear relation identify and explain any restrictions on the variables in a linear relation arising from a realistic situation 	<ul style="list-style-type: none"> describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses identify, through investigations, some properties of linear relations, and apply these properties to determine whether a relation is linear or non-linear determine the equation of a line of best fit using an informal process

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relations
<ul style="list-style-type: none"> see Connections Across the Grades, p. 3 	<ul style="list-style-type: none"> determine the relationship of optimal values of rectangles, e.g., maximum area given perimeter; minimum perimeter given area relate the geometric representation of the Pythagorean theorem to the algebraic representation $a^2 + b^2 = c^2$ develop formulas for volumes of pyramids, cones, and spheres investigate geometric relationships, using a variety of tools 	<ul style="list-style-type: none"> describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses identify, through investigation, some properties of linear relations, and apply these properties to determine whether a relation is linear or non-linear describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied

Grade 10 Applied

Measurement and Trigonometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
<ul style="list-style-type: none"> determine, through investigation, the trigonometric ratios of sine, cosine, and tangent develop the formula for surface area of pyramids 	<ul style="list-style-type: none"> use patterns to identify the properties of slope of lines and line segments, explain the geometric significance of m and b in the equation of a line, $y = mx + b$, and connect the special cases $x = a$ and $y = b$ to $y = mx + b$ 	<ul style="list-style-type: none"> move between the numerical, graphical, and algebraic representations for quadratic relations connect the graphical representations, $y = x^2 + bx + c$ and $y = (x - r)(x - s)$ of quadratic relations

Name:

Date:

Expectation – Patterning and Algebra, 7m61:

Make predictions about linear growing patterns, through investigation with concrete materials.

Knowledge and Understanding

(Facts and Procedures)

Helena created a table to look for a pattern in the given figures. The figure number in the last row is 500.

Complete Helena's table.

Figure number	Number of sides
1	3
2	5
3	7
4	
5	
500	



Figure 1



Figure 2

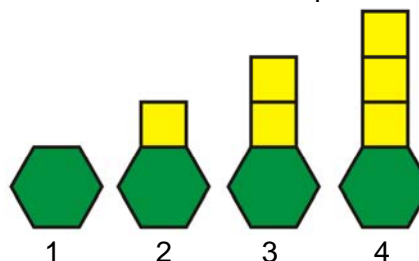


Figure 3

Knowledge and Understanding

(Conceptual Understanding)

The picture shows 4 stages in the construction of a walkway. The walkway starts with a hexagon and continues with squares.



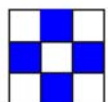
Ryan created a table:

Stage	Perimeter	Ryan's pattern
1	6	6
2	8	$6 - 1 + 3$
3	10	$6 - 1 + 2 + 3$
4	12	$6 - 1 + 2 + 2 + 3$

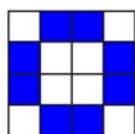
Explain Ryan's pattern. How could you use this pattern to determine the perimeter at any stage?

Problem Solving

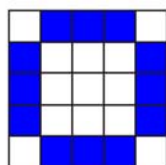
(Reasoning and Proving, Connecting)



3 × 3 design
4 dark tiles



4 × 4 design
8 dark tiles



5 × 5 design
12 dark tiles

A tiling company specializes in multi-colour tile patterns. A small hotel is interested in the pattern above for its square-shaped reception area. How many dark-coloured tiles will there be if the reception area needs 18 tiles on each side?

Show your work.

Problem Solving

(Reasoning and Proving, Reflecting)

The picture shows rows of houses constructed using toothpicks.

Note: The walls connecting adjacent houses are constructed using one toothpick.



Chandra has 50 toothpicks. What is the greatest number of houses in 1 row that she can construct? Will any toothpicks be left over?

Explain your steps.

Name:

Date:

Expectation – Patterning and Algebra, 8m56:

Represent, through investigation with concrete materials, the general term of a linear pattern, using one or more algebraic expressions.

Knowledge and Understanding (Facts and Procedures)

Helena created a table to look for a pattern in the number of toothpicks used to make the given figures.



Figure 1



Figure 2



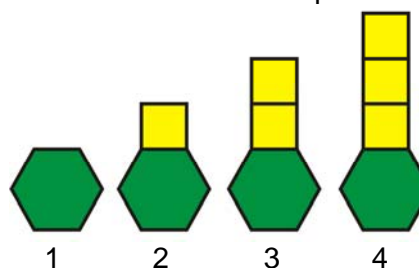
Figure 3

Complete Helena's table.

Figure number	Number of toothpicks
1	3
2	5
3	7
4	
5	
n	

Knowledge and Understanding (Conceptual Understanding)

The picture shows 4 stages in the construction of a walkway. The walkway starts with a hexagon and continues with squares.



Complete the table.

Stage number	Perimeter
1	
2	
3	
4	

Determine the general term for the perimeter.

Explain your steps.

Problem Solving (Representing)

Jaclyn found $6n - 1$ written on the board when she came into class. On her table were pattern blocks, toothpicks, and interlocking cubes.

Use one of these materials to represent the first three terms of the pattern created by the general term $6n - 1$.

Explain why you chose this representation.

Problem Solving (Reasoning and Proving, Representing)

Two patterns are shown below.

Pattern A



Figure 1



Figure 2



Figure 3

Pattern B



Figure 1



Figure 2



Figure 3

Harpreet worked with one of the patterns and determined the perimeter of the n^{th} figure to be $P = 3 + 3n - 1$.

