TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Continuum and Connections

Fractions

Overview

Context Connections

- Positions fractions in a larger context and shows connections to everyday situations, careers, and tasks
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers
- Includes relevant specific expectations for each grade
- Summarizes prior and future learning

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied
- Includes advice on how to help students develop understanding

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (fractions) as an organizer

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question*.
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated
- Serves as a model for developing other questions relevant to the learning expected for the grade/course

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate
- Focuses on problem-solving strategies, involving multiple mathematical processes
- Provides an opportunity for students to engage with the problem at many levels
- Provides problems appropriate for students in Grades 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical processes Reasoning and Proving, and Reflecting

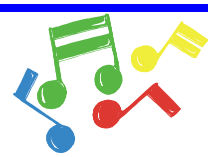
Fractions

Context

- Fractions have a variety of meanings. The fraction $\frac{2}{5}$ can be interpreted as 2 parts of a whole that has been divided into 5 equal parts (part of a whole). This fraction also expresses 2 parts of a group of 5 (part of a set) where the elements of the set are not necessarily identical, e.g., 2 out of 5 books on a shelf. As ratios or rates, fractions are used for comparisons.
- A fraction can represent a division or a measurement.
- Fractions are used daily in construction, cooking, sewing, investments, time, sports, etc. Since many occupations require workers to think about and use fractions in many different ways, it is important to develop a good understanding of fractions. Using manipulatives and posing higher-level thinking questions helps build understanding of what fractions represent.
- Understanding builds when students are challenged to use a variety of representations for the same fraction (or operation) and when students connect fractions to ratios, rates of change, percents, or decimals.

Context Connections

Cooking

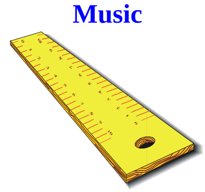


Music

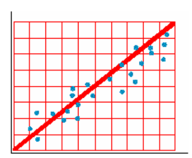
Sharing

Retail/Shopping

Interest Rates

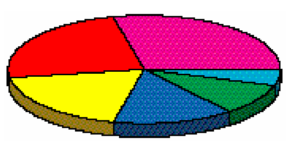


Measurement

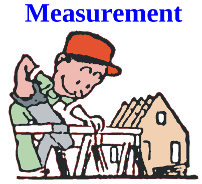


Slope

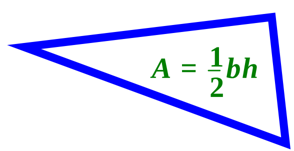
Time



Circle Graphs



Construction



Formulas

Other Connections

Manipulatives

- cubes
- colour tiles
- pattern blocks
- geoboards
- tangrams
- coloured rods
- grid paper
- base ten blocks

Technology

- spreadsheet software
- The Geometer's Sketchpad®4
- calculators/graphing calculators
- word processing software

Other Resources

http://www.uwinnipeg.ca/~jameis/New%20Pages/MYR21.html
http://www.standards.nctm.org/document/chapter6/numb.htm
http://math.rice.edu/~lanius/proportions/rate9.html
http://mmmproject.org/number.htm
http://matti.usu.edu/nlvm/nav/category_g_2_t_1.html

Connections Across Grades

Selected results of word search using the *Ontario Curriculum Unit Planner*

Search Words: *rational, fraction, ratio, rate, denominator, numerator, multiple, factor*

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
▪ represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation; ▪ represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation; ▪ determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions, decimal numbers, and percents.	▪ represent, compare, and order decimals to hundredths and fractions, using a variety of tools; ▪ select and justify the most appropriate representation of a quantity (i.e., fraction, decimal, percent) for a given context; ▪ divide whole numbers by simple fractions and by decimal numbers to hundredths, using concrete materials; ▪ use a variety of mental strategies to solve problems involving the addition and subtraction of fractions and decimals; ▪ add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms; ▪ demonstrate, using concrete materials, the relationship between the repeated addition of fractions and the multiplication of that fraction by a whole number; ▪ determine, through investigation, the relationships among fractions, decimals, percents, and ratios; ▪ research and report on everyday applications of probabilities expressed in fraction, decimal, and percent form.	▪ represent, compare, and order rational numbers; ▪ translate between equivalent forms of a number; ▪ use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution; ▪ represent the multiplication and division of fractions, using a variety of tools and strategies; ▪ solve problems involving addition, subtraction, multiplication, and division with simple fractions.	**Applied and Academic** ▪ substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases); ▪ simplify numerical expressions involving integers and rational numbers, with and without the use of technology. **Academic** ▪ solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate and proportion; ▪ solve first-degree equations involving one variable, including equations with fractional coefficients. **Applied** ▪ represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations; ▪ solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms.	**Applied** ▪ solve first-degree equations involving one variable, including equations with fractional coefficients; ▪ determine the value of a variable in the first degree, using a formula.

Summary of Prior Learning

In earlier years, students:

- relate fractions to decimals and percents;
- compare and order fractional amounts, with like and unlike denominators (including proper and improper fractions and mixed numbers);
- explain the concept of equivalent fractions, with and without concrete materials;
- represent fractions using words, concrete materials, and notations.

In Grade 7, students:

- continue to develop proficiency by using fractions in mental strategies and in selecting and justifying use;
- develop proficiency in adding and subtracting simple fractions;
- use fractions to solve problems involving the area of a trapezoid;
- describe the reduction of two-dimensional shapes to create similar shapes and to describe reductions in dilatations;
- write fractions to determine the theoretical probability of an event or two independent events.

In Grade 8, students:

- develop proficiency in comparing, ordering, and representing fractions;
- develop proficiency in operations with fractions with and without concrete materials;
- solve problems involving addition, subtraction, multiplication, and division of simple fractions;
- substitute fractions for the variables in algebraic expressions up to three terms;
- continue to write fractions to determine the theoretical probability of an event, e.g., complementary event.

