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Leading  
MATH  
Success

TIPS4RM

**TIPS4RM** Targeted Implementation  
and Planning Supports for  
Revised Mathematics

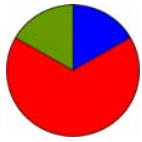
# Mathematical Processes

## Sample Adjusted Lessons

Grade 7

 Ontario

<b>Mathematical Process</b>	<b>Grade 7 TIPS4RM Lesson</b>
<b>Reasoning and Proving</b>	Unit 10 Day 10
<b>Reflecting</b>	Unit 6 Day 1
<b>Selecting Tools and Computational Strategies</b>	Unit 2 Day 11
<b>Connecting</b>	Unit 7 Day 11
<b>Representing</b>	Unit 2 Day 1



**Math Learning Goals**

- Investigate the relationship between surface area and volume of rectangular prisms.

**Materials**

- BLM 10.10.1
- interlocking cubes

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Use GSP<sup>®</sup>4 file Paper Folding To Investigate Triangular Prisms to check student responses and investigate additional scenarios (Day 9 Home Activity).

[PaperPrism.gsp](#)

Provides a dynamic model of the paper folding activity.

**Action! Pairs → Investigation**

Pose the question:

If two rectangular prisms have the same volume, do they have the same surface area?

Students might benefit from having interlocking cubes to help them visualize the various shapes and sizes of boxes.

Students investigate, using BLM 10.10.1:

- For prisms with the same volume, is the surface area also the same? (*no*)
- What shape of rectangular prism has the largest surface area for a given volume?

**Individual → Written Report**

Students individually prepare a written report of their findings.

**Communicating/Presentation/Rating Scale:** Assess students' ability to communicate in writing and visually their understanding of surface area and volume as a result of their investigation.

**Solution**

The more elongated the prism, the greater the surface area. The closer the prism becomes to being cube-shaped or spherical, the less surface area it has.

**Consolidate Debrief Whole Class → Student Presentations**

Students present their findings and apply the mathematics learned in the investigation to answer this question:

Why would a Husky dog curl up in the winter to protect himself from the cold winds when he is sleeping outdoors? (If the dog remains “long and skinny” he has greater surface area exposed to the cold. If he curls up, he has less surface area exposed to the cold, and thus he would lose much less body heat. Although his volume stays the same, his surface area decreases as he becomes more “cube-ish,” or spherical.)

**Home Activity or Further Classroom Consolidation**

*Concept Practice* Complete the practice questions.

Provide students with appropriate practice questions.



**Mathematical Process Goals**

- Hypothesize whether rectangular prisms of fixed volume have surface areas that vary or not.
- Reason inductively to prove a hypothesis.

**Materials**

- BLM 10.10.1(A)
- interlocking cubes

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Use GSP<sup>®</sup>4 Paper Folding To Investigate Triangular Prisms to check student responses and investigate additional scenarios (Day 9 Home Activity).

**Mathematical Process Focus:**  
Reasoning and Proving

See TIPS4RM Mathematical Processes package pp. 3–4.

**Action! Pairs → Investigation**

Students make a prediction about the surface area of two rectangular prisms that have the same volume.

Students investigate, using BLM 10.10.1(A).

**Individual → Written Report**

Students individually prepare a logical and organized written report of their findings.

Possible guiding questions:

- How did you refine your prediction as evidence was gathered?
- What details are needed in your report so that your argument is convincing?

**Mathematical Process/Reasoning and Proving/Checklist:** Observe students as they communicate their solutions, noting the correct use of mathematical terminology as they share their reasoning.

**Consolidate Debrief Whole Class → Discussion**

Pose the following question:

- What similarities and differences did you notice about the surface area and the volume in the question of Part A and Part B. Is this always true?

**Home Activity or Further Classroom Consolidation**

Predict, verify, and conclude the shape of the rectangular prism that will have the least amount of surface area if the rectangular prism has a volume of:

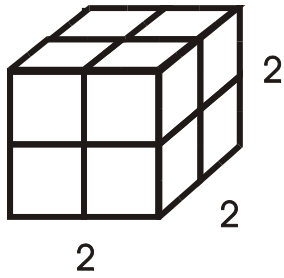
- 24 cubic units;
- 12 cubic units;
- 16 cubic units.