Identify Harpreet's pattern.

Justify your choice.

Name:

Date:

Expectation – Linear Relations, LR2.03:

Identify, through investigation, some properties of linear relations and apply these properties to determine whether a relation is linear or non-linear.

Knowledge and Understanding (Facts and Procedures)

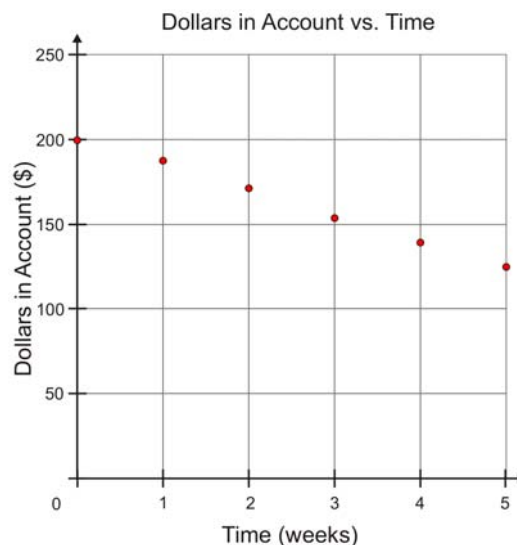
Calculate first differences to determine if the height of a baseball and the time since it was hit is a linear or non-linear relationship.

Give reasons for your answer.

Time (s)	Height (m)	First Difference
0	1	
1	16	
2	21	
3	16	
4	1	

Knowledge and Understanding (Conceptual Understanding)

Sebastian graphed the relationship between the amount of money in his bank account and time in weeks.



Determine the relationship between the money in Sebastian's account and time in more than one way.

Problem Solving (Reasoning and Proving, Reflecting)

Reyna made an error when she copied a table from the board. Terry copied the table correctly.

Reyna's Table

Time (h)	Pay (\$)
2	14
4	28
6	42
9	56

Terry's Table

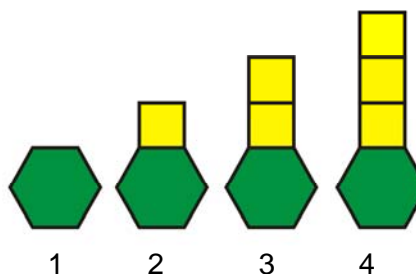
Time (h)	Pay (\$)
2	14
4	28
6	42
8	56

Reyna says the relationship between time and pay is non-linear. Terry says the relationship between time and pay is linear.

Explain why both answers show good reasoning.

Problem Solving (Reasoning and Proving, Representing)

The picture shows 4 stages in the construction of a pathway.



Is the relationship between the stage number and the perimeter of the pathway linear or non-linear?

Give reasons for your answer.

Name:

Date:

Expectation – Modelling Linear Relations, ML2.01:
 Connect the rate of change of a linear relation to the slope of the line, and define the slope as the ratio $m = \frac{\text{rise}}{\text{run}}$.

Knowledge and Understanding
 (Facts and Procedures)

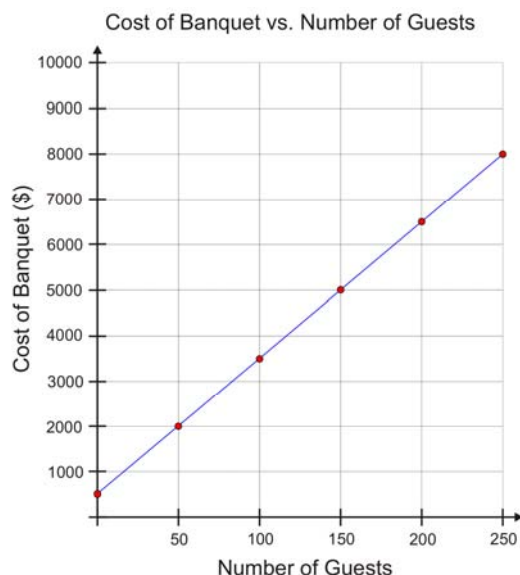
Determine the slope of the line that passes through the points A(-5,7) and B(-2,3).

Knowledge and Understanding
 (Conceptual Understanding)

Given the points D(2,-4) and E(6,8).
 Determine the slope of the line joining D and E. Find another point on this line, F, and verify that DF and EF have the same slope.

Problem Solving
 (Connecting)

The relationship between cost and the number of guests at a banquet is graphed below.



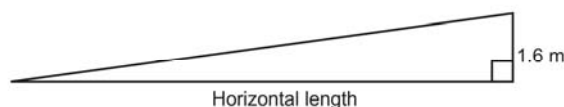
Determine the slope of this line.
 Explain how the slope of this line is related to the cost of the banquet.

Problem Solving
 (Selecting Tools and Computational Strategies, Reasoning and Proving, Connecting)

The slope of a ramp to a certain loading dock must be less than 0.125.

The dock is 1.6 m above ground level.
 Determine the minimum horizontal length of an acceptable ramp.

Show your work.



Name:

Date:

Find at least two different ways to determine how many toothpicks there would be in the 50th term of the pattern. Give reasons for your answer.



1.	2.
----	----

3.	4.
----	----


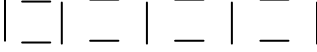
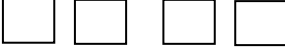

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1.