In Grade 9 Applied, students:

- continue to solve problems involving percents, fractions, and decimals;
- use rationals as bases when evaluating expressions involving natural number exponents;
- use fractions in solving problems involving area of composite figures with triangles and/or trapezoids, and in solving problems involving volumes of pyramids or cones.

In Grade 10 Applied, students:

- solve first degree equations and rearrange formulas involving variable in the first degree, involving fractional coefficients;
- use ratios and proportions in problem solving when working with similar triangles and trigonometry;
- rearrange measurement formulas involving surface area and volume to solve problems;
- graph equations of lines, with fractional coefficients.

In later years

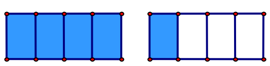
Students' choice of courses will determine the degree to which they apply their understanding of concepts related to fractions.

Instruction Connections

Suggested Instructional Strategies

Grade 7

- Students continue to build their understanding in illustrating, comparing, and ordering fractions, e.g. use zero, one-half and one as benchmarks. Ask students to state which benchmark a fraction is closer to, and to name a fraction that is even closer to the benchmark.
- Continue to build a variety of comparison techniques through patterning, e.g., $\frac{3}{5}$ is less than $\frac{4}{5}$ because there are fewer of the same size parts, whereas $\frac{3}{5}$ is larger than $\frac{3}{8}$ because although there are the same number of parts, the size of each part in the first number is larger than the size of each part in the second number – notice that common denominators or common numerators are helpful when comparing fractions.
- Give students varied opportunities to build understanding of the addition and subtraction of fractions using concrete materials and drawings before moving to symbols.
  Pose questions like: How may different ways can you express five-quarters as the sum or difference of two fractions? Use a visual representation to help students see a variety of solutions.



$\frac{5}{4} = 1 + \frac{1}{4} = \frac{1}{2} + \frac{3}{4} = \frac{7}{4} - \frac{1}{2} = \frac{1}{8} + \frac{9}{8}$

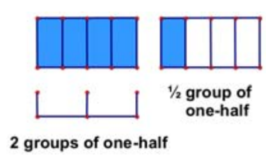
- Ask students to explain why each addition or subtraction represents the same fraction using concrete materials or diagrams.
- Build on their prior knowledge that a repeated addition can be expressed as a multiplication. Connect the multiplication of fractions to this prior knowledge.

Grade 8

- Encourage understanding of division by considering division problems as grouping, as repeated subtraction, and as sharing problems:

  For example, $\frac{5}{4} \div \frac{1}{2}$

  How many groups (or sets) of one-half are there in five-quarters?
  How many times can you subtract $\frac{1}{2}$ from $\frac{5}{4}$?



The drawing shows that there are $2\frac{1}{2}$ groups (of one-half).

- Have students explain their solutions using multiple representations – both concrete and symbolic.
- Ensure that students are able to interpret fractional answers in context, e.g., If I had a half-cup measure, I could fill it two and a half times and I would have one and a quarter cups of flour.
- When students demonstrate understanding of the four operations with fractions, have them apply the operations to order of operations questions involving brackets, using a maximum of three operations.

Helping to Develop Understanding

- To determine equivalent fractions or find common denominators, students must understand the concepts of factors and multiples. They need to understand that multiplying or dividing numerator and denominator by the same quantity is the same as multiplying or dividing by 1.
- Keep denominators simple so students can easily make drawings, use concrete materials, explain their solutions, and build understanding.
- When expressing a fraction in simplest form or in lowest terms, it is not necessary to change an improper fraction into a mixed fraction. Students need to understand that $\frac{5}{4}$ is between 1 and 2 and can be represented visually as shown to the left, or shown on a number line at $1\frac{1}{4}$.
- Improper fractions rather than mixed fractions are used to express slopes of lines.
- If asked to place a number of fractions with different denominators on a number line, students may need help deciding on the number of divisions to make between units.
- Count fractions aloud on a number line, e.g.,
  $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}$.
  Then, express in simplified form:
  $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \frac{7}{6}, \frac{4}{3}$.

Suggested Instructional Strategies	Helping to Develop Understanding
Grade 9 Applied • Extend students' working knowledge of positive fractions to negative fractions, e.g., negative rates of change. • Introduce the new terminology *rational number*, i.e., any number that can be expressed as a positive or negative fraction and written as a terminating or repeating decimal number. • Encourage estimation in problem solving to verify accuracy of solutions. • Encourage the conversions between fractions, decimals, and percents wherever appropriate to simplify a problem. • Consolidate understanding of fractions within the context of the course. **Grade 10 Applied** • Consolidate operations with rational numbers within the context of the course. • Encourage students whenever appropriate to leave numbers presented in the question in fraction form and to express answers in fraction form.	

Connections Across Strands

Note

Summary or synthesis of curriculum expectations is in plain font.

Verbatim curriculum expectations are in italics.

