

Unit C

Volume of Right Prisms and Cylinders

Combined Grades 7 and 8

Lesson Outline

BIG PICTURE

Students will:

- develop and apply the standard formula $\text{Volume} = \text{area of base} \times \text{height}$ to calculate volume of right prisms (Grade 7) and cylinders (Grade 8);
- investigate the volume of a variety of right prisms (Grade 7) and cylinders (Grade 8);
- make conversions between metric units of volume and capacity;
- sketch different prisms that have the same volume.

Day	Grade 7 Math Learning Goals	Grade 8 Math Learning Goals	Expectations
1	<ul style="list-style-type: none"> • Build models of prisms using linking cubes. • Determine the volume of a prism by counting the number of cubes in its structure. • Develop and apply the formula for volume of a prism. • Relate exponential notation to volume, e.g., explain why volume is measured in cubic units. 		7m17, 7m35, 7m40 8m32 CGE 5d, 5e
2	<ul style="list-style-type: none"> • Explore with centimetre cubes to determine the number of cubic centimetres that entirely fill a cubic decimetre, e.g., determine the number of cm^3 that cover the base. How many layers are needed to fill the whole dm^3? • Determine how many dm^3 fill a m^3 and use this to determine how many cm^3 are in a m^3. • Solve problems that require conversion between metric units of volume. 		7m35 8m33 CGE 3b, 4a
3	<ul style="list-style-type: none"> • Explore the relationship between cm^3 and litres, e.g., cut a 2-litre milk carton horizontally in half to make a 1-litre container that measures $10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$. This container holds 1-litre or 1000 cm^3. • Determine that 1 cm^3 holds 1 millilitre. • Solve problems that require conversion between metric units of volume and capacity. 		7m35 8m33 CGE 3b, 4a
4	<ul style="list-style-type: none"> • Develop and apply the formula for volume of a rectangular prism. • Solve problems involving volume of a rectangular prism that require conversion between metric measures of volume and capacity. • Begin research on applications of volume and capacity (Grade 8). 		7m35, 7m40 8m33 CGE 4b, 4c
5	<ul style="list-style-type: none"> • Investigate, develop, and apply the formula for volume of triangular prisms. 	<ul style="list-style-type: none"> • Investigate, develop, and apply the formula for volume of triangular prisms and volume of a variety of prisms. • Solve problems involving volume of a triangular prism that require conversion between metric measures of volume. • Research applications of volume and capacity. 	7m35, 7m40 8m50 CGE 3c, 5d
6	<ul style="list-style-type: none"> • Calculate the volume of a parallelogram-based prism by decomposing it into two triangular prisms. 	<ul style="list-style-type: none"> • Investigate volume of a cylinder by comparing the capacity of the cylinder to the volume calculated using area of the base \times height. 	7m35, 7m40 8m33, 8m37 CGE 5f

Day	Grade 7 Math Learning Goals	Grade 8 Math Learning Goals	Expectations
7	<ul style="list-style-type: none"> Determine the volume of a trapezoidal-based prism by decomposing it into two triangular prisms and a rectangular prism. Solve problems involving volume of a trapezoidal-based prism 	<ul style="list-style-type: none"> Solve problems involving the volume of a cylinder. 	7m35, 7m38, 7m40 8m37, 8m39 CGE 5f
8	<ul style="list-style-type: none"> Determine the volume of right prisms with bases that are composite figures by decomposing the prism into triangular and rectangular prisms (Grade 7) and cylinders (Grade 8). Research applications of volume and capacity. 		7m35, 7m40 8m39 CGE 3b
9	<ul style="list-style-type: none"> Apply volume and area formulas to explore the relationship between triangular prisms (and cylinders in Grade 8) with the same surface area. Estimate volumes. 		7m42 8m39 CGE 5e, 5f
10	<ul style="list-style-type: none"> Apply volume and area formulas to explore the relationship between rectangular prisms (and cylinders in Grade 8) that have the same volume. 		7m42 8m38, 8m39 CGE 4c, 5a
11	<ul style="list-style-type: none"> Demonstrate knowledge and understanding of volume of prisms with polygon bases (and cylinders in Grade 8). 		7m34, 7m35, 7m42 8m39 CGE 3a, 3c



Math Learning Goals

- Use linking cubes to build models of prisms.
- Determine the volume of a prism by counting the number of cubes in its structure.
- Develop and apply the formula for volume of a prism.
- Relate exponential notation to volume, e.g., explain why volume is measured in cubic units.

Materials

- linking cubes
- BLM C.1.1, C.1.2
- isometric dot paper

Assessment Opportunities

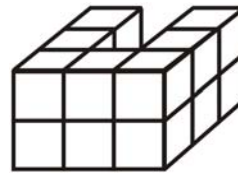
Minds On... Whole Class → Guided Discussion

Show a cube and ask: If the length of one side is 1 unit –

- What is the surface area of one face? (1 unit²)
- What is the volume? (1 unit³)
- Why is volume measured in cubic units?

Using a “building tower” constructed from linking cubes, lead students through a discussion based on the model:

- Why is this a prism?
- What is the surface area of the base?
- What is the height of the building?



Count the cubes to determine the volume of the building.

A prism has at least one pair of congruent, parallel faces.

Students gain better insight into the development of volume concepts and formulas by actually constructing the objects and counting the cubic units.

Action! Pairs (Students in the Same Grade) → Investigation

Students create several more irregular prisms of various sizes. Invite students to ask clarifying questions about the investigation and display their findings in the table (BLM C.1.1).

After investigating the problem with several samples, they generalize the formula for the volume of a prism.

$$\text{Volume} = \text{area of base} \times \text{height}$$

Students test their formula for accuracy by constructing two other towers.

Curriculum Expectations/Observation/Checkbric: Observe students and assess their inquiry skills and learning skills (BLM C.1.2.). **Note:** This is the first of several days where the same processes can be observed in small groups.

Invite students to ask clarifying questions about the investigation.

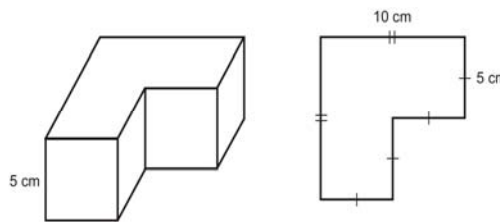
Provide some support for students who are having difficulty generalizing the formula.

Consolidate Debrief Whole Class → Student Presentation

As students present their findings, summarize the results of the investigation on a class chart.