Problem-Solving Strategies:

- Build a model
- Draw a diagram
- Look for a pattern

Visualization	Description	Understanding
	Start with 4 toothpicks for the first term, then add 3 toothpicks for each subsequent term.	4 $4 + 3$ $4 + 3 + 3$... The 50 th term will be: $4 + 49 \times 3 = 151$
	Two horizontal toothpicks and 1 vertical toothpick for each term number, plus 1 more vertical toothpick than the number of the term.	$2 + 1 + 1$ $2 \times 2 + 2 \times 1 + 1$ $2 \times 3 + 3 \times 1 + 1$... The 50 th term will be: $2 \times 50 + 50 \times 1 + 1 = 151$
	Four toothpicks times the number of the term, take away the extra vertical toothpicks in the interior.	4 $4 + 4 - 1$ $4 + 4 + 4 - 1 - 1$... The 50 th term will be: $50 \times 4 - 49 \times 1 = 151$
	Think of there being 1 vertical toothpick there before starting to build the squares, and add 3 toothpicks for each term.	$1 + 3$ $1 + 3 + 3$ $1 + 3 + 3 + 3$... The 50 th term will be: $1 + 50 \times 3 = 151$

2.

Pattern	Description	Understanding
4, 7, 10, 13, ...	Find the general term and then determine the 50 th term.	General term = $1 + 3n$ 50 th term = 151

Problem-Solving Strategies:

- Use algebraic reasoning
- Look for a pattern

3.

Figure Number	Number of Toothpicks	Number Pattern
1	4	$4 + 3(0)$
2	7	$4 + 3(1)$
3	10	$4 + 3(2)$
4	13	$4 + 3(3)$
⋮	⋮	⋮
50		$4 + 3(49) = 151$

Problem-Solving Strategies:

- Make an organized list
- Look for a pattern

Note:

Multiple representations of a pattern can motivate the study of algebraic equivalence. All of the expressions in the right-most column are equivalent since they all represent the same pattern.

Grade 9

Students' solutions could include any of the Grades 7 and 8 answers.

1. Use a table to determine the rate of change and initial value. Create an equation relating the figure number, n , to number of toothpicks, T , and then substitute in $n = 50$ to evaluate the total number of toothpicks.

Problem-Solving Strategies:

- Make a model
- Work backwards
- Use algebra
- Look for a pattern

Figure	Number of Toothpicks	First Difference
1	4	3
2	7	
3	10	3
4	13	3

3 is the rate of change
 1 is the initial value (extending chart "backwards")

Equation is
 $T = 1 + 3n$
 $T = 1 + 3(50)$
 $T = 151$

Grade 9

2. Graph the points from the chart.
Determine the rate of change and initial value.

Problem-Solving Strategies:

- Make a model
- Draw a graph
- Use algebra

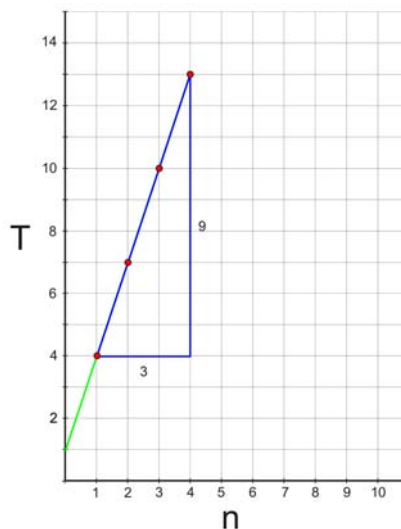
$$\begin{aligned}\text{Rate of change} &= \frac{9}{3} \\ &= 3\end{aligned}$$

$$\text{Initial value} = 1$$

$$\text{Equation: } T = 3n + 1$$

$$T = 3(50) + 1$$

$$T = 151$$



Grade 10

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Use linear relationship to derive equation relating figure number, n , to number of toothpicks, T , and then substitute in $n = 50$.
The equation is derived by determining slope and y -intercept.

Problem-Solving Strategies:

- Build a model
- Use algebra

Figure	Number of Toothpicks
1	4
2	7
3	10
4	13

1 { } 3

$$m = 3 \text{ since slope, } m = \frac{\Delta T}{\Delta n} = \frac{3}{1} = 3 \text{ (using slope formula)}$$

$$b = 1, \text{ since } y\text{-intercept is found when } n = 0,$$

(using table and working backwards – Grade 9, or using $y = mx + b$ and substituting in known values – Grade 10)

Equation

$$T = 3n + 1$$

$$T = 3(50) + 1$$

$$T = 151$$

Name:

Date:

Ten people are attending a party. Each person shakes hands with every one else at the party. Find two different ways to find the total number of handshakes.

1.

2.

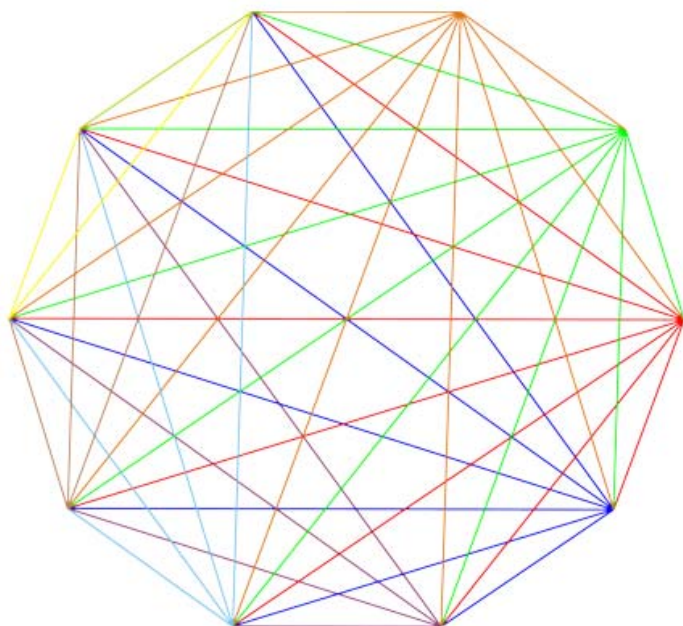
Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1. Students can determine the total number of handshakes by drawing a diagram connecting each of the 10 students in the problem, then counting the number of lines very carefully.
This results in: $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ handshakes.