*Concept Practice*

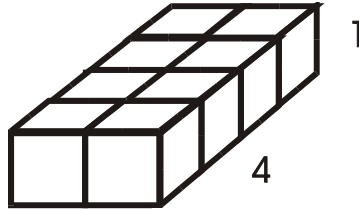
## 10.10.1(A): Wrapping Packages

### Part A:

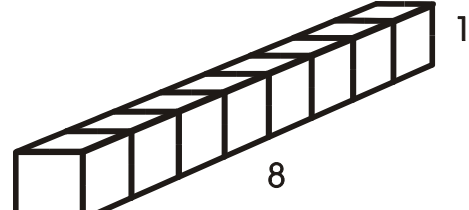
Three different rectangular prism-shaped boxes each have a volume of 8 cubic units.



$$2 \times 2 \times 2$$



$$2 \times 4 \times 1$$



$$1 \times 8 \times 1$$

1. **Make a prediction:**

Does each box require the same amount of paper to wrap?

2. Determine the amount of paper required for each by calculating the surface area. (Ignore the overlapping pieces of paper you would need.)

3. Describe your findings. Do they support your prediction?

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## 10.10.1(A): Wrapping Packages (continued)

### Part B:

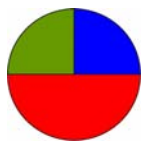
1. Construct three different rectangular prism-shaped boxes such that each has a volume of 27 cubic units. Sketch the boxes indicating the dimensions.

2. **Make a prediction:**

Does each box require the same amount of paper to wrap?

3. Determine the amount of paper required for each by calculating the surface area. (Ignore the overlapping pieces of paper you would need.)

4. Describe your findings. Do they support your prediction?

**Math Learning Goals**

- Construct acute, obtuse, right, and reflex angles.
- Estimate angle sizes and measure with a protractor.
- Bisect angles using a variety of methods, e.g., protractor, compass, paper folding, Mira.

**Materials**

- compasses
- protractors
- Miras
- BLM 6.1.1, 6.1.2, 6.1.3

**Assessment Opportunities****Minds On... Whole Class → Demonstration**

Develop four different ways to describe a straight angle using the headings: mathematical characteristics, everyday examples, diagram, and explanation. (See BLM 6.1.1 for sample responses.)

**Groups of 4 → Exploring Angles**

Post eight pieces of chart paper around the room. In groups of four, students focus on a specific angle, i.e., acute, right, obtuse, and reflex. Each angle is done twice. They define the angle and show examples, using available resources, books, Internet, etc.

Facilitate a class discussion using prompts such as:

- How did each group classify the angle? (by its degree range)
- Which angle(s) seems most common in the everyday world?
- Reflect on and explain why. (responses will vary)

**Action!****Groups of 4 → Practice**

Students complete Part A (BLM 6.1.2) and reflect after each measurement:

- Do we need to revise our estimates?
- Are our estimates within  $10^\circ$ ?

**Whole Class → Demonstration**

Demonstrate how to bisect using a Mira, a compass, paper folding, and a protractor and mark equal angles using proper notation. Students complete each bisection, marking equal angles on BLM 6.1.2, Part B.

**Individual → Reflection**

Students reflect, using guiding questions:

- What happened to the original angle? (bisected)
- What does *bisect* mean? (divides angle into two equal parts)
- How does this method compare to the others, i.e., compass, Mira, paper folding, and protractor? (responses will vary)

**Consolidate Debrief****Individual → Practice: Bisecting Angles**

Students complete BLM 6.1.3, Part C.

Ask:

- What do you notice about the two new angles created after bisecting the original angle? (They are equal.)
- What conclusions can you draw? (Bisecting an angle divides it into two new equal angles.)

**Curriculum Expectations/Observation/Mental Note:** Assess students' ability to bisect angles using at least two methods.

Demonstrate paper folding using a prepared angle on a piece of paper.

**Home Activity or Further Classroom Consolidation**

Using a protractor, a compass, and paper folding, complete the worksheet 6.1.3.

Alternatively, use the Frayer model (BLM 5.1.1).

It is important that all four angles are represented.

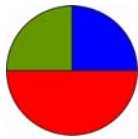
Word Wall

- bisect
- acute angle
- obtuse angle
- right angle
- reflex angle
- estimate

Lesson may vary depending on what protractors are available ( $360^\circ$  or  $180^\circ$ ).

Copy protractors on overhead acetates and cut up for Home Activity.

Practice



**Mathematical Process Goals**

- Use a graphic organizer to reflect on knowledge of various types of angles.
- Consider the reasonableness of answers.