Orally complete a few examples calculating volume of prisms given a diagram.

Reinforce the concept of cubic units.



The poster Metric – Capacity and Mass provides a good reference for this unit (BLM C.1.3).

Home Activity or Further Classroom Consolidation

Both Grades: A prism has a volume of 24 cm³. Draw prisms with this volume. How many possible prisms are there with a volume of 24 cm³?

Grade 8: Include several prisms with fractional and decimal dimensions.

If students use decimal and fractional measures, an infinite number of prisms are possible.

*Concept Practice
Application
Skill Drill*

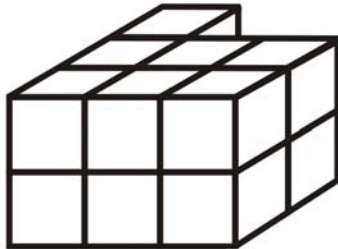
C.1.1: Building Towers

Name:

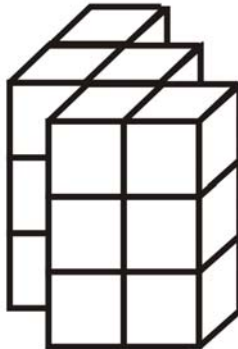
Date:

Each tower pictured here is a prism. Build each prism and determine the volume of each building by counting cubes.

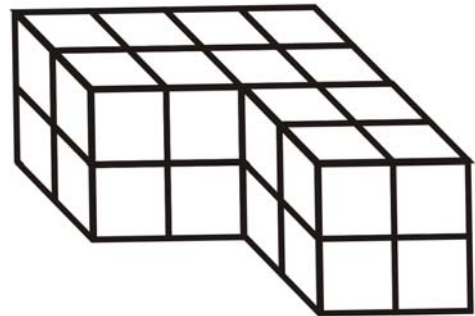
Tower A



Tower B



Tower C



Complete the table of measures for each tower.

Tower	Area of Base	Height of Tower	Volume (by counting cubes)
A			
B			
C			

What relationship do you notice between volume, area of the base, and height?

State a formula that might be true for calculating volume of a prism when you know the area of the base and the height of the prism.

Test your formula for accuracy by building two other prism towers and determining the volume. Sketch your towers and show calculations on this table.

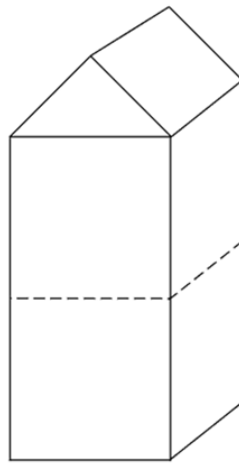
Tower	Area of Base	Height	Volume (by counting cubes)	Volume (using your formula)
D				
E				

Describe the accuracy of your formula and explain any adjustments that you made.

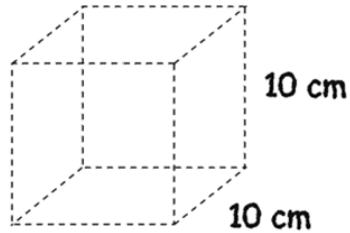
C.1.2: Checkbric

Learning Skills	Needs Improving	Satisfactory	Good	Excellent
Independent Work				
<ul style="list-style-type: none"> follows routines and instruction without supervision 				
<ul style="list-style-type: none"> persists with tasks 				
Initiative				
<ul style="list-style-type: none"> responds to challenges 				
<ul style="list-style-type: none"> demonstrates positive attitude towards learning 				
<ul style="list-style-type: none"> develops original ideas and innovative procedures 				
<ul style="list-style-type: none"> seeks assistance when necessary 				
Use of Information				
<ul style="list-style-type: none"> organizes information logically 				
<ul style="list-style-type: none"> asks questions to clarify meaning and ensure understanding 				

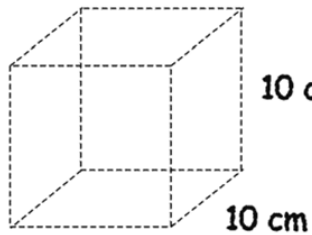
C.1.3: Metric – Capacity and Mass



Two-litre milk carton

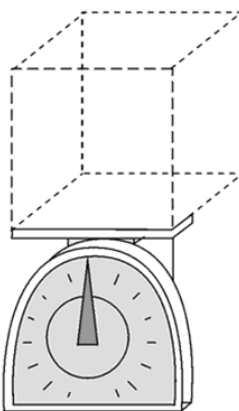


10 cm



10 cm

1000 cm³ holds 1 litre



1 litre of water has a mass of 1 kg

****1cm³ holds 1 ml and has a mass of 1 g**



Math Learning Goals

- **Grade 7:** Develop and apply the formula for volume of a rectangular prism. Solve problems involving volume that requires conversion between metric measures of volume and capacity.
- **Grade 8:** Begin research on application of volume and capacity.

Materials

- models of rectangular prisms
- calculators

Assessment Opportunities

Minds On... Whole Class → Sharing

Students share their diagrams and solutions for prisms with a volume of 24 cm³ completed on Day 1.

Whole Class → Discussion

Using a concrete sample of a rectangular prism, ask students:

- What can be altered in the volume of a prism formula to make the formula specific for a rectangular prism?
- Will the volume be the same or different when the prisms are oriented vertically or horizontally?
- What do we mean by “dimension of a prism?”
- How is the volume of a prism related to its capacity?

Grade 8: If you know the capacity of a rectangular prism, how can you determine possible dimensions of the prism?

Action!

Grade 7 Pairs → Investigation

Students determine the volume of a rectangular prism.

Grade 8 Pairs → Problem Solving

Students use a calculator to find multiple solutions to this problem:

An aquarium in the shape of a rectangular prism has a capacity of 36 000 mL. Determine possible dimensions of the aquarium.

Curriculum Expectations/Observation/Checkbric: Observe students and assess their inquiry skills and learning skills, using BLM C.1.2. **Note:** This checkbric can be used over several days in this unit where the same processes can be observed.

Consolidate Debrief

Grade 7 Students → Reflection

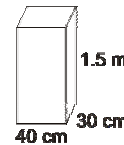
Students share their investigation and justify their explanations, using diagrams and calculations.

Grade 8 Students → Presentation

Students present their list of possible measures for the rectangular prism aquarium. Make connections to the factors of 36 000.

Whole Class → Problem Solving

Find the volume of the plastic bin illustrated in the shape of a rectangular prism. Express the volume in m³ and cm³. What is the capacity of the plastic bin?