Problem-Solving Strategies:

- Draw a diagram or picture by hand or with the use of technology (e.g., The Geometer's Sketchpad®4)



2. If students are working in a group, they might act the problem out, working with another group or groups, if necessary. A simple count would achieve the correct answer of 45.

Problem-Solving Strategies:

- Act it out

3. Some students might work this problem out logically, realizing that each of 10 students will shake the hand of 9 other students, i.e., $10 \times 9 = 90$. Further logical thinking will determine that this would include Student A shaking the hand of Student B, and Student B shaking the hand of Student A, so the total must be divided by 2 to get the correct answer of 45.

Problem-Solving Strategies:

- Use logical reasoning

Grades 7 and 8

4. Students might try looking at the same problem with a smaller number of students or by looking at a diagram (question 1) with fewer sides, then finding the pattern and extending it. The number of handshakes for up to 4 students could be determined fairly easily. (They might act it out in groups of 4.)

Problem-Solving Strategies:

- Make a simpler but similar problem
- Look for a pattern
- Make an organized list
- Act it out

Students might organize the data in a table of values.

Number of Students	Number of Handshakes
1	0
2	1
3	3 (1 + 2)
4	6 (1 + 2 + 3)

At this point some students will see a pattern (number of handshakes increasing by 1, 2, 3, ...) and might continue the pattern.

Number of Students	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45

5. Some students may have been exposed to Pascal's Triangle and may recognize that the pattern they find if they model smaller problems of up to 4 or 5 handshakes is indeed in Pascal's Triangle. If they have a picture of the triangle in their text or on the classroom wall, they may refer to it to solve the problem.

Problem-Solving Strategies:

- Make a similar problem
- Find a pattern
- Recognize a pattern from a previous problem

$$\begin{array}{c}
 1 \\
 1 \ 1 \\
 1 \ 2 \ 1 \\
 1 \ 3 \ 3 \ 1 \\
 1 \ 4 \ 6 \ 4 \ 1 \\
 1 \ 5 \ 10 \ 10 \ 5 \ 1
 \end{array}$$

6. Although it is not an expectation that students in these grades create a non-linear algebraic model, some students will determine the general term through inductive reasoning. They will (as noted above) notice that each of 10 students shakes hands with each of the other 9, and the result must be divided by 2 to get the correct answer. Hence, if n is the number of students, the number of handshakes can be determined by:

Problem-Solving Strategies:

- Create an algebraic model
- Use logical reasoning

$$\frac{n(n-1)}{2}$$

Students' solutions could include any of the Grades 7 and 8 answers.

Students might put the data in a table of values (see Grades 7 and 8 solutions) and check for first differences to see if the relation is linear or non-linear.

Problem-Solving Strategies:

- Find a pattern
- Make an organized list

Number of Students	Number of Handshakes	First Difference
1	0	1
2	1	2
3	3	3
4	6	

First differences are not constant, so the relation is non-linear.

Using first differences would make the pattern obvious and the table would be completed easily.

Number of Students	Number of Handshakes	First Difference
1	0	1
2	1	2
3	3	3
4	6	4
5	10	5
6	15	6
7	21	7
8	28	8
9	36	9
10	45	

Students' solutions could include any of the Grades 7, 8, and 9 answers.

- Students look at second differences to determine if a relation is quadratic.

Problem-Solving Strategies:

- Find a pattern
- Make an organized list

Number of Students	Number of Handshakes	First Difference	Second Difference
1	0		
2	1	1	1
3	3	2	1
4	6	3	1
5	10	4	

Second differences are constant so the relation is quadratic. Students complete the table.

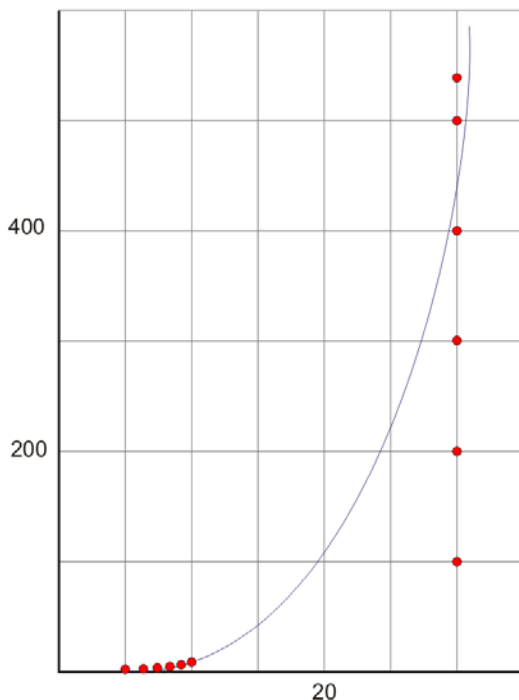
Number of Students	Number of Handshakes	First Difference	Second Difference
1	0		
2	1	1	1
3	3	2	1
4	6	3	1
5	10	4	1
6	15	5	1
7	21	6	1
8	28	7	1
9	36	8	1
10	45	9	

Grade 10

2. If the question is extended for Grade 10 to a larger number, e.g., 30 students in a classroom, students might try to graph the relation using graphing software or a graphing calculator in order to find the number of handshakes. They could find the answer by finding an intersection with the line $x = 30$. This would also give them the value $y = 435$.