**Materials**

- compasses
- protractors
- BLM 6.1.2(A)

**Assessment Opportunities**

**Minds On...**

**Whole Class → Demonstration**

Develop four different ways to describe a straight angle. (See TIPS4RM BLM 6.1.1 Teacher for sample responses.) An alternative everyday example is a road sign that points in two opposing directions with the distance indicated for the two places.

**Groups of 4 → Reflecting**

Assign each group a different angle type. Each angle is completed twice. Students reflect on prior knowledge of acute, right, obtuse, and reflex angles (See TIPS4RM BLM 6.1.1 for sample answers.)

**Whole Class → Discussion**

Facilitate a class discussion using prompts such as:

- Which angle(s) seems most common in our everyday world?
- Why do you think that angle occurs most frequently?
- What angles are we most familiar with? Recall anchor angles of 90°, 45°, and 180°.

**Action!**

**Individually → Estimating**

Each student estimates the angles on BLM 6.1.2(A) Part A.

Students reflect on the choice of strategy used to determine an estimate for the angle measure. Students do not measure the angle until after the sharing.

**Groups of 3 → Sharing**

Students share their estimates and their strategies for obtaining estimates with other group members.

They reflect on the input from group members to make any adjustments to their estimates.

**Individually → Measuring**

Using a protractor, students measure the angles on BLM 6.1.2(A) Part A.

They reflect on the strategy they used and the reasonableness of their estimates using BLM 6.1.2(A) Part B. Was your strategy effective? Why or why not?

**Mathematical Process/Communicating/Checklist:** Observe students as they communicate their reflections, noting the correct use of mathematical terminology.

**Mathematical Process Focus:**  
Reflecting

See TIPS4RM, Mathematical Processes package, Reflecting, p. 5

Circulate among groups asking guiding questions:

After sharing within the group, students consider feedback and rewrite estimates.

Review the use of a protractor, particularly for reflex angles, as needed.

**Consolidate Debrief**

**Whole Class → Discussion**

Pose the following questions:

- What is the relationship between the reflex angle and the acute angle in the diagrams?
- How does knowing this help you estimate a reflex angle?

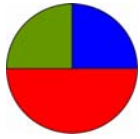
**Home Activity or Further Classroom Consolidation**

Identify two non-right angles in your environment and sketch a diagram of each, using a straight edge. Identify the type of angle, estimate, and determine the angle measure, using a protractor.

*Application Practice*







**Mathematical Process Goals**

- Reflect on strategies for bisecting angles.

**Materials**

- compasses
- protractors
- Miras
- BLM 6.1.3(A)

**Assessment Opportunities**



**Minds On...**

**Small Group → Sharing**

Students present their two angles from previous day’s home activity. Students describe where the angle exists in their environment and what strategy they used to estimate the angle measure.

**Mathematical Process Focus:**  
Reflecting

See TIPS4RM, Mathematical Processes package, Reflecting, p. 5



**Action!**

**Whole Class → Demonstration**

Demonstrate how to bisect an angle using a Mira, a compass, paper folding, and a protractor. Mark equal angles using proper notation.

**Individual → Bisecting Practice**

Students use each of the strategies to bisect angles on BLM 6.1.3(A). Students write down their reflections on page 2 of the task.



**Consolidate Debrief**

**Whole Class → Reflection**

Pose the following questions orally:

- What do you notice about the two new angles created after bisecting the original angle? (*They are equal.*)
- What are the advantages and disadvantages of the different methods of bisecting an angle?
- Consider, if there are any situations where one method would work better than others.

**Home Activity or Further Classroom Consolidation**

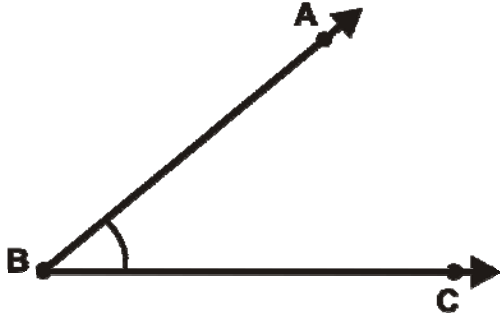
Using a protractor, a compass, and paper folding, complete page 3 of worksheet 6.1.3A.