Home Activity or Further Classroom Consolidation

Both Grades:

How many rectangular prisms with whole number dimensions can you sketch with volume: a) 27 cm³? b) 48 cm³?

- Why are there many more solutions for 48 cm³?
- Choose a volume for a rectangular prism that can be generated by several different sets of measurements. List several different dimensions.

Grade 8: Create a cover page for a portfolio of applications of volume and capacity measurement.

For any prism:
 $V = \text{Area of base} \times \text{height}$

For rectangular prisms:
 $V = l \times w \times h$

When calculating volume of a rectangular prism, any of its faces can be thought of as the “base.”

The capacity of a prism is the maximum amount that can be contained in the prism in mL. The volume of a prism is the amount of space that the prism occupies in cm³.

Some Grade 8 students may need support to see that 36 000 mL is equivalent to a volume of 36 000 cm³.

Some of the many possible dimensions for the Grade 8 problem:
30 × 30 × 40
20 × 30 × 60
60 × 15 × 40
15 × 15 × 160
15 × 30 × 80

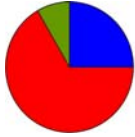
Link measurement to the use of factors and multiples: The more factors a number has, the greater the number of rectangular prisms that will have that volume.

*Exploration
Concept Practice*

Unit C: Day 5: Investigating Volume of a Triangular Prism (Grade 7)

Investigating Volume of a Variety of Prisms (Grade 8)

Grades 7 and 8



Math Learning Goals

- Investigate, develop, and apply the formula for volume of triangular prisms ([Grade 7](#)) and other prisms ([Grade 8](#)).
- [Both Grades](#): Solve problems involving volume of a triangular prism that require conversion between metric measures of volume.
- [Grade 8](#): Solve problems involving volume of any prisms that require conversion between metric measures of volume.

Materials

- models of triangular prisms
- BLM C.5.1

Minds On...

Whole Class → Sharing

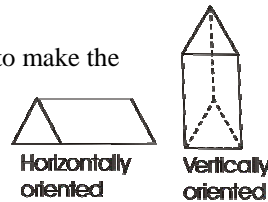
Students share their sketches of prisms with volumes 27 cm^3 and 48 cm^3 .

Generate a list of rectangular prisms that can be generated by several different sets of measurements. Discuss the relationship of these measures to the factors of a number.

Whole Class → Discussion

Using concrete samples of a triangular prism, ask:

- What can be altered in the volume of a prism formula to make the formula specific for a triangular prism?
- Will the volume be the same or different when the prism is orientated vertically or horizontally?
- What do we need to think about when applying the volume formula to a triangular prism?



Assessment Opportunities

For any prism:
 $V = \text{Area of base} \times \text{height}$

For triangular prisms:
 $V = \frac{1}{2}bh \times H$

When calculating volume of a rectangular prism, any of its faces can be thought of as the 'base.' However, when calculating the volume of a triangular prism, its 'base' is one of the triangles, not one of the rectangles.

Action!

Grade 7 Pairs → Investigation

Students use a triangular prism to develop a formula specific to their prism (based on the general formula of volume = area of base \times height). They investigate how to use their specific formulas to calculate volumes of horizontally and vertically oriented prisms, including several examples and show their calculations to justify their conclusions.

Grade 8 Pairs → Investigation

Determine the volume of prisms with a variety of bases (triangular, parallelogram, trapezoid).

Curriculum Expectations/Observations/Checkbric: Observe students and assess their inquiry skills and learning skills. **Note:** This is one of several days where the same processes can be observed.

Some students may need to physically orient triangular prisms vertically in order to visualize and determine the volume.

Consolidate Debrief

Grade 7 Students → Reflection

Students share their findings. Focus discussion on the need to identify the triangular face as the 'base' when using the formula $V = \text{area of base} \times \text{height}$ for a triangular prism.

Discuss the need for h and H in the formula for volume, the fact that h is perpendicular to b , and the fact that H is perpendicular to the triangular base.

Discuss each of these ideas in relationship to rectangular prisms.

Grade 8 Students → Presentation

Students present the solution to the problem they worked on.

Home Activity or Further Classroom Consolidation

Sketch a triangular prism whose whole number dimensions will produce a volume that is: a) an even number b) an odd number c) a decimal value.

Explain your thinking in each case.

[Grade 7:](#) Complete the practice questions on worksheet C.5.1.

[Grade 8:](#) Research at least 3 applications of volume and capacity measurement for your portfolio.

Exploration
Concept Practice

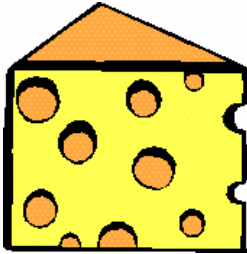
C.5.1: Volume of Triangular Prisms

Grade 7

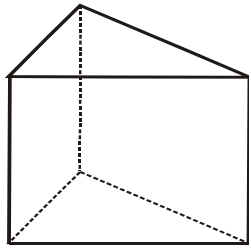
Show your work using good form and be prepared to tell how you solved the problem.

1. Determine the volume of the piece of cheese (ignore the holes!).
Create a problem based on the volume.

Picture

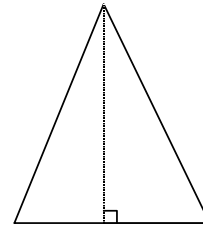


Skeleton



height of prism = 5.0 cm
length of rectangle = 6.3 cm

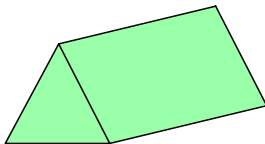
Base



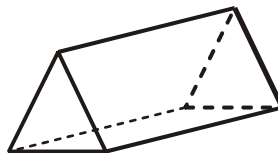
height of triangle = 6.0 cm
base of triangle = 4.0 cm

2. Determine the volume of the nutrition bar.
Create a problem based on the volume.

Picture

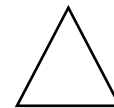


Skeleton



length of rectangle = 5.0 cm

Base



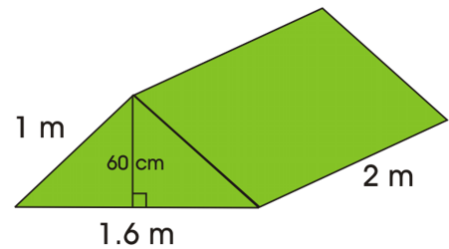
equilateral triangle with:
height = 3.0 cm
base = 3.5 cm

C.5.1: Volume of Triangular Prisms (continued)

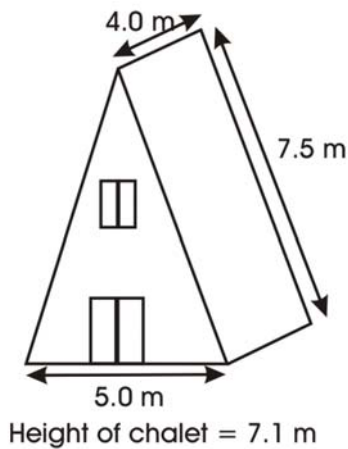
Grade 7

3. Determine the volume of air space in the tent.
The front of the tent has the shape of an isosceles triangle.

Create a problem based on the volume.