Problem-Solving Strategies:

- Make a graph
- Use technology
- Use algebra



Intersecting point is shown at (30, 435).

3. Students have become familiar with quadratic relations. Students could determine the quadratic relation $T = \frac{n(n-1)}{2}$ using technology and apply the “formula.”

Problem-Solving Strategies:

- Use technology
- Use algebra

Name:

Date:

Pizza Palace charges a fixed amount for a basic pizza plus an additional cost per topping. Pizza Palace charges \$13 for its 4-topping pizza and \$16.75 for its 7-topping pizza. Find two different ways to determine the cost of a deluxe 12-topping pizza.

1.

2.

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1. Determine the cost per topping:

$$\begin{array}{r} \$16.75 - \$13.00 \\ \hline 7 - 4 \text{ toppings} \\ \hline = \$3.75 \\ \hline 3 \text{ toppings} \\ \hline = \$1.25 \\ \hline \text{topping} \end{array}$$

Problem-Solving Strategies:

- Use logical reasoning
- Work backwards

Work backwards to determine the cost for a basic pizza:

- \$13.00 for a 4-topping pizza
- \$11.75 for a 3-topping pizza
- \$10.50 for a 2-topping pizza
- \$9.25 for a 1-topping pizza
- \$8.00 for a basic pizza

Determine the cost of a 12-topping pizza by adding the cost of a basic pizza (\$8.00) plus the cost of 12 toppings ($12 \times \$1.25$):

$$\begin{aligned} \text{Cost} &= \$8.00 + 12 \times \$1.25 \\ \text{Cost} &= \$8.00 + \$15.00 \\ \text{Cost} &= \$23.00 \end{aligned}$$

Therefore, the cost of a 12-topping pizza is \$23.00.

2. Organize the given information in a table.

Use guess and check or $\frac{(16.75 - 13.00)}{3}$ to determine that the cost per topping is \$1.25.

Number of Toppings	Cost (\$)
4	13.00
5	
6	
7	16.75

Problem-Solving Strategies:

- Create a numerical model
- Look for a pattern
- Guess and check

As the number of toppings increases by 1, the cost increases by \$1.25.

Begin to complete the table. Verify that the cost of a 7-topping pizza is \$16.75.

Number of Toppings	Cost (\$)
4	13.00
5	14.25
6	15.50
7	16.75

Extend the table to determine that the cost of a 12-topping pizza is \$23.00.

Number of Toppings	Cost (\$)
4	13.00
5	14.25
6	15.50
7	16.75
8	18.00
9	19.25
10	20.50
11	21.75
12	23.00

3. Graph the relationship between cost of the pizza and the number of toppings by plotting the points (4,13.00) and (7,16.75).

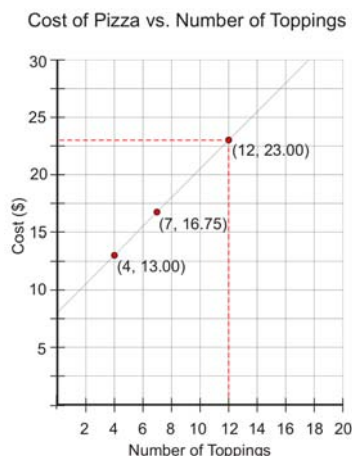
Problem-Solving Strategies:

- Create a graphical model and extrapolate

Draw the line that passes through the points (4,13.00) and (7,16.75). Extend this line. This line models the relationship between cost of the pizza and the number of toppings on the pizza.

Use the graph to determine (extrapolate) that the cost of a 12-topping pizza is approximately \$23.00.

Therefore, the cost of a 12-topping pizza is \$23.00.



Grade 9

Students’ solutions could include any of the Grades 7 and 8 answers.

Students in Grade 9 learn to determine both graphical and algebraic models given a numeric model (two points).

Graph the relationship between cost of the pizza and the number of toppings by plotting the points (4,13.00) and (7,16.75).

Draw the line that passes through the points (4,13.00) and (7,16.75).

Using the graph, determine the rate of change (cost per topping) and the initial cost (the cost of a basic pizza).

Problem-Solving Strategies:

- Create a graphical and algebraic model
- Use a formula

$$\begin{aligned} \text{Cost of a basic pizza} \\ = \$8.00 \end{aligned}$$

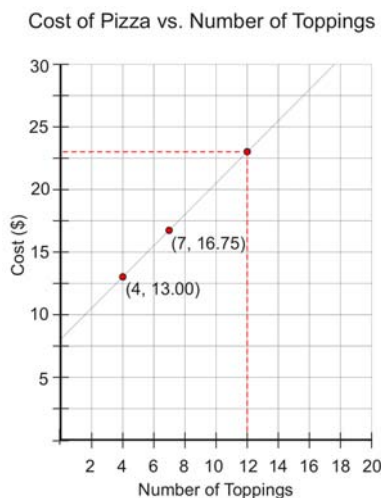
$$\begin{aligned} \text{Rate of Change} \\ = \frac{16.75 - 13.00}{7 - 4} \\ = \frac{3.75}{3} \\ = \$1.25/\text{topping} \end{aligned}$$