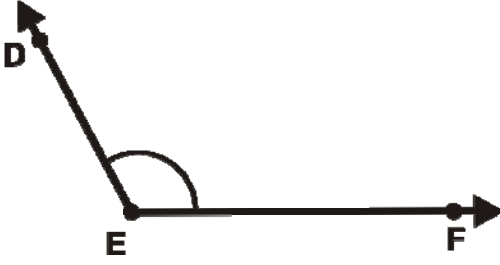
If there are Miras that students can take home, include this strategy in the **Home Activity**.

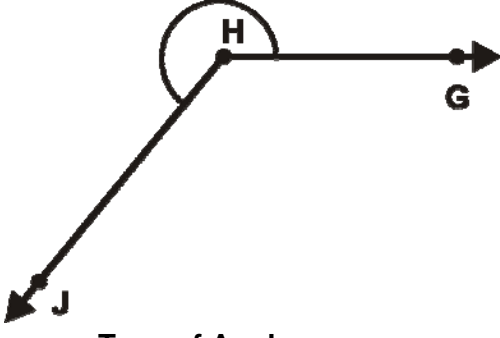
*Practice*

## BLM 6.1.2(A): Estimating and Measuring Angles

### Part A

 <p>Type of Angle: _____</p> <p>Estimate: <math>\angle ABC</math> _____<math>^\circ</math></p>	<p>Strategy</p> <p>Actual: <math>\angle ABC =</math> _____<math>^\circ</math></p>
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 <p>Type of Angle: _____</p> <p>Estimate: <math>\angle DEF</math> _____<math>^\circ</math></p>	<p>Strategy</p> <p>Actual: <math>\angle DEF =</math> _____<math>^\circ</math></p>
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 <p>Type of Angle: _____</p> <p>Estimate: <math>\angle GHJ</math> _____<math>^\circ</math></p>	<p>Strategy</p> <p>Actual: <math>\angle GHJ =</math> _____<math>^\circ</math></p>
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## BLM 6.1.2(A): Estimating and Measuring Angles

### Part B

#### Reflection

Compare your estimate with the actual measure for each of the angles.

1. For which angle was your estimate closest to the actual measure?

What strategy did you use to arrive at your estimate?

2. For which angle was your estimate furthest from the actual measure?

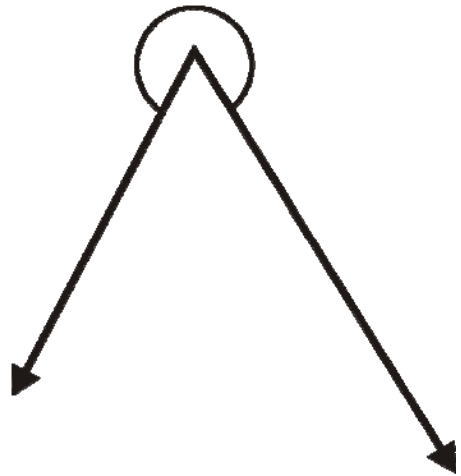
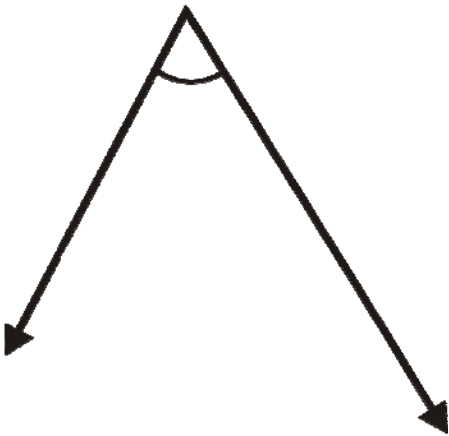
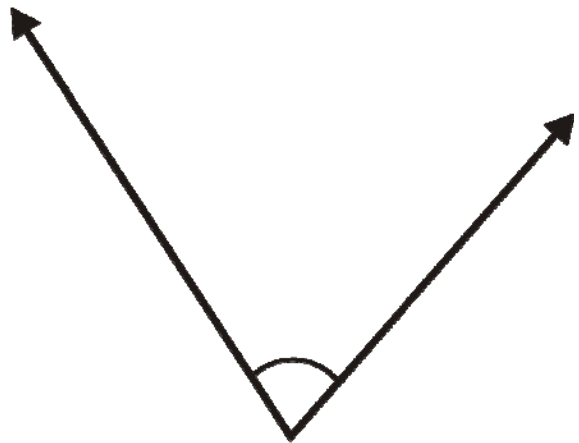
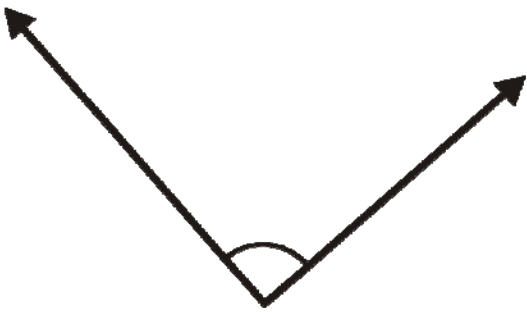
What strategy did you use?