4. a) If you could only have 1 person per 15 m^3 to meet fire safety standards, how many people could stay in this chalet?



Hint:

Think about whether the height of the chalet is the same as the height of the prism.

Which measurements are unnecessary for this question?

- b) How much longer would the chalet need to be to meet the safety requirement to accommodate 16 people?

Unit C: Day 6: Investigating Volume of Right Prisms with Parallelogram Bases (Grade 7)

Investigating Volume of a Cylinder (Grade 8)

Grades 7 and 8



Math Learning Goals

- **Grade 7:** Calculate the volume of a parallelogram-based prism by decomposing it into two triangular prisms.
- **Grade 8:** Investigate volume of a cylinder by comparing the capacity of the cylinder to the volume calculated, using area of the base \times height.

Materials

- BLM C.6.1, C.6.2
- calculators
- cans, measuring cups
- tape measures

Assessment Opportunities

Minds On... Whole Class \rightarrow Discussion

Display a collection of cans and prisms with labels still attached. Students read aloud the capacity on various cans (in mL) and mentally convert the measures to a volume in cm^3 .

Ask students how knowing the capacity of a cylinder or prism can help to determine its volume.



Grade 7 Students \rightarrow Demonstration

Display two congruent triangular prisms (BLM C.6.1) fitted together to make a parallelogram-based prism. To determine the formula for volume of a parallelogram-based prism, follow the lesson detailed in TIPS4RM Grade 7 Unit 10 Day 6.

Action!

Grade 7 Pairs \rightarrow Investigation

Refer to TIPS4RM Grade 7 Unit 10 Day 6.

Grade 8 Pairs \rightarrow Investigation

Students determine the volume of a cylinder using the general formula for volume of a prism (BLM C.6.1). They verify their volume calculation by determining the capacity of the cylinder.

Curriculum Expectations/Observation/Checkbric: Observe students and assess their inquiry skills and learning skills. **Note:** This is one of several days where the same processes can be observed in small groups.

Comparing capacity in mL to volume in cm^3 is the basis for determining the formula for volume of a cylinder.

An Anticipation Guide could be used before Grade 8 students receive BLM C.6.2. See *Think Literacy: Cross Curricular Approaches – Mathematics* for examples.

Students in both grades will benefit from investigating volume, using concrete materials to help them understand the basis of the formulas that they create.

Consolidate Debrief

Grade 7 Students \rightarrow Discussion

Debrief the students' findings. Students should understand that volume of a parallelogram-based prism can be determined by decomposing the parallelogram into its composite triangles and finding the sum of the volumes. The volume of a parallelogram-based prism can also be determined using the standard volume formula of area of the base \times height of the prism.

Grade 8 Students \rightarrow Discussion

Debrief the students' findings. Discuss why errors in measurement might occur. Students should understand that the volume of a cylinder can be determined using the same general formula as volume of a prism. Discuss the similarities between a cylinder and a right prism.

Right prisms and cylinders both have parallel and congruent bases and tops; lateral faces are perpendicular to the base.

The cylinder does not have faces that are rectangular, but its curved face can be "unrolled" to form a rectangle.

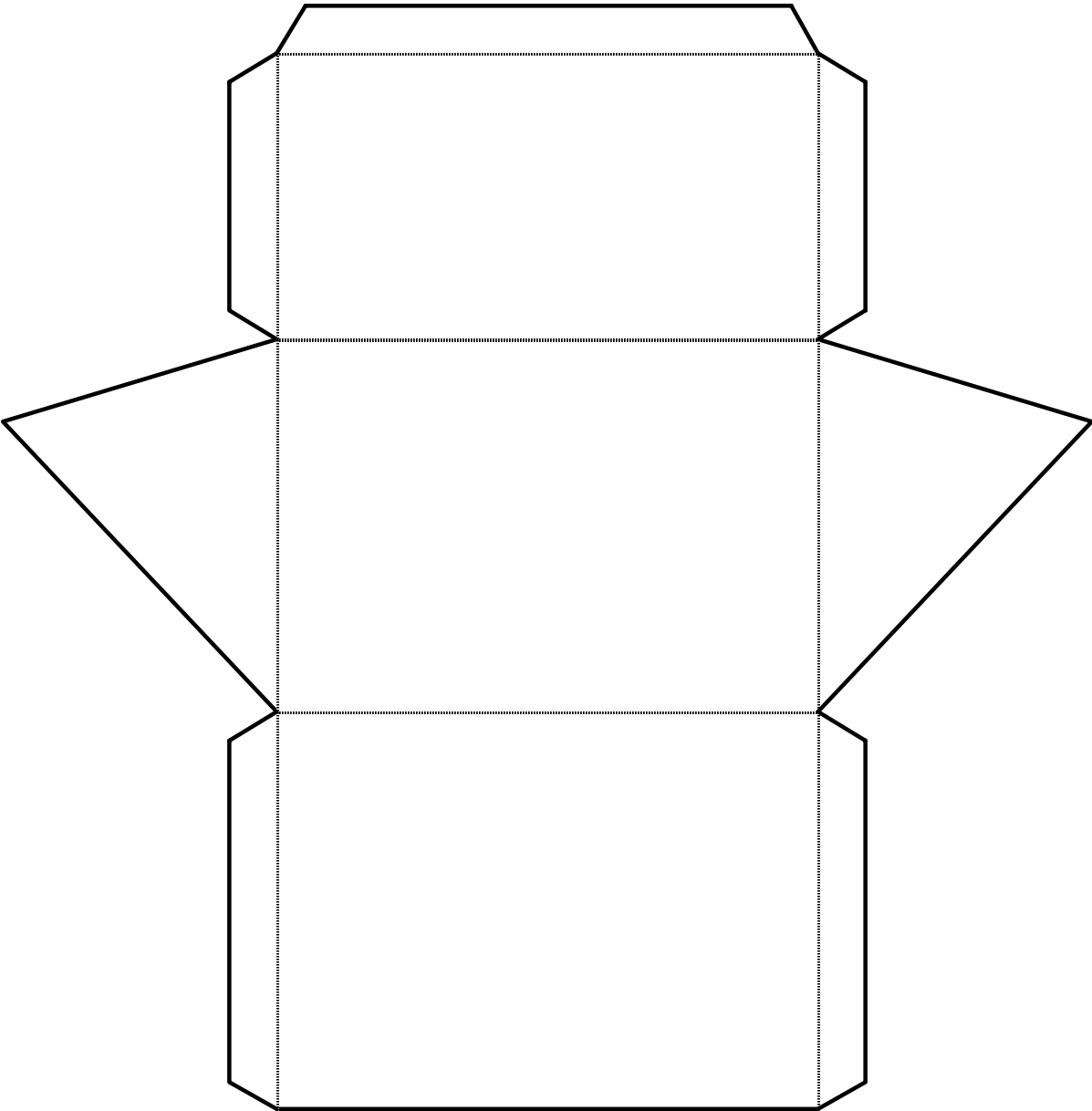
Home Activity or Further Classroom Consolidation