Write an equation that models the relationship between the cost of a pizza (C) and the number of toppings (n).

$$C = 8.00 + 1.25n$$

Use the equation to determine that the cost of a 12-topping pizza is \$23.00.

$$\begin{aligned} C &= 8.00 + 1.25n \\ C &= 8.00 + 1.25(12) \\ C &= 23.00 \end{aligned}$$



Grade 10

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Students may choose to solve the system of equations algebraically.

$$C = I + nP$$

C = Cost

$$16.75 = I + 7P \quad \text{①}$$

I = Initial cost

$$\underline{13.00 = I + 4P} \quad \text{②}$$

P = Price per
topping

$$3.75 = 3P$$

(Subtract ② from ①)

$$1.25 = P$$

n = Number of
toppings

$$13.00 = I + 4(1.25)$$

$$8.00 = I$$

(Substitute value for P into ①)

General equation is $C = 8.00 + 1.25P$

Cost is $C = 8.00 + 1.25(12)$

$$= \$23.00$$

Problem-Solving Strategies:

- Use algebra

Note:

Students may use slightly different terminology, e.g., *slope* instead of *rate of change*, *y-intercept* instead of *initial value*.

Is This Always True? (Reflecting, Reasoning and Proving)

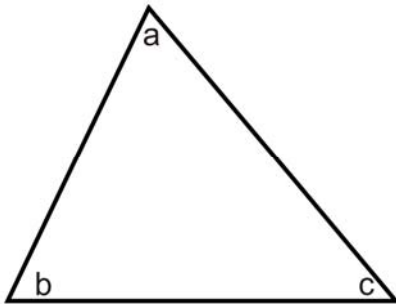
Grades 7–10

Name:

Date:

Emily says that the sum of the interior angles of any polygon is equal to 360° less than the product of 180° and the number of sides of the polygon.

For example, for a triangle:



$$\begin{aligned} &180^\circ \times \text{number of sides} - 360^\circ \\ &= 180^\circ \times 3 - 360^\circ \\ &= 540^\circ - 360 \\ &= 180^\circ \end{aligned}$$

$$a + b + c = 180^\circ$$

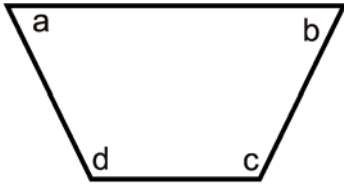
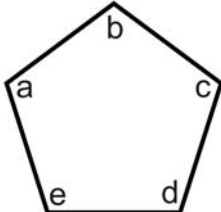
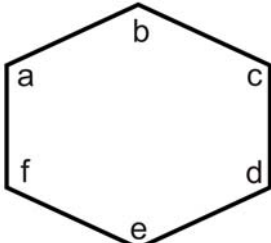
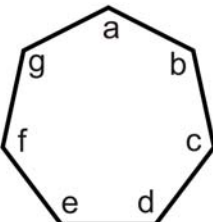
The sum of the interior angles in a triangle is 180° .

Is this true for all polygons?

1. Use a protractor or technology to determine the sum of the interior angles in various polygons and verify that Emily’s statement remains true.

Problem-Solving Strategies:

- Use technology/tools
- Draw a diagram
- Look for a pattern
- Use inductive reasoning

Number of Sides	Sum of Interior Angles (using a protractor or technology)	Emily’s Statement
4 (quadrilateral)	 $a + b + c + d = 360^\circ$ <p>The sum of the interior angles in a 4-sided polygon is 360°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 4 - 360^\circ$ $= 720^\circ - 360^\circ$ $= 360^\circ$
5 (pentagon)	 $a + b + c + d + e = 540^\circ$ <p>The sum of the interior angles in a 5-sided polygon is 540°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 5 - 360^\circ$ $= 900^\circ - 360^\circ$ $= 540^\circ$
6 (hexagon)	 $a + b + c + d + e + f = 720^\circ$ <p>The sum of the interior angles in a 6-sided polygon is 720°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 6 - 360^\circ$ $= 1080^\circ - 360^\circ$ $= 720^\circ$
7 (heptagon)	 $a + b + c + d + e + f + g = 900^\circ$ <p>The sum of the interior angles in a 7-sided polygon is 900°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 7 - 360^\circ$ $= 1260^\circ - 360^\circ$ $= 900^\circ$

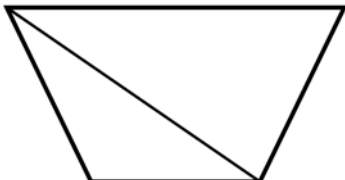
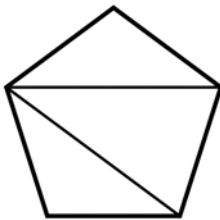
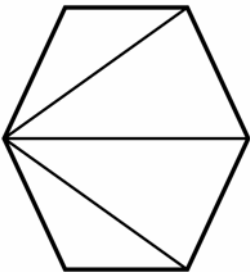
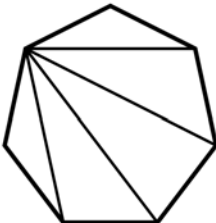
The statement appears to be true.

Grades 7 and 8

2. Divide various polygons into non-overlapping triangles to determine the sum of the interior angles of the polygons and verify that Emily's statement remains true.