What other strategy could you have used to make your estimate closer to the actual?

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## BLM 6.1.3(A): Bisecting Angles

1. Bisect each angle using a different method (Mira, protractor, folding paper, compass).



2. Compare the methods you used for bisecting angles. Which do you prefer? Explain why.



**Math Learning Goals**

- Consolidate integer addition with integer tiles.
- Add integers using number lines.
- Compare the two methods for addition of integers.

**Materials**

- BLM 2.11.1
- sets of integer tiles
- large cards with numbers -4 to 4

**Assessment Opportunities**

**Minds On...**

**Whole Class → Problem Solving**

Pose the problem: If a spider climbs 3 metres up a water spout during the day, then slides back down 2 metres every night, how many days does it take to reach the top of a 10-metre spout?

Discuss multiple ways to model and solve this problem. Using the integer addition sentence  $(+10) + (+20)$ , prompt students to ask a question related to everyday life whose answer could be determined by this addition sentence, e.g., if the spider climbed 10 metres up the water spout today, and 20 m tomorrow, how high will the spider be?

**Pairs → Connecting**

Write five symbolic representations of addition sentences on the board. In pairs, students write corresponding questions.

**Action!**

**Whole Class → Modelling/Discussion**

Nine volunteers line up, evenly spaced, facing the class to form a human number line. The 5<sup>th</sup> (middle) person represents 0. Students display numbers corresponding to their position. (-4 through 4)

Another student stands in front of the person represents 0 and then walks three places in the positive direction to stand in front of the person at +3.

Ask: What integer can represent the move so far? (+3) Record the response. This student walks one more place in the positive direction. Ask: What integer can represent this second move? (+1) Record this beside the previous answer. Demonstrate that the “trip” so far can be represented by the addition sentence  $(+3) + (+1)$ , whose answer can be determined by looking at the volunteer’s current location. (+4)

Use a similar procedure for demonstrating addition of two negative integers, then a positive and a negative integer.

Connect the use of a number line to show integer addition to the questions on BLM 2.11.1 – always start at 0, use red arrows pointing to the right for positive integers, and blue arrows pointing to the left for negative integers.

**Individual → Problem Solving**

Students complete BLM 2.11.1, representing the addition questions with blue and/or red arrows, and determining answers.

**Consolidate Debrief**

**Small Groups → Discussion**

Students compare each of their answers against those of other group members and share their strategy for addition. Discuss as a class. Compare and connect to the strategies students developed on Day 6.

**Curriculum Expectations/Self-Assessment/Checklist:** Students reflect on their competency with addition of positive and negative integers, using a number line.

**Home Activity or Further Classroom Consolidation**

Explain to another person the similarities and contrasts between using number lines vs. integer tiles to perform integer addition. Record thoughts in your math journal, along with your personal preference.

*Concept Practice Reflection Problem Solving*

Answer:  
It takes eight days for the spider to reach the top.

A diagram, number line, integer tiles, integer addition, and graphs are useful.

Technology alternative for any part of lesson:  
[Integer.gsp](#)

Add further visual cues, such as having the +3 person and the +1 person hold their hands up.  
OR  
model the trip with a visual drawing on the board.

Addition on the number line: start at 0, show first arrow, second arrow begins where first one ends, resulting destination is the sum.

See *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle, p. 425, for more information on the coloured arrow techniques.

Students should use the word *sum* as the result of addition.



**Mathematical Process Goals**

- Use a variety of tools to consolidate integer addition.
- Select and use a tool to solve a problem involving the addition of integers
- Share computational strategies to add integers.

**Materials**

- BLM 2.11.1(A), 2.11.2(A)

**Assessment Opportunities**

**Minds On... Whole Class → Demonstration**

Pose the problem:

In the morning the outdoor thermometer read  $-5^{\circ}\text{C}$ . It was stated on the radio that the temperature would rise  $15^{\circ}\text{C}$  by noon. What will the temperature be at noon? If the temperature then drops  $8^{\circ}\text{C}$  by midnight. What will the temperature be at midnight?