Grade 7: Complete the practice problems.

Grade 8: Your friend frequently mixes up the formula for area of a circle and the formula for volume of a cylinder. In your journal, use diagrams to explain the differences between the two formulas and the units used to measure them.

Provide students with appropriate practice questions.

Concept Practice Reflection



C.6.2: Determining the Volume of a Cylinder

Grade 8

Can the formula for volume of a right prism, **Volume = Area of base × Height**, be used to determine the volume of a cylinder?

My hypothesis is:

Yes

No

Sometimes

Let's Investigate!

Select a cylindrical can.

Determine the capacity of the cylinder by filling the can with water.

Pour the water into a measuring cup or graduated beaker. Record the capacity in mL.



Use a tape measure to determine the diameter and height of the cylinder in centimetres.

Calculate the volume of a cylinder using the formula for volume of a right prism. According to your volume calculation, what is the capacity of the cylinder?

Place your results in the table. Repeat the experiment with two more cans.

Can	Capacity (determined by measuring with water)	Dimensions of Cans	Area of the Circular Base	Height of Cylinder	Volume: Area of Base × Height	Capacity (determined from volume calculation)
A						
B						
C						

Examine the results on your table. Was your hypothesis correct?

Write about your findings. Consider the following as you analyse your data:

- Should the two capacity measures be the same for each can? Are they the same?
- Why might there be slight discrepancies?
- Can the formula for volume of a right prism also be use to calculate volume of a cylinder?

Unit C: Day 7: Volume of Right Prisms with Trapezoid Bases (Grade 7) Volume of Cylinders (Grade 8)

Grades 7 and 8



Math Learning Goals

- **Grade 7:** Determine the volume of a trapezoidal-based prism by decomposing it into two triangular prisms and a rectangular prism. Solve problems involving volume of a trapezoidal-based prism.
- **Grade 8:** Solve problems involving the volume of a cylinder.

Materials

- calculators
- box with can marks on bottom
- BLM C.7.1, C.7.2

Assessment Opportunities

Minds On... **Small Groups → Peer Tutoring**

Grade 7: Students discuss the homework questions that they found the most challenging. They peer edit each other's solutions. Assign one or two groups a question to present to the class.

Grade 8: Share journal entries about area of a circle and volume of a cylinder; also share portfolios, and applications.

Action!

Pairs → Investigation

Grade 7: Prepare students to investigate the volume of a trapezoid-based right prism by decomposing the trapezoid into triangles and rectangles. Refer to TIPS4RM Grade 7 Unit 10 Day 7 for detailed lesson plan.

Students complete their investigation. Prompt students who are having difficulty decomposing the trapezoid by suggesting some of these possibilities:



Grade 8: Display a cardboard grocery box that has contained cans. Examine the can marks on the bottom of the box. Discuss how cylinders are packaged in rectangular prism boxes. Students begin the packaging problems on BLM C.7.2. Students complete BLM C.7.2.

Curriculum Expectations/Observation/Checkbric: Observe students and assess their inquiry skill and learning skills. **Note:** This is one of several days in this unit where the same processes can be observed in small groups.

Consolidate Debrief **Grade 7 Students → Discussion**

Debrief the students' findings. Discuss the need for h and H in the formula. Focus the discussion on the fact that the standard formula volume = area of base \times height of the prism always works for right prisms. Volume can also be calculated by decomposing into composite prisms. Ask a student to read aloud the problem on BLM C.7.1. Ask other students to think aloud about the problem.

Grade 8 Students → Discussion

Debrief the activity. Note that doubling the diameter will more than double the volume. Discuss the effect of squaring the radius vs. doubling.

Home Activity or Further Classroom Consolidation

Grade 7: Complete worksheet C.7.1

Grade 8:

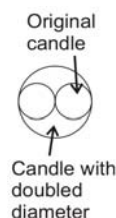
- A new pop can is designed to be twice as tall as the original can. How are the volumes of the cans related?
- Complete the practice questions on volume and capacity of cylinder problems.

Concept Practice

The most common ways to decompose a trapezoid are into:
a) one rectangle and two triangles;
b) two triangles;
c) one parallelogram and one triangle.

See *Think Literacy: Cross Curriculum Approaches – Mathematics, Visualizing*, pp. 54–53.

Grade 8 question 5:



The candle top and bottom are four times as big when the diameter is doubled.

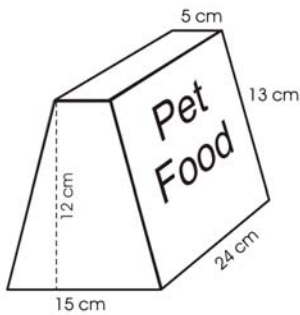
Provide students in Grade 8 with appropriate practice questions on volume and capacity of cylinders.

C.7.1: Designing a Box

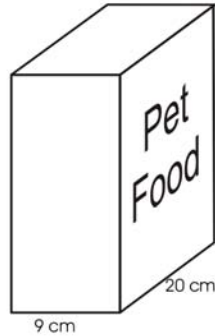
Grade 7

A local pet food company wishes to package its new product in a box. The preliminary design is shown as Box A.