Problem-Solving Strategies:

- Draw a diagram
- Use logic

Number of Sides	Sum of Interior Angles	Emily's Statement
4 (quadrilateral)	 <p>2 triangles = $2 \times 180^\circ = 360^\circ$ The sum of the interior angles in a 4-sided polygon is 360°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 4 - 360^\circ$ $= 720^\circ - 360^\circ$ $= 360^\circ$
5 (pentagon)	 <p>3 triangles = $3 \times 180^\circ = 540^\circ$ The sum of the interior angles in a 5-sided polygon is 540°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 5 - 360^\circ$ $= 900^\circ - 360^\circ$ $= 540^\circ$
6 (hexagon)	 <p>4 triangles = $4 \times 180^\circ = 720^\circ$ The sum of the interior angles in a 6-sided polygon is 720°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 6 - 360^\circ$ $= 1080^\circ - 360^\circ$ $= 720^\circ$
7 (heptagon)	 <p>5 triangles = $5 \times 180^\circ = 900^\circ$ The sum of the interior angles in a 7-sided polygon is 900°.</p>	$180^\circ \times \text{number of sides} - 360^\circ$ $= 180^\circ \times 7 - 360^\circ$ $= 1260^\circ - 360^\circ$ $= 900^\circ$

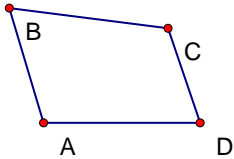
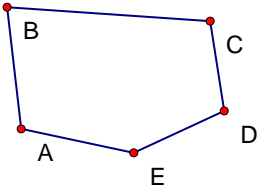
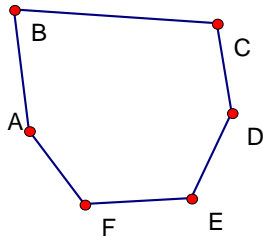
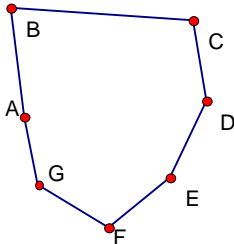
The statement appears to be true.

Grades 7 and 8

3. Use The Geometer's Sketchpad[®] 4 to determine the sum of the interior angles in various polygons and verify that Emily's statement remains true.

Problem-Solving Strategies:

- Use technology
- Use inductive reasoning

Number of Sides	Sum of Interior Angles	Emily's Statement
4	$m\angle ABC = 66.18^\circ$ $m\angle BCD = 116.01^\circ$ $m\angle CDA = 71.20^\circ$ $m\angle DAB = 106.61^\circ$ $m\angle ABC + m\angle BCD + m\angle CDA + m\angle DAB = 360.00^\circ$ 	$180^\circ \times 4 - 360^\circ$ $= 720^\circ - 360^\circ$ $= 360^\circ$
5	$m\angle ABC = 79.49^\circ$ $m\angle BCD = 102.81^\circ$ $m\angle CDE = 106.18^\circ$ $m\angle DEA = 142.89^\circ$ $m\angle EAB = 108.64^\circ$ $m\angle ABC + m\angle BCD + m\angle DEA + m\angle CDE + m\angle EAB = 540.00^\circ$ 	$= 180^\circ \times 5 - 360^\circ$ $= 900^\circ - 360^\circ$ $= 540^\circ$
6	$m\angle ABC = 79.49^\circ$ $m\angle BCD = 102.81^\circ$ $m\angle CDE = 146.21^\circ$ $m\angle DEF = 118.18^\circ$ $m\angle EFA = 123.88^\circ$ $m\angle FAB = 149.43^\circ$ $m\angle ABC + m\angle CDE + m\angle BCD + m\angle DEF + m\angle EFA + m\angle FAB = 720.00^\circ$ 	$= 180^\circ \times 6 - 360^\circ$ $= 1080^\circ - 360^\circ$ $= 720^\circ$
7	$m\angle ABC = 79.49^\circ$ $m\angle BCD = 102.81^\circ$ $m\angle CDE = 146.21^\circ$ $m\angle DEF = 154.42^\circ$ $m\angle EFG = 108.92^\circ$ $m\angle FGA = 133.50^\circ$ $m\angle GAB = 174.66^\circ$ $m\angle GAB + m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEF + m\angle EFG + m\angle FGA = 900.00^\circ$ 	$= 180^\circ \times 7 - 360^\circ$ $= 1260^\circ - 360^\circ$ $= 900^\circ$

The statement appears to be true.

Students' solutions could include any of the Grades 7 and 8 answers.

- Students determine both graphical and algebraic models given a numeric model (two points).

Problem-Solving Strategies:

- Look for a pattern
- Use a graph to write an equation
- Use algebra

Students create a table that shows the relationship between the sum of the interior angles and the number of sides in a polygon.

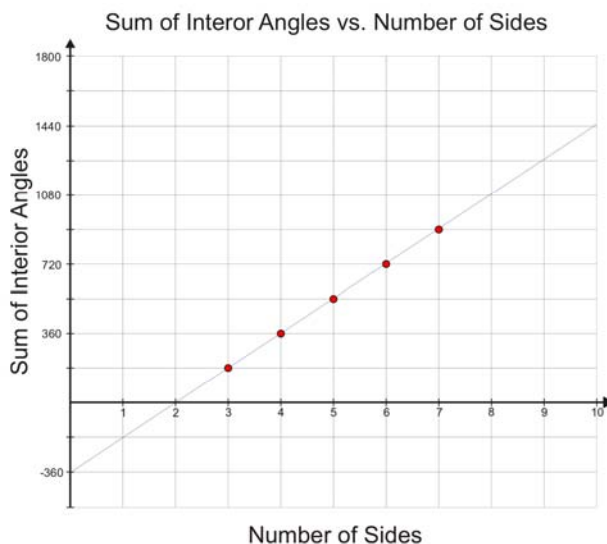
Number of Sides	Sum of Interior Angles
3	180°
4	360°
5	540°
6	720°
7	900°

They graph this relationship.