Demonstrate how to use the vertical number line to model the above problem.

**Small Groups → Modelling**

Students create four new models to represent the problem with tools other than the vertical number line.

Select four students (each of whom used a different tool) to share their models with the class.

**Action! Pairs → Problem Solving**

Pairs of students work on a different number or word problem requiring the addition of integers (BLM 2.11.1A)). Pairs solve the problem using at least two different tools, and write their solutions (BLM 2.11.2(A)).

**Group → Sharing**

Two or three pairs of students who have chosen different tools to solve their problem join together. Each pair presents their problem to the other pair(s), and communicates their solution, including reasons for selecting the particular tools they used to model and solve the problem.

**Mathematical Process/Communicating/Checklist:** Observe students as they communicate their solutions, noting the correct use of mathematical terminology.

**Consolidate Debrief Whole Class → Presentation**

Each of the four groups shares one of the pair’s questions and solutions with the class and includes the rationale for its choice of tools.

Highlight from the presentations the variety of tools and computational strategies used by various groups to solve their problem.

**Curriculum Expectations/Presentation/Anecdotal Note:** Note students’ understanding of representing the addition of integers using a variety of tools.

**Home Activity or Further Classroom Consolidation**

Create three number problems involving the addition of integers. At least one question must have a negative answer. Solve one of your problems using two different tools. Explain why you chose those tools to solve the problem, using worksheet 2.11.2(A).

**Mathematical Process Focus:**

Selecting Tools and Computational Strategies

See TIPS4RM Mathematical Processes package pp. 6–7.

Reinforce the zero principle, as appropriate.

Provide a supply of the following tools from which to select:

- vertical and horizontal number lines
- linking cubes
- bingo dabbers
- colour tiles
- integer tiles
- deck of cards
- play money
- geoboards

Key questions:

- How did the learning tool you chose contribute to your understanding and solving the problem?
- What other tools did you consider using? Why?

Provide students with a copy of BLM 2.11.2(A).

*Concept Practice  
Reflection  
Problem Solving*

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## BLM 2.11.1(A): Integer Addition Problems

1.  $(-3) + (-4)$
2. If the temperature is  $5^{\circ}\text{C}$  at six o'clock in the evening but drops  $6\text{C}^{\circ}$  during the night, what is the temperature at six o'clock in the morning?
3.  $-7 + (+2)$
4. If you came to school with a toonie in your pocket and donated it to the Environment Club at lunch, how much money did you go home with?
5.  $-5 + (+4) + (-3)$
6. On a partly cloudy day, the temperature started out at  $1^{\circ}\text{C}$ , it dropped  $2\text{C}^{\circ}$  with the cloud cover, rose  $3\text{C}^{\circ}$  when the sun came out later in the morning, then dropped  $4\text{C}^{\circ}$  when the clouds came back and finally rose  $5\text{C}^{\circ}$  in the afternoon when the sun shone again. What was the temperature in the afternoon?
7. If a spider climbs 3 metres up a water spout during the day, then slides back down 2 metres every night, how many days does it take to reach the top of a 10-metre spout?

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## BLM 2.11.2(A): Integer Addition Problems: Record Sheet

Solve your problem using at least two different tools and/or computational strategies. Indicate the tool/strategy you selected.

Show or describe your solution in the space provided, explaining how you used the tool/strategy.

<b>State Problem:</b>	
<b>Tool/Strategy Used is:</b> _____ <b>Solution to the problem is:</b>	<b>Tool/Strategy Used is:</b> _____ <b>Solution to the problem is:</b>





**Math Learning Goals**

- Explore the relationship between fractions and decimals.

**Materials**

- BLM 7.11.1, 7.11.2

**Assessment Opportunities**

**Minds On...**

**Whole Class → Review and Introduce New Problem**

Ask students to think of any two fractions that are “really close.” Record a few of their suggestions on the board.

Challenge them to choose one pair of fractions from the board and to find two numbers that are between the two listed. Ask what types of numbers they might use to solve this problem. Identify that they could use fractions or decimals.

**Action!**

**Pairs → Problem Solving**

Students find two numbers between one pair of fractions listed on the board. Pairs develop their own strategies and methods independently, share their solutions to the problem, and their reasoning in finding the two numbers. If they use decimals, they should make the connection to fractions.

**Communicating/Observation/Anecdotal Note:** Assess students’ ability to communicate their thinking using correct mathematical terminology.