1. Determine the volume of Box A. Verify your calculation using an alternative method of calculation.



Box A



Box B

2. Box B has the same volume as Box A. What is the height of Box B? Explain how you know.

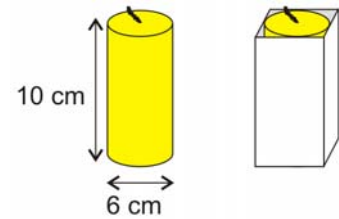
3. Design a new box, Box C, with the same volume as the two boxes above.

C.7.2: A Packaging Problem

Grade 8

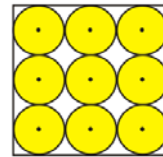
A candle company wishes to package its cylindrical candles in boxes.

Each candle is placed in a box that is a square-based prism. The candles fit tightly into the boxes in order to avoid damage from rolling.



1. Determine the volume of the candle.
2. Determine the dimensions and capacity of the box that will fit tightly against the candle.
3. What percentage of the box's capacity is unused by the candle?

4. If 9 candles are being packaged in one box, what percentage of the box's capacity is unused? Explain your answer.



5. The candlemaker decided to double the diameter of the candle.
 - a) Will the volume of the candle be doubled? Explain how you know.
 - b) Will the surface area of the top be doubled? Justify your answer.
6. What are the dimensions and capacity of the box that will contain 9 of these new shape candles?



Math Learning Goals

- Determine the volume of right prisms with bases that are composite figures by decomposing the prism into triangular and rectangular prisms (Grade 7) and cylinders (Grade 8).

Materials

- calculators
- BLM C.8.1
- graph or grid paper

Assessment Opportunities

Minds On... Whole Class → Brainstorming

Use a mind map and brainstorm a list of shapes that could form the base of a right prism. Students sketch the 2-D shapes on the board or have some diagrams prepared on an overhead. Include pentagons, hexagons, quadrilaterals, and composite shapes (Grade 7) and circles (Grade 8).

Same Grade Pairs → Problem Solving

Students decompose the shapes displayed on the board/overhead into triangles and rectangles and (Grade 8) circles. They discuss how they would determine the area of the shape in order to calculate the volume of a prism with that shape for a base.

Action! Same Grade Pairs → Problem Solving

Students calculate the volume of the boxes. Students present different solutions to the class (BLM C.8.1).

Pairs → Game

Select a secret volume (say 925 cm³) and write it in a hidden location. In pairs students design a polygon-based prism and calculate its volume. Students may create several different figures, but each polygon base must include a different set of composite shapes.

Grade 8: Pairs must include circles and cylinders.

Same Grade Pairs → Game

Students use dot paper or grid paper to design right prism figures whose bases are composite shapes. They sketch the base and the side view, with measurements labelled on the diagram (as illustrated on BLM C.8.1). They calculate the volume of their figures and write it beside the sketch.

Curriculum Expectations/Observation/Checkbric: Observe students and assess the inquiry skill and learning skills. **Note:** This is one of several days in this unit where the same processes can be observed in small groups.

Consolidate Debrief Grade Groups → Conferencing

Reveal the target volume and the pairs with that volume present their figure. The grade groups check the calculations for accuracy.

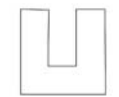
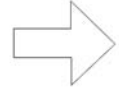
Home Activity or Further Classroom Consolidation

Both Grades: Design two right prisms with bases that are composite figures. The prisms must have an approximate capacity of 1000 mL.

Grade 8: Research two more volume and capacity applications to add to your portfolio.

Concept Practice

Suggested composite shapes (Grades 7 and 8):



(Grade 8 only)



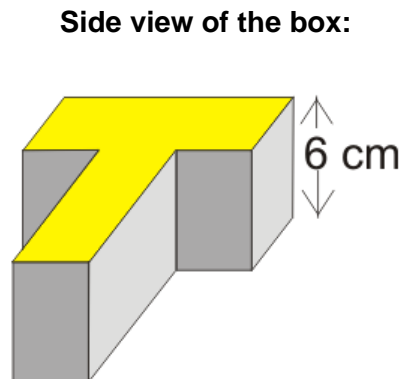
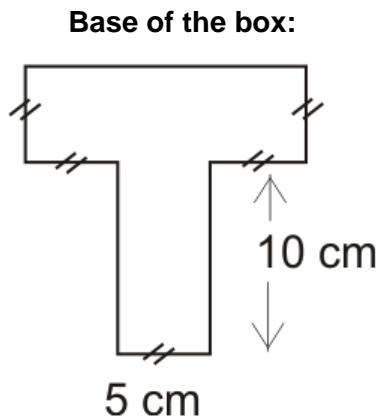
Have some of the previously constructed figures available for student reference, e.g., Unit B Day 6 Activity “Designing a Gift Box.”

Some students may still need to build the 3-D figure to visualize the volume.

C.8.1: Gift Boxes

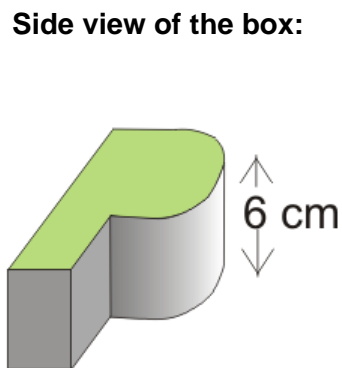
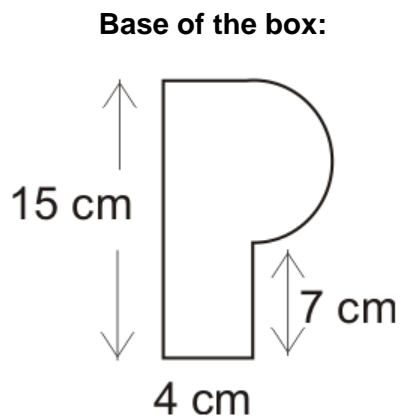
Grade 7

Determine the volume of the gift boxes designed by the students from Trillium Park School. Verify your answer by decomposing the figure in a different way and re-calculating the volume. Include diagrams.



Grade 8

Determine the volume of the gift boxes designed by the students from Trillium Park School. Verify your answer by decomposing the figure in a different way and re-calculating the volume. Include diagrams.





Math Learning Goals

- Apply volume and area formulas to explore the relationship between triangular prisms (both grades) and cylinders (Grade 8 only).
- Estimate volumes.