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• *generate multiples and factors of given numbers* • solve problems involving whole number percents • demonstrate an understanding of rate • solve problems involving unit rates • see Connections Across the Grades, p. 3	• use fraction skills in solving problems involving measurement, e.g., the area of a trapezoid	• use fractions to describe reductions in dilatation and in reducing two-dimensional shapes to create similar figures • use fractions to describe related lines, e.g., perpendicular lines meet at 90° which is $\frac{1}{2}$ of 180° • plot points on the Cartesian plane with simple fractional coordinates	• model everyday relationships involving rates • translate phrases into algebraic expressions	• use fractions to express the experimental and theoretical probability of an event • *research and report on real-world applications of probabilities expressed in fraction, decimal, and percent form* • *determine the theoretical probability of a specific outcome involving two independent events*

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• determine common factors and multiples • solve problems involving proportions • solve problems involving percent • see Connections Across the Grades, p. 3	• use fraction skills in solving problems involving measurement, e.g., the area of a circle	• graph the image of a point on the Cartesian coordinate plane with simple fractional coordinates • determine relationships: area, perimeter, and side length of similar shapes, e.g., if 2 triangles are similar and the perimeter of one is $\frac{1}{2}$ the perimeter of the other, compare their areas	• *evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables* • translate statements into algebraic expressions and equations	• use fractions to express the experimental and theoretical probability of an event • *identify the complementary event for a given event, and calculate the theoretical probability that a given event will not occur*

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relationships
• see Connections Across the Grades, p. 3	• solve problems involving area of composite figures, involving triangles and/or trapezoids • *develop, through investigation, the formulas for the volume of a pyramid or cone* • solve problems involving the volume of pyramids or cones	• interpret points on a scatterplot • collect data; describe trends • construct tables of values and graphs for data • determine and describe rates of change • use initial value and rate of change to express a linear relation • determine graphically a point of intersection

Grade 10 Applied

Measurement and Trigonometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
• use proportional reasoning (by setting two ratios written as fractions equal) to solve similar triangle problems • determine, through investigation, the trigonometric ratios of sine, cosine, and tangent as ratios represented by fractions • solve problems involving the measures in right-angled triangles • rearrange measurement formulas when solving problems involving surface area and volume, including combination of these figures in the metric or imperial system • perform conversions between imperial and metric systems • solve problems involving surface area and volume of pyramids and cylinders	• graph lines (with fractional coefficients) by hand, using a variety of techniques • solve equations involving fractional coefficients • connect rate of change and slope • determine the equations of a line • determine graphically the point of intersection of 2 lines	• collect data that can be represented as a quadratic relation • solve problems involving quadratic relations

Developing Proficiency

Grade 7

Name:
Date:

Expectation – Number Sense and Numeration, 7m24: Add and subtract fractions with simple like and unlike denominators using a variety of tools and algorithms.

Knowledge and Understanding
(Facts and Procedures)

a) $\frac{1}{3}+\frac{5}{6}$

b) $\frac{7}{2}-\frac{2}{3}$

c) $5\frac{3}{8}+4\frac{1}{8}$

Knowledge and Understanding
(Conceptual Understanding)

Use a model to represent the following addition:

$$\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$$

Record a drawing of your model.

Problem Solving
(Reasoning and Proving)

Three friends, Ahmed, Anja, and Eric, have a lemonade stand. They decide to share any profits in the following way.

Ahmed will get $\frac{2}{3}$ of the total profits.

Anja will get $\frac{1}{4}$ of the total profits.

Explain why Eric will get $\frac{1}{12}$ of the total profits.

Give reasons for your answer.

Problem Solving
(Connecting, Representing)

Vesna used 8 eggs from a new carton of one dozen eggs. Vesna says she used $\frac{2}{3}$ of the carton

and her friend, Sue, says she has $\frac{4}{12}$ of the carton remaining.

Explain why they are both right, using a diagram or concrete materials.

Developing Proficiency

Grade 8

Name:
Date:

Expectation – Number Sense and Numeration, 8m19: Represent the multiplication and division of fractions, using a variety of tools and strategies.

Knowledge and Understanding
(Facts and Procedures)

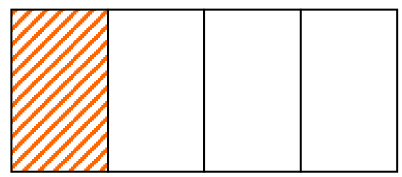
a) $\frac{3}{4} \times \frac{1}{8}$

b) $4 \div \frac{4}{9}$

Knowledge and Understanding
(Conceptual Understanding)

Complete the diagram to illustrate:

$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



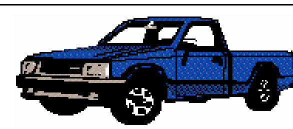
Problem Solving
(Reflecting, Reasoning and Proving)

Jason incorrectly thinks that 6 divided by one-half is 3.

Show that $6 \div \frac{1}{2} = 12$.

Illustrate your answer with a diagram.

Problem Solving
(Connecting)



Sanjay's father went on a short trip in his truck. The 60-litre gas tank was full when he started his trip.

He used about $\frac{4}{5}$ of the gasoline during the trip. How many litres of gas does he have left in his tank?

Show your work.

Developing Proficiency

Grade 9 Applied

Name:
Date:

Expectation – Number Sense and Algebra, NA1.06:
Solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms.

Knowledge and Understanding
(Facts and Procedures)

Bradley's homework question was to put 55%, 0.45, and $\frac{3}{5}$ in order from smallest to largest.
He drew this chart to help his thinking.

Complete the following table for Bradley.

Percent	Decimal	Fraction
55%		
	0.45	
		$\frac{3}{5}$

Knowledge and Understanding
(Conceptual Understanding)

Explain why $\frac{1}{2}$ of $\frac{4}{5}$ is $\frac{2}{5}$.

Illustrate with a diagram or concrete materials.

Problem Solving
(Reasoning and Proving, Connecting)

Ahmed, Anja, and Eric invested in a business. They each invested a different amount of money.

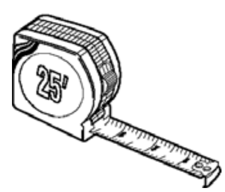
Ahmed invested $30 000.
Anja invested $20 000.
Eric invested $10 000.

In the first year of the business, the profit is $180 000. What should each person's share of the profits be?

Give reasons for your answer.

Problem Solving
(Connecting, Selecting Tools and Computational Strategies)

In shop class, you are asked to cut a round tabletop with a diameter of $72\frac{3}{4}$ cm.