Students determine an equation that represents the relationship between the sum of the interior angles (S) and the number of sides in a polygon (n):

$$\begin{aligned}
 &\text{Rate of Change} && \text{Initial Value} \\
 &= \frac{180^\circ}{1 \text{ side}} && = -360^\circ \\
 &= 180^\circ/\text{side}
 \end{aligned}$$

Equation: $S = 180n - 360$



Therefore, Emily's statement is true: The sum of the interior angles in a polygon is equal to 360° less than the product of 180° and the number of sides in the polygon.

2. Using methods 1–3 (pp. 29–31) or previous knowledge, students create a table that shows the relationship between the sum of the interior angles and the number of sides in a polygon.

Problem-Solving Strategies:

- Use a table to write an equation
- Work backwards

Number of Sides	Sum of Interior Angles	First Differences
3	180°	$360^\circ - 180^\circ = 180^\circ$ $540^\circ - 360^\circ = 180^\circ$ $720^\circ - 540^\circ = 180^\circ$ $900^\circ - 720^\circ = 180^\circ$
4	360°	
5	540°	
6	720°	
7	900°	

Rate of change = 180°/side

Number of Sides	Sum of Interior Angles
0	-360°
1	-180°
2	0°
3	180°
4	360°
5	540°
6	720°
7	900°

Initial value = *S*-intercept = -360° (work backwards)

They write an equation that models the relationship between the sum of the interior angles (*S*) and the number of sides in a polygon (*n*): $S = 180n - 360$.

Therefore, Emily’s statement is true: The sum of the interior angles in a polygon is equal to 360° less than the product of 180° and the number of sides in the polygon.

Is This Always True?

(Reflecting, Reasoning and Proving)

Grades 7–10

Name:

Date:

Is it always true that, starting with 1, the sum of any number of consecutive odd natural numbers is a perfect square? For example, $1 + 3 + 5 + 7 = 16 = 4^2$

Grades 7 and 8

Sample Solutions

1. Students could make a table of values and infer that the fact remains true that the sum of any number of consecutive odd natural numbers is a perfect square. Student may continue for a larger number of odd numbers.

Problem-Solving Strategies:

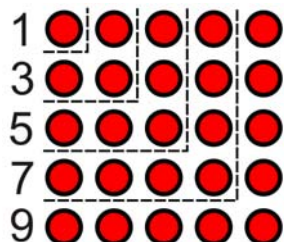
- Make an organized list
- Look for a pattern
- Use inductive reasoning

Consecutive Odds	Sum
1	1
1 + 3	4
1 + 3 + 5	9
1 + 3 + 5 + 7	16
1 + 3 + 5 + 7 + 9	25

2. Use a dot diagram or interlocking cubes to visually represent the addition of the next odd natural number. As each odd number is added, a square is always created.

Problem-Solving Strategies:

- Draw a diagram
- Use concrete materials



Grade 9

Create a table looking at the number of odd integers that make up the sum and the sum of the consecutive odd natural numbers. The sum is always equal to the number of integers squared.

Problem-Solving Strategies:

- Make an organized list
- Look for a pattern
- Use inductive reasoning

Number of Integers	Sum
1	$1 = 1^2$
2 (1 + 3)	$4 = 2^2$
3 (1+3+5)	$9 = 3^2$
4 (1+3+5+7)	$16 = 4^2$
5 (1+3+5+7+9)	$25 = 5^2$

Grade 10

Represent the data in a table and determine the first and second differences.

Problem-Solving Strategies:
• Use algebraic reasoning

Use the constant second difference to conclude that the relationship is quadratic and therefore, the relationship will involve degree two or square power. Students can then determine its equation.

Consecutive Odds	Sum	First Difference	Second Difference
1	1		
1 + 3	4	3	2
1 + 3 + 5	9	5	2
1 + 3 + 5 + 7	16	7	2
1 + 3 + 5 + 7 + 9	25	9	

Is This Always True?

(Reflecting, Reasoning and Proving)

Grades 7–10

Name:

Date:

Farid says that he can always predict a pattern from the first three terms, e.g., The pattern 2, 4, 6, ... goes up by 2 each time and can be modelled by the expression $2n$. The pattern 1, 4, 9 ... shows consecutive squares and can be modelled by the expression n^2 .

Can a pattern always be determined by the first three terms?

Grades 7–10

Sample Solutions

Many students will think this is true, but some patterns are not quite as predictable as the examples mentioned. Only one counter-example is needed.

Problem-Solving Strategies:

- Find a counter example

1, 1, 2, ...

This pattern could be extended in a number of ways:

1, 1, 2, 2, 3, 3, 4, 4, ...

1, 1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, ...

1, 1, 2, 3, 5, 8, 13, ... i.e., the Fibonacci sequence

1, 1, 2, 4, 7, 11, ...

1, 1, 2, 6, 24, 120, ...

As any of these sequences are logical extensions of the first three terms, Farid's contention is not true.