Materials

- rope, rectangular tarp or sheet
- connecting cubes
- fill material (sand, bird seed, Styrofoam chips)
- BLM C.9.1, C.9.2, C.9.3

Assessment Opportunities

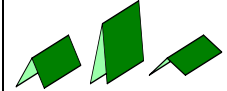
Minds On... Small Groups → Discussion and Presentation

Students share solutions for homework questions assigned on Day 8 for volume of right prisms with composite bases. Collect Grade 8 portfolios for assessment.

Whole Class → Activity

This activity might be done outside or in a gymnasium. Place a large tarp on the floor/ground. Students become vertices of a triangular prism tent. Gradually raise the tarp until a tent is formed. Invite two or three other students to become campers. Their task is to verbalize observations about the tent’s capacity as the tent’s height is increased and decreased. Ask: Does it feel like there is more or less room? The surface area has remained the same, but has the volume?

Consider using a rope to hold the peak of the tent in place.



Students might investigate changes when the “fold” is moved from lengthwise to widthwise.

Action! Whole Class → Instructions

Ask: If two triangular prisms (or two cylinders) have the same surface area, do they have the same volume?

Students record their hypothesis in their journal before beginning the investigation.

Same Grade Pairs → Experimenting

Grade 7: Complete the investigation of triangular prisms (BLM C.9.1).

Grade 8: Complete the investigation of cylinders (BLM C.9.2).

Learning Expectations/Observation/Checkbric: Assessing students’ ability to stay on task.

Pairs → Discussion

In pairs, students respond to the question: Is the following statement sometimes, always, or never true?

Two triangular prisms (or cylinders – Grade 8) with the same surface area also have the same volume.

While circulating, ask probing questions to determine if students understand that a constant surface does not have a constant volume. Ask students if their conclusion would be the same for closed and open-ended prisms.

From the tent making activity, students should have a sense that the statement is not always true.

Consolidate Debrief Whole Class → Discussion

Discuss how an experiment can be used to help confirm or deny a hypothesis about the relationship between surface area and volume.

Home Activity or Further Classroom Consolidation

In your journal, write about the volume of triangular prism (or cylinders – Grade 8) that have the same surface areas. Explain how you initially thought the volume would be related to the surface area.

- Did your thinking change? Explain why.
- In what jobs does this relationship matter?

GSP[®]4 C.9.3 provides a dynamic model of the paper folding activity.

Skill Practice

C.9.1: Surface Area and Volume of Triangular Prisms

Grade 7

Use this entire paper to make Tent A by folding along the two solid lines below. Form a triangular prism. Imagine that it also has paper on the two triangular ends. Sketch the prism and its net. Take the measurements needed to calculate the surface area (including the two triangular ends) and volume. Label the diagrams with the measurements. Calculate the surface area and volume.

Repeat the process to make the prism named Tent B by folding along the two broken lines.



Both prisms, Tent A and Tent B have the same surface area. Do they have the same volume? Discuss your findings.

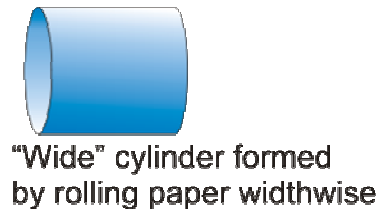
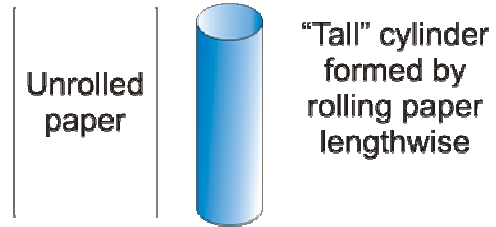
C.9.2: Surface Area and Volume of Cylinders

Grade 8

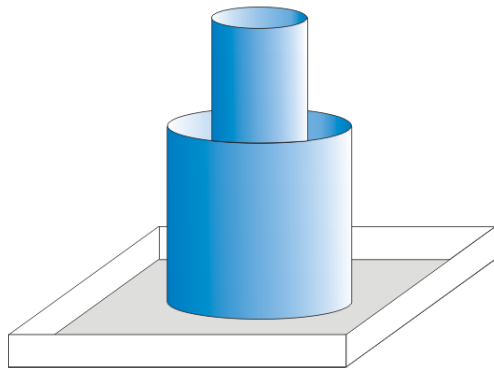
Make two different cylinders by rolling a rectangular sheet of paper two ways, lengthwise and widthwise. Tape the edges with no overlap.

The surface areas are the same. How do the volumes compare? Write your hypothesis.

I think that when the surface areas are the same, the volume...



Place the tall cylinder in a large flat box. Fill the tall cylinder.



Place the wide cylinder around it.

Make a prediction about how full the wide cylinder will be if the tall cylinder is lifted out so that the fill material goes into the wide cylinder.

Record your prediction **before** doing the experiment.

I predict that...

Compare your hypothesis and prediction to what actually happened in the experiment.

What conclusions can you make about the volume of cylinders that have the same surface areas?

C.9.3: The Geometer's Sketchpad[®] 4 Dynamic Model

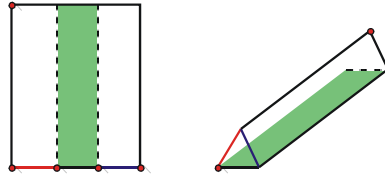
Paper Folding to Investigate Triangular Prisms

If two triangular prisms have the same surface area will they have the same volume?

The dynamic model below shows a piece of paper with two fold lines. If possible, the folded sections are joined so that the paper becomes an open-ended triangular prism.

Change the prism by changing the location of the folds.

Any red point in the dynamic model can be dragged.



Next Page: Measurements

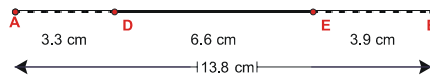
Paper Triangular Prisms

Fold Paper

Unfold Paper

Show Area of Triangle

Imagine you are looking at the edge of the paper.
The paper is folded at point D and point E.



Next Page: Volume

Paper Folding to Investigate Triangular Prisms

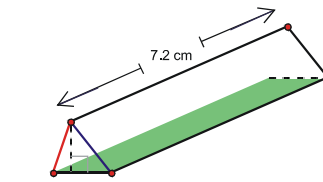
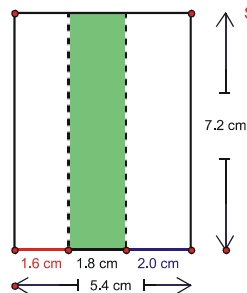
- 1) What paper size are you using?
- 2) Where do you want to place your folds?
- 3) If two triangular prisms have the same surface area do they have the same volume?