What is the area of the tabletop?

Show your work.

Developing Proficiency

Grade 10 Applied

Name:
Date:

Expectation – Modelling Linear Relations, ML1.01: Solve first-degree equations involving one variable, including equations with fractional coefficients.

Knowledge and Understanding
(Facts and Procedures)

Solve for the variable.

a) $\frac{4}{5}n - \frac{2}{5} = \frac{10}{5}$

b) $\frac{3}{4} + \frac{t}{3} = 5$

Knowledge and Understanding
(Conceptual Understanding)

Explain why the solution to $\frac{2}{5}x + 3 = \frac{1}{2}$ is equivalent to the solution of $4x + 30 = 5$.

Problem Solving
(Reflecting, Reasoning and Proving)

Philip knows that a pyramid has a height of 6 m and volume 900 m^3. He determines the area of base as:

$$V = \frac{1}{3}(\textit{area of base})(\textit{height})$$

$$900 = \frac{1}{3}(\textit{area of base})(6)$$

$$\frac{900}{3} = (\textit{area of base})(6)$$

$$300 - 6 = \textit{area of base}$$

Area of base of pyramid is 294 m.
Prove that this is incorrect and state Philip's error.

Problem Solving
(Connecting, Selecting Tools and Computational Strategies)

A small order of popcorn at the local fair is sold in a paper cone-shaped container. If the volume of the cone is 500 cm^3 and the radius is 6 cm, determine the height of the paper cone.

Problem Solving Across the Grades

Sample 1

Name:
Date:

Betty cut $\frac{3}{4}$ m from a piece of rope $2\frac{2}{3}$ m long.

Is there enough rope left for two pieces $\frac{5}{6}$ m long each?

Show your answer in more than one way.

1.	**2.**

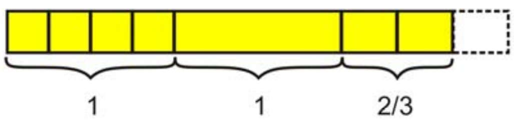
3.	**4.**

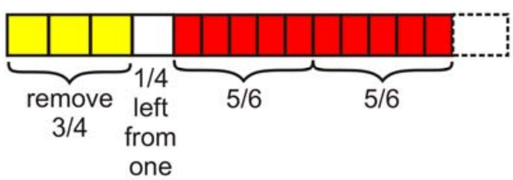
Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1a. A student operating at the concrete stage may use coloured strips of paper. A visual learner may draw a picture of the lengths of rope. The following diagrams illustrate what the first student may cut out and the second student may draw.

Problem-Solving Strategies:
- Use manipulative representation
- Draw a diagram

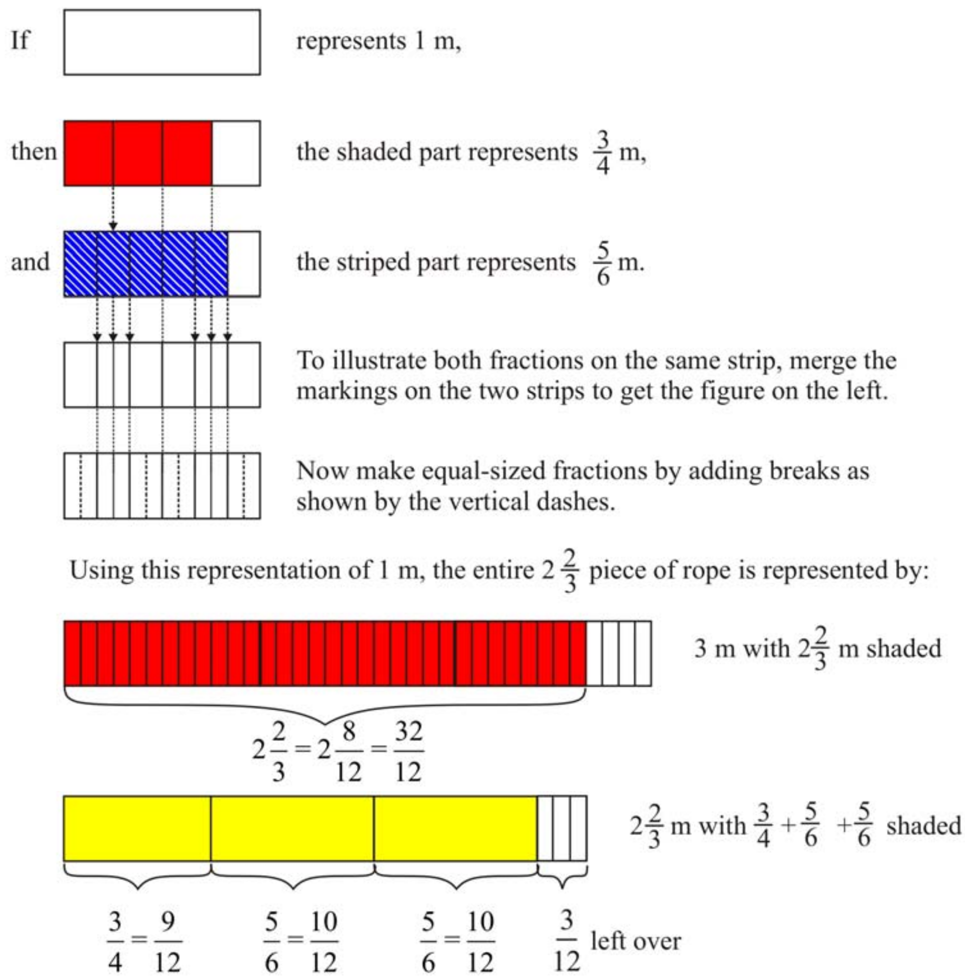




Divide $1\frac{2}{3}$ into sixths

Therefore, there is enough rope left for two pieces $\frac{5}{6}$ m long. There will be $\frac{1}{4}$ m left over.