Two methods to change a fraction to a decimal on a calculator are:

- divide the numerator by the denominator
- enter the fraction using the fraction key ( $a \frac{b}{c}$ ), press ENTER, then press the fraction key again

The definition of *multiple* may need to be reviewed with students.

**Consolidate Debrief**

**Whole Class → Sharing**

Some discussion around the connection between fractions and decimals and how to use a calculator to convert fractions to decimals would be useful. Include number systems, common relationships that students are familiar with, and applications/appropriateness of each in daily contexts.

**Pairs → Practice**

Reinforce understanding of the fraction-decimal relationship (BLM 7.11.1).

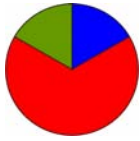
**Home Activity or Further Classroom Consolidation**

Create three determine-the-decimal questions. Each one should have either two or three clues and all the clues should be needed to determine the decimal.

Complete the practice questions.

Provide students with appropriate practice questions for exploring the relationship of fractions to decimals.

*Concept Practice  
Exploration  
Reflection*



**Mathematical Process Goals**

- Make connections between new and prior knowledge of fractions and decimals.
- Make connections between conversions of fractions and decimals, and solve problems in context.
- Communicate understanding of connections between fractions and decimals, and their application within a life context.

**Materials**

- NHL statistics: 2005–2006 Regular Season (Goalie – Summary – [www.nhl.com/nhlstats/stats](http://www.nhl.com/nhlstats/stats))
- cue cards
- exit card

**Assessment Opportunities**

**Minds On... Groups of 4 → Placemat**

Present the challenge:  
How do we decide who is the Top NHL Goaltender?  
Using NHL statistics provided students choose the top goalie based on specific data from the summary table. They explain the reasoning behind their choice and the strategy they used. Students record work in their area of the placemat. Groups decide from the four cases presented, which data and strategy should be used to determine the top NHL goaltender. They record their response in the centre of the placemat. Groups present arguments to the class.  
Guide groups to explore the concept of using a fraction and/or decimal to determine who the top NHL goaltender is, by comparing sets of data, e.g. W/GP, Sv/SA, etc. [Guide students to choose part to whole relationships and not part-to-part ratios.]

**Action! Pairs → Problem Solving**

Give pairs a set of two cue cards, each with a fraction representing the “save results”  $\left(\frac{Sv}{SA}\right)^*$  of an NHL goaltender. Each fraction has a different denominator as each goaltender will have a different number of total shots against (SA) for the season.

Students create two sets of goaltender data (Sv and SA). Goaltender A and Goaltender B “save results” must rank in between the original two goaltenders (from cue cards). Students present strategies for finding Goaltender A and Goaltender B “save results” on chart paper.

\*Sv = Total Number of Saves; SA = Total Shots Against

**Consolidate Debrief Whole Class → Sharing**

Pairs present their strategies to the class.

**Pairs → Practice**

Reinforce understanding of the fraction-decimal relationship (TIPS4RM BLM 7.11.1).

**Individual → Exit Card: Decimals and Fractions**

- How is a decimal like a fraction?
- How are they different?
- How did thinking about conversions between fractions and decimals within this context help you to understand?

**Mathematical Process/Connecting/Checklist:** Observe how the students communicate their understanding of how the concepts are connected.

**Home Activity or Further Classroom Consolidation**

- In your journal:
- Describe the discoveries you made today about the connections between fractions and decimals.
  - Where could conversions between fractions and decimals be used in your daily life? (e.g., calculating the discount of a sale item)

**Mathematical Process Focus:** Connecting

See TIPS4RM Mathematical Processes package p. 8

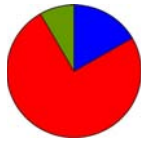
Making Connections to the Data Management strand: Encourage students to make inferences and convincing arguments that are based on the analysis of charts and tables to determine which goaltender is “top” in the NHL.

Allow students to explore their own procedures and algorithms, monitoring these for correctness.

Possible guiding questions:

- What other math have you studied that has the same principle or procedure?
- How do these different representations connect to one another?

*Application  
Concept Practice  
Differentiated  
Reflection*



**Math Learning Goals**

- Represent linear growing patterns.
- Make predictions about growing patterns.
- Explore multiple representations.

**Materials**

- toothpicks
- BLM 2.1.1

**Assessment Opportunities**

**Minds On ...**

**Whole Class → Discussion**

Students contribute to a class concept map about patterning. Based on their experiences with patterning, they may identify types of patterns, materials for patterns, sample numerical or geometrical patterns, or applications of patterns.