Hide Text

Volume of Prism = 9.8 cm^3

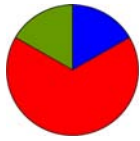
Surface Area of Prism (without triangles) = 38.6 cm^2

Surface Area of Prism (with triangles) = 41.3 cm^2



Area of Triangular Face = 1.4 cm^2

Return to page 1



Math Learning Goals

- Apply volume and area formulas to explore the relationship between rectangular prisms (Grade 7) and triangular prisms (Grade 8) that have the same volume.

Materials

- BLM C.10.1
- interlocking cubes
- calculators

Assessment Opportunities

Minds On... Whole Class → Discussion

Review the findings from Day 9 (BLM C.9.3). The dynamic model can be quickly used to investigate additional scenarios of paper-folding triangular prisms.

Whole Class → Anticipation Guide

Pose this question: If two rectangular prisms have the same volume, do they have the same surface area?

Students complete an Anticipation Guide with their answer.

Action! Pairs (same grade) → Investigation

Grade 7: Introduce the problem using BLM C.10.1. Students investigate:

- For prisms with the same volume, is the surface area also the same? (No)
- What shape of rectangular prism has the largest surface area for a given volume?

Grade 8:

- What are the dimensions of a cylinder with volume 3600 cm^3 that has the smallest surface area? (Hint: Try various sizes of r and calculate h . Collect data in an organized list.)

Curriculum Expectations/Demonstration/Checkbric: Observe students and assess their inquiry skills and learning skills.

Individual → Written Report

Students individually prepare a written report of their findings.

Grade 7: Students investigate the length of ribbon needed to tie around boxes with the same volume.

Grade 8: How are the height and radius related in a cylinder that has the smallest surface area for a fixed volume? (Answer: $h = 2r$)

Consolidate Debrief Whole Class → Student Presentations

Students present their findings. Students should understand that prisms with the same volumes do not have the same surface area.

Students apply the mathematics learned to answer this question:

Why would a Husky dog curl up in the winter to protect itself from the cold winds when it is sleeping outdoors?

(If the dog remains “long and skinny,” it has greater surface exposed to the cold. If it curls up, it has less surface area exposed to the cold, and thus will lose much less body heat. Although its volume stays the same, its surface area decreases as it becomes more “cube-ish” or spherical.)

Home Activity or Further Classroom Consolidation

Grades 7 and 8: Complete the assigned surface area and volume practice questions in preparation for a skills test.

Grade 8: Explain why a snake coils up in a tight loop when it hibernates. Justify your answer using the mathematics of cylinders.

Refer to *Think Literacy: Cross Curricular Approaches – Mathematics*. Anticipation Guide, p. 10.

Students in Grade 7 may benefit from having interlocking cubes to help them visualize the various shapes and sizes of boxes.



Solution

The more elongated the prism, the greater the surface area. The closer the prism becomes to being cube-shaped or spherical, the less surface area it has.

Concept Practice Extension

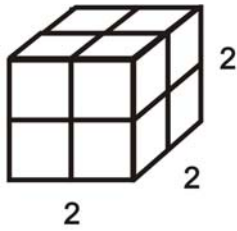
Provide students with appropriate practice questions.

C.10.1: Wrapping Gifts

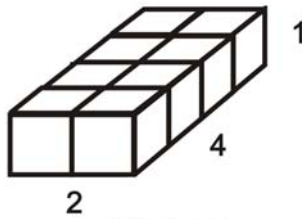
Grade 7

Three different rectangular prism-shaped boxes each have a volume of 8 cubic units, i.e., the prisms have a “fixed” volume.

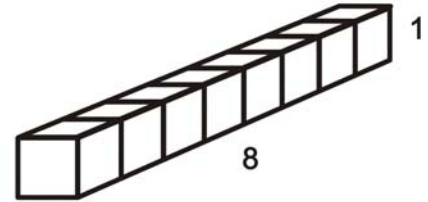
Does each box require the same amount of paper to wrap? Let’s investigate!



$2 \times 2 \times 2$



$2 \times 4 \times 1$



1

$1 \times 8 \times 1$

1. Verify that each rectangular prism illustrated above has a volume of 8 cubic units.
 - a) Draw the net for each rectangular prism box.
 - b) Determine the amount of paper required by calculating the surface area. (Ignore the overlapping pieces of paper you would need.)
 - c) Describe your findings.
2. List the dimensions of different rectangular prism boxes that can be designed to have a volume of 24 cubic units?
 - a) Draw several of the boxes, labelling the dimensions.
 - b) How much paper is required to wrap each box?
 - c) Describe your findings.
3. Investigate wrapping rectangular prism boxes with a volume of 36 cubic units. Determine the dimensions of the rectangular prism with the greatest surface area.
4. Write a report of your findings. Include the following information, justifying your statements.
 - Describe how surface area and volume are related, when the volume remains the same.
 - What shape of rectangular prism box uses the most paper?
 - What shape of rectangular prism box uses the least paper?



Math Learning Goals

- Assess students’ knowledge and understanding of volume of prisms with polygon bases.

Materials

- BLM C.11.1, C.11.2

Assessment Opportunities

Minds On... Whole Class → Review Discussion

Lead a discussion on the decomposition of complex solids. Make geosolids available as a visualization aid. Discuss the difference between metric measures of area, volume, and capacity.

Read the instructions on BLM C.11.1 (Grade 7) or BLM C.11.2 (Grade 8). Display some “fancy” glass bottles or magazine photos of perfume bottles to illustrate the unusual shapes found in perfume bottles.

Encourage students to make only one rectangular prism; using different polygon bases for their two other designs.

Give students an opportunity to clarify instructions. Ensure that they have a clear understanding of what the question is asking.

Action! Individual → Assessment

Students complete the task individually.

Curriculum Expectations/Observation/Anecdotal Notes: Circulate and help student as needed. Note strengths, area of improvement and next steps to give oral feedback.

Curriculum Expectations/Demonstration/Rubric: Collect student work and score using a rubric.



Based on the results of the assessment, it may be necessary to provide additional practice and feedback for some students.

Consolidate Debrief Whole Class → Reflection

Students share their methods and results orally.

Home Activity or Further Classroom Consolidation

Access Prior Knowledge

In your journal, describe careers where it will be important to package products safely and economically. Why will a knowledge of surface area and volume relationships be important in these careers?