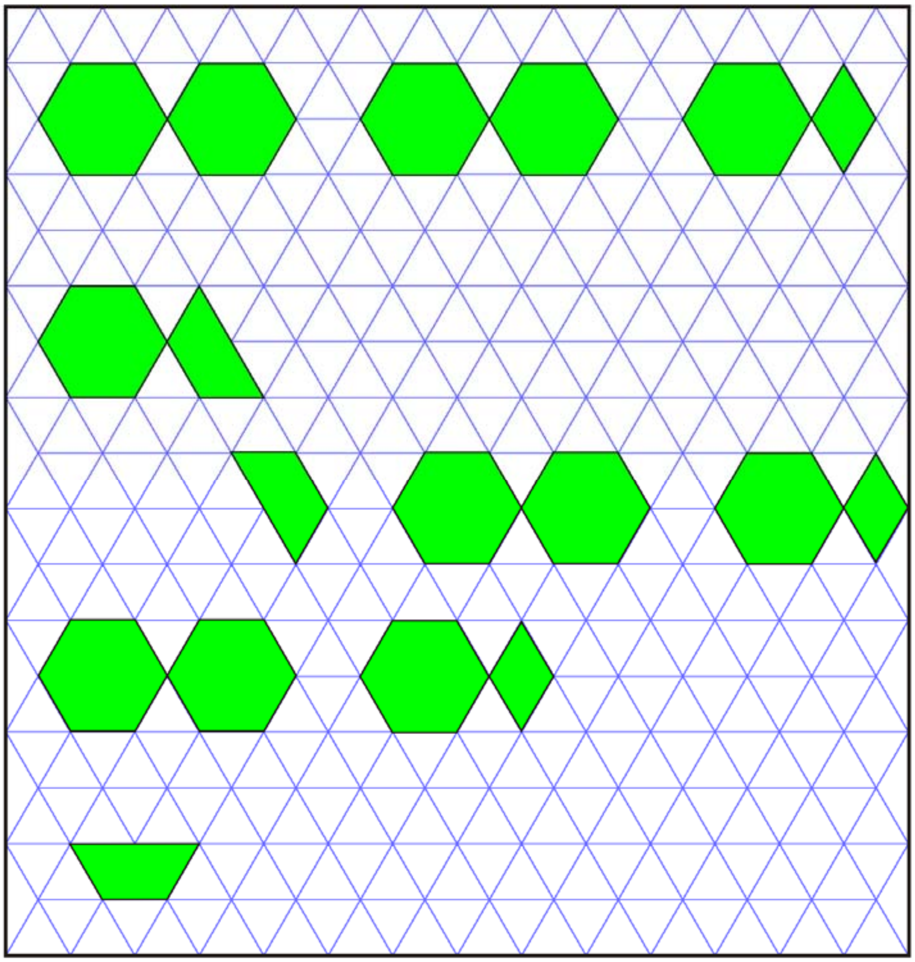
1b. A student operating at the concrete stage may use coloured strips of paper. A visual learner may draw a picture of the lengths of rope. The following diagrams illustrate what the first student may cut out and the second student may draw.



The total rope cut off is $\frac{29}{12}$ with $\frac{3}{12}$ or $\frac{1}{4}$ left over.

There is enough rope for the two $\frac{5}{6}$m lengths.

1c. Use concrete materials such as pattern blocks.



$2\frac{2}{3}$

Subtract $\frac{3}{4}$

Remaining

Subtract 2 pieces of $\frac{5}{6}$

Remaining

Note: a double hexagon represents 1.

2. Some students may already be comfortable using symbols and operations with fractions.

$$2\frac{2}{3}-\frac{3}{4}-\frac{5}{6}-\frac{5}{6}$$
$$=\frac{8}{3}-\frac{3}{4}-\frac{5}{6}-\frac{5}{6}$$
$$=\frac{32}{12}-\frac{9}{12}-\frac{10}{12}-\frac{10}{12}$$
$$=\frac{3}{12}$$
$$=\frac{1}{4}$$

There is enough rope for all three pieces.

Problem-Solving Strategies:
- Use symbolic representation

Grade 7 — Sample Solutions

3. Some students using symbols may approach the solution differently.

Problem-Solving Strategies:
- Use symbolic representation

$$2\frac{2}{3}-\frac{3}{4} = \frac{8}{3}-\frac{3}{4} = \frac{32}{12}-\frac{9}{12} = \frac{23}{12}$$

$$\frac{5}{6}+\frac{5}{6} = \frac{10}{6} = \frac{20}{12}$$

$$\frac{23}{12} > \frac{20}{12}$$

There is enough rope for all three pieces.

4. Some students may convert fractions to decimals.

Problem-Solving Strategies:
- Use symbolic representation

$$2\frac{2}{3} = 2.6666\ldots \qquad \frac{3}{4} = 0.75 \qquad \frac{5}{6} = 0.8333\ldots$$

$$\begin{array}{r} 2.6666\ldots \\ -0.75 \\ \hline 1.9166\ldots \end{array} \qquad \begin{array}{r} 0.8333\ldots \\ +0.8333\ldots \\ \hline 1.6666\ldots \end{array}$$

$$\begin{array}{r} 1.9166\ldots \\ -1.6666\ldots \\ \hline 0.25 \end{array}$$

There is enough rope for all three pieces.

Grade 8 — Sample Solutions

Students' solutions could include any of the Grade 7 answers.