Discuss why the ability to identify and discuss patterns is important. Ask a student to present a pattern on the board and another student to present a different type of pattern. Other students add the next term to each pattern and explain their thinking. Include different types of patterns, e.g., number, geometric, colour.

**Curriculum Expectations/Oral Questioning/Mental Note:** Assess students' understanding of patterns, and their confidence in using them to help plan further instruction.

Distinguish between a growing or diminishing pattern and a constant design.

People use patterns to investigate and represent complex relationships existing in many areas, including nature and science.

**Action!**

**Pairs → Exploration**

On an overhead, create the first two terms of the toothpick pattern presented on BLM 2.1.1. Ask a student to create the third term.

In pairs, students continue the pattern with their toothpicks, and complete BLM 2.1.1.

Encourage students to look at different ways to build the 5<sup>th</sup> term, the 25<sup>th</sup> term, the  $n^{\text{th}}$  term, etc. There is no right way to formulate the construction of a term.

Students discuss solutions with their partners. Stress that each partner may have a different entry in the Understanding column but should have the same value in the Number of Toothpicks column (BLM 2.1.1).

See LMS Library, My Professional Practice, Multiple Representations – Pattern Building.

**Note:** The “ $n^{\text{th}}$  term” might be new to students.

**Consolidate Debrief**

**Whole Class → Discussion**

Students share their approaches.

Discuss different entries in the Understanding column, highlighting the validity of all representations. Students should represent their patterns using words and numbers, but may not be using variables at this point.

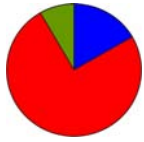
It is important that students understand the limitation of recursive representations, e.g., add three to the last term. Students should move to more sophisticated patterning, e.g., predicting the number of toothpicks required by consideration of the term number.

**Home Activity or Further Classroom Consolidation**

Design another toothpick pattern, building and recording the first three terms. Explain your pattern.

Consider how many toothpicks would be required to build the 100<sup>th</sup> term in the pattern.

*Concept Practice*

**Mathematical Process Goals**

- Represent linear growing patterns using concrete materials, pictures, words, tables of values, algebraic expressions, and graphs.

**Materials**

- toothpicks
- BLM 2.1.1(A)

**Assessment Opportunities****Minds On... Whole Class → Discussion**

Develop a class concept map about patterning, using SMART Ideas™ software.

**Mathematical Process Focus:**  
Representing

See TIPS4RM  
Mathematical Processes package  
p. 9

**Action!****Whole Class → Guided Representation**

On an overhead, create the first two terms of each of the toothpick patterns presented on BLM 2.1.1(A). Students create the next three terms for each.

Support students with filling in question c.

Possible guiding questions:

- How does each representation help you to see the pattern?
- What is a different way to represent this pattern without using toothpicks?
- How could we represent this pattern with the Term Number using a variable such as 'x' or 'n'? (only for linear patterns)
- Which representation would you use to determine the 10<sup>th</sup> term? 100<sup>th</sup> term?

Facilitate a graphical representation of the pattern, plotting 'Term Number' on the horizontal axis and the 'Number of Toothpicks' on the vertical axis. Do the first two together, then have students complete the rest individually.

**Mathematical Process/Representing/Checklist:** Observe students as they complete the chart noting comfort level using different representations.

**Consolidate Debrief Whole Class → Discussion**

Students share their representations with the class.

Pose the following to the students:

- Looking at various representations, describe what you know about the pattern.
- Using various representations, how can you determine if the pattern is linear?
- How would the representations be different if the pattern is non-linear?

**Home Activity or Further Classroom Consolidation**

Design a toothpick pattern that is linear. Build the first three terms and explain your pattern. Show at least three representations.

Design a toothpick pattern that is non-linear, build the first three or four terms.

Explain your pattern and show at least three representations.

*Concept Practice*

## 2.1.1(A): Toothpick Patterns

For each of the following patterns:

- Build the terms shown using toothpicks.
- Build the next three terms in the pattern.



Term 1



Term 2



Term 3



Term 1



Term 2



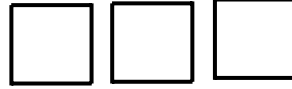
Term 3



Term 1



Term 2



Term 3



Term 1



Term 2



Term 3



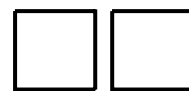
Term 4



Term 1



Term 2



Term 3



Term 4