Problem-Solving Strategies:
- Use symbolic representation

$$2\frac{2}{3}-\frac{3}{4} = \frac{8}{3}-\frac{3}{4} = \frac{32}{12}-\frac{9}{12} = \frac{23}{12}$$

$$\frac{5}{6}(2) = \frac{10}{6} = \frac{20}{12}$$

$$\frac{23}{12}-\frac{20}{12} = \frac{3}{12} \text{ or } \frac{23}{12} > \frac{20}{12}$$

Therefore, there is enough rope.

Note: Students in Grade 8 multiply whole numbers and fractions.

Students' solutions could include any of the Grades 7 and 8 answers.

1. Some students may choose to use a scientific calculator with a fraction key.

Problem-Solving Strategies:
- Use a scientific calculator

$$2\ \boxed{a\,{}^{b}\!/_{c}}\ 2\ \boxed{a\,{}^{b}\!/_{c}}\ 3\ \boxed{-}\ 3\ \boxed{a\,{}^{b}\!/_{c}}\ 4\ \boxed{-}\ \boxed{(}\ 5\ \boxed{a\,{}^{b}\!/_{c}}\ 6\ \boxed{+}\ 5\ \boxed{a\,{}^{b}\!/_{c}}\ 6\ \boxed{)}\ \boxed{=}$$

Since the display shows 1 ⌋4, meaning $\frac{1}{4}$, there is some rope left over, indicating there is enough for all three pieces.

Note: Different calculators may show fractions differently.

2. Some students may represent the problem numerically using order of operations as follows:

Problem-Solving Strategies:
- Use symbolic representation

$$\begin{aligned}
&2\frac{2}{3}-\left[\frac{3}{4}+2\left(\frac{5}{6}\right)\right]\\
&=2\frac{2}{3}-\left[\frac{3}{4}+\frac{5}{3}\right]\\
&=\frac{8}{3}-\left(\frac{9}{12}+\frac{20}{12}\right)\\
&=\frac{8}{3}-\frac{29}{12}\\
&=\frac{32}{12}-\frac{29}{12}\\
&=\frac{3}{12}\\
&=\frac{1}{4}\ \text{(which is greater than zero, indicating there is more than enough rope for all three pieces)}
\end{aligned}$$

Problem Solving Across the Grades — Sample 2

Name:
Date:

So-Jung has a collection of 550 building blocks, whose sides measure $1\frac{3}{5}$ cm. She wants to store them in boxes with dimensions of $9\frac{1}{10}$ cm by $9\frac{1}{10}$ cm by $9\frac{1}{10}$ cm.

How many of these boxes will So-Jung need to store her collection?

Show your answer in two ways.

1.	**2.**

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

1. Students may build a ratio table using doubling.

Problem-Solving Strategies:
- Use a ratio table
- Use logic

Number of Blocks	Measure	Comments
1	$1\frac{3}{5}$	
2	$2\frac{6}{5} = 3\frac{1}{5}$	double
4	$6\frac{2}{5}$	double again
5	$6\frac{2}{5} + 1\frac{3}{5} = 7\frac{5}{5}$ $= 8$	add 1 block

$9\frac{1}{10} - 8 = 1\frac{1}{10}$ 1 more block $= 1\frac{3}{5}$ $1\frac{3}{5} > 1\frac{1}{10}$

There isn't enough room for 6 blocks. Therefore, the bottom layer has 5 × 5 = 25 blocks.

How many layers in 550 blocks? 550 ÷ 25 = 22

Since you can only have 5 layers in a box, there would be 4 boxes with 2 layers left.
So-Jung will need 5 boxes.

2. It is possible that coloured rods could be used with the 10 rods as the unit. However, this might be a bit unwieldy given that the length of the box would have to be represented using 11 rods. Some students may still need the comfort of concrete materials though. (See solution 3.) The rods could be used to determine the number of building blocks that would line up in one dimension only. That number, 5, would then have to be cubed. The rods would not need to be used for this operation.

Problem-Solving Strategies:
- Use manipulative representation
- Draw a diagram
- Use logic

$$5 \times 5 \times 5 = 125$$

So, 125 building blocks would fit into one box. Then, the students will determine the number of boxes needed for 550 blocks. They could do this in a number of ways:

a) 550 ÷ 125 = 4.4 Students should interpret this as 5 boxes.
b) 125 + 125 + 125 + 125 + 50 = 550 Students should interpret this as 5 boxes.

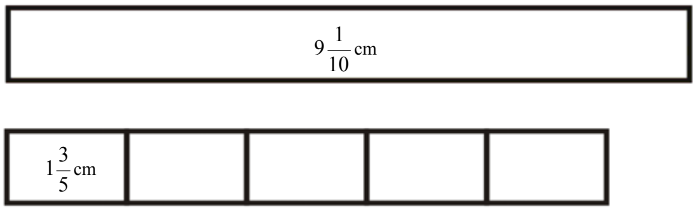
Note: It is assumed in all answers that the building blocks will be placed in the box face to face in layers and that there will be some air space in each of the three dimensions.

Grade 7 — Sample Solutions

> **Problem-Solving Strategies:**
> - Use a scale model

3. A student might use a scale model in one dimension.

Cut out a strip of paper $9\frac{1}{10}$ cm long. Cut out several strips $1\frac{3}{5}$ cm long.



Five is the maximum number of $1\frac{3}{5}$ cm strips that can be placed in $9\frac{1}{10}$ cm or less.
So, 5 building blocks will fit along each inside edge of the box. Since the box has three dimensions, the number building blocks are 5 × 5 × 5 = 125.

Note: A student may employ the logic in this answer but use diagrams of rectangles instead of cut-outs.

Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:

a) 550 ÷ 125 = 4.4   Students should interpret this as 5 boxes.
b) 125 + 125 + 125 + 125 + 50 = 550   Students should interpret this as 5 boxes.

Grade 8 — Sample Solutions

Students' solutions could include any of the Grade 7 answers.

> **Problem-Solving Strategies:**
> - Guess and check
> - Use representation
> - Draw a scaled diagram

1. A student might use symbols, beginning by guessing and checking.

Will 4 blocks fit in one dimension?

$$4\times 1\frac{3}{5}=4\times\frac{8}{5}=\frac{32}{5}=6\frac{2}{5}\left(<9\frac{1}{10}\right)$$

4 building blocks will fit and leave enough room for 1 more building block

Try 5 blocks:

$$5\times 1\frac{3}{5}=5\times\frac{8}{5}=8\left(<9\frac{1}{10}\right)$$

5 building blocks will fit and the $9\frac{1}{10}-8=1\frac{1}{10}$ cm left is not enough to fit in a 6^{th} building block

Try 6 blocks:

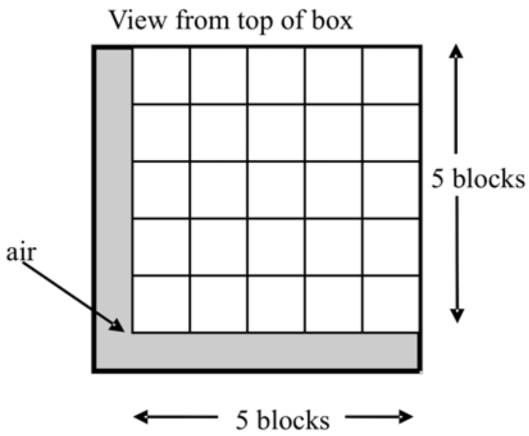
$$6\times 1\frac{3}{5}=6\times\frac{8}{5}=\frac{48}{5}=9\frac{3}{5}\left(>9\frac{1}{10}\right)$$

The maximum number of blocks for each dimension is 5.

Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:
a) 550 ÷ 125 = 4.4   They should interpret this as 5 boxes.
b) 125 + 125 + 125 + 125 + 50 = 550   They should interpret this as 5 boxes.

2. A student might use a scale drawing in two dimensions.

Problem-Solving Strategies:
- Use a scale drawing



$5 \times 5 = 25$
25 building blocks per layer

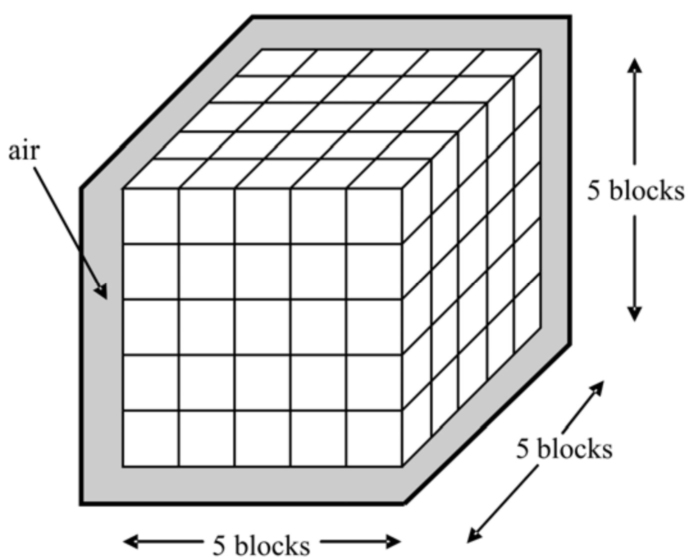
There are five layers like this.
$5 \times 25 = 125$

The maximum number of building blocks that will fit in the box is 125. Then, the students will determine the number of boxes needed for 550 blocks. They could do this in a number of ways:

a) $550 \div 125 = 4.4$ They should interpret this as 5 boxes.
b) $125 + 125 + 125 + 125 + 50 = 550$ They should interpret this as 5 boxes.

3. A student might use a scale drawing in three dimensions.
The student could determine that there are 5 building blocks per dimension as was done in previous answers.

Problem-Solving Strategies:
- Use a scale drawing



Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:

a) $550 \div 125 = 4.4$ They should interpret this as 5 boxes.
b) $125 + 125 + 125 + 125 + 50 = 550$ They should interpret this as 5 boxes.

Students' solutions could include any of the Grades 7 and 8 answers.

1. Some students might choose to use exponents and fractions to solve the problem.

> **Problem-Solving Strategies:**
> • Use symbolic representation

$$\left(9\frac{1}{10}\right)^3 \div \left(1\frac{3}{5}\right)^3$$

$$=\left(\frac{91}{10}\right)^3 \div \left(\frac{8}{5}\right)^3$$

$$=\frac{91^3}{10^3} \div \frac{8^3}{5^3} \quad \text{OR} \quad \left(\frac{91}{10} \div \frac{8}{5}\right)^3$$

Note: a calculator would be used from this point on.

$$=\frac{753571}{1000} \div \frac{512}{125}$$

$$=\frac{753571}{\cancel{1000}_{8}} \times \frac{\cancel{125}^{1}}{512}$$

$$=\frac{753571}{4096}$$

$$=183\frac{4003}{4096}$$

OR

$$=\left(\frac{91}{\cancel{10}_{2}} \times \frac{\cancel{5}^{1}}{8}\right)^3$$

$$=\left(\frac{91}{16}\right)^3$$

$$=\frac{91^3}{10^3}$$

Note: a calculator would be used from this point on.

$$=\frac{753571}{4096}$$

$$=183\frac{4003}{4096}$$

Therefore, she can fit 183 building blocks into one box.

Note: Students would need to think about the practical context to realize that this method gives an answer that is too big since it assumes use of space in the box that cannot accommodate the building blocks. If this solution is used, guide the students to determine the maximum number of building blocks that would fit along one side of the box. To determine the maximum number of building blocks, students could evaluate $\sqrt[3]{\frac{753571}{4096}}$ and see that it equals 5.6875.

So, five building blocks will fit along each inside edge of the box. Since the box has three dimensions, the number of building blocks is 5^3 or 125.

Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:

a) $550 \div 125 = 4.4$ They should interpret this as 5 boxes.
b) $125 + 125 + 125 + 125 + 50 = 550$ They should interpret this as 5 boxes.

2. Some students might choose to use decimals.

Problem-Solving Strategies:
- Use decimals

$$9\frac{1}{10} = 9.1 \qquad 1\frac{3}{5} = 1.6$$

$9.1 \div 1.6 = 5.6875$

So, 5 building blocks can fit in one dimension. Since the box has three dimensions, the number of cubes is 5^3 or 125.

Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:

a) $550 \div 125 = 4.4$ They should interpret this as 5 boxes.
b) $125 + 125 + 125 + 125 + 50 = 550$ They should interpret this as 5 boxes.

3. A student could choose to use a scientific calculator with a fraction key.

Problem-Solving Strategies:
- Use a scientific calculator

$$9\ \boxed{a\,{}^{b}\!/_{c}}\ 1\ \boxed{a\,{}^{b}\!/_{c}}\ 1\,0\ \boxed{\div}\ 1\ \boxed{a\,{}^{b}\!/_{c}}\ 3\ \boxed{a\,{}^{b}\!/_{c}}\ 5\ \boxed{=}$$

Since the display shows 5 ⌋ 11 ⌋ 16, meaning $5\frac{11}{16}$, So-Jung can fit 5 along one of the edges. Since the box has three dimensions, the number of cubes is 5^3 or 125.

Then, the students will determine the number of boxes needed for 550 blocks.
They could do this in a number of ways:

a) $550 \div 125 = 4.4$ They should interpret this as 5 boxes.
b) $125 + 125 + 125 + 125 + 50 = 550$ They should interpret this as 5 boxes.

Is This Always True?

Grades 7–10

Name:
Date:

1. Jagdeep claims that if you increase both the numerator and denominator of a fraction by the same amount, the result will always be greater than the original fraction.

   For example, start with $\frac{2}{3}$. Now add, let's say 5, to the numerator and denominator.

   $\frac{2+5}{3+5}=\frac{7}{8}$ which is greater than $\frac{2}{3}$.

   Is Jagdeep's claim true for all fractions?

2. Jeff declares that if you subtract a smaller fraction from a larger fraction, the result will always be between the two original fractions. For example, $\frac{9}{10}-\frac{1}{10}=\frac{8}{10}$

   Is Jeff's statement true for all fractions?

3. Graham states that if you add $\frac{1}{2}$ to any fraction, the common denominator will always be even.

   Is Graham's statement true for all fractions?

Sample Solutions

1. **No**. Counter-example: start with $\frac{3}{2}$, add 2 to the numerator and denominator: $\frac{3+2}{2+2}=\frac{5}{4}$

   $\frac{5}{4}$ (which is $1\frac{1}{4}$) is smaller than $\frac{3}{2}$ (which is $1\frac{1}{2}$)

   Follow-up question: Under what conditions will this statement be true?

   Problem-Solving Strategies:
   • Find a counter-example

2. **No**. Counter-example: $\frac{9}{10}-\frac{8}{10}=\frac{1}{10}$ (not between the two original fractions)

   Problem-Solving Strategies:
   • Find a counter-example

3. **Yes**. If the denominator of the first fraction is odd, you will need to multiply it by 2 to get the lowest common denominator, so the common denominator will be a multiple of two and therefore, even. If the denominator of the first fraction is even, then it will be the common denominator since two will divide evenly into every even number.

   Problem-Solving Strategies:
   • Use logical reasoning

Is This Always True?

Grades 8–10

Name:
Date:

1. Is it always true that if you multiply two fractions the product will be smaller than the original two fractions? For example, $\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)=\frac{1}{6}$

2. Anuroop asserts that the square of any rational number always results in a smaller rational number.

   Is Anuroop's statement always true?

3. Ana argues that if you multiply the numerator and denominator of a fraction by the same number, the original fraction is unchanged.

   Is Ana's statement always true?

4. Susan notices that the formula for the area of a trapezoid is stated as:

   $$A=h\left(\frac{a+b}{2}\right),\qquad A=\frac{h}{2}(a+b),\qquad A=\frac{1}{2}h(a+b)$$

   Are the 3 formulas always equivalent to each other?

Sample Solutions

1. **No**. Counter-example: $\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)=\frac{35}{2}$, which is larger than the original fractions.

   Problem-Solving Strategies:
   - Find a counter-example

2. **No**. Counter-example: $\left(\frac{2}{1}\right)^2=\frac{2}{1}\times\frac{2}{1}=\frac{4}{1}=4$, (4 is larger than 2) or

   $\left(\frac{3}{2}\right)^2=\frac{3}{2}\times\frac{3}{2}=\frac{9}{4}=2\frac{1}{4}$, $\quad$ ($\frac{3}{2}=1\frac{1}{2}$ is smaller than $\frac{9}{4}=2\frac{1}{4}$).

   Problem-Solving Strategies:
   - Find a counter-example

3. **Yes**. If you multiply the numerator and denominator by the same number, it is the same as multiplying by the number 1. Multiplication by 1 never changes a number.

   Problem-Solving Strategies:
   - Use logical reasoning

4. **Yes**. $\frac{1}{2}$ multiplied by h is the same as $\frac{h}{2}$.

   So, $A=h\left(\frac{a+b}{2}\right)=\frac{h}{2}(a+b)=\frac{1}{2}h(a+b)$.

   Problem-Solving Strategies:
   - Use equivalent statements