Leading MATH Success

TIPS4RM

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Mathematical Processes

Sample Adjusted Lessons

Grade 10 Applied

Ontario

Mathematical Process	Grade 10 Applied TIPS4RM Lesson
Reasoning and Proving	Unit 4 Day 3
Reflecting	Unit 2 Day 4
Selecting Tools and Computational Strategies	Unit 4 Day 8
Connecting	Unit 7 Day 2
Representing	Unit 7 Day 8

Unit 4: Day 3: Making a Difference

Grade 10 Applied

75 min

Math Learning Goals

- Determine that if the table of values yields a constant second difference the curve is parabolic and vice versa.
- Realize that there are other non-linear relationships that are not parabolic.
- Develop a word wall of new vocabulary related to the quadratic.

Materials

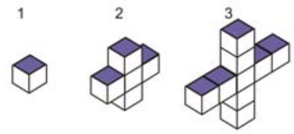
- connecting cubes
- graph paper

Assessment Opportunities

Minds On…

Whole Class → Investigation

Demonstrate a linear relationship, using the following construction.



Students individually complete a table of values with the headings Model Number, Number of Cubes, and First Difference and create a graph of the information. They connect the first difference with the slope of the line and reflect on the value of difference to determine if the relationship is linear or non-linear, and provide reasons for their conclusions.

Action!

Pairs → Summarizing

Students refer to data from any two experiments completed on Day 1. Engage student thinking by providing prompts: Prove that the data is non-linear.
Further prompting: We know all the graphs are parabolas; we call these quadratic relations.
Ask:

- How could we know that they are parabolas just from the table of values?
- What pattern do you see in the work you've done on the tables?

Students complete the statement: The graph of a table of values will be a parabola if....

Before continuing, students should have the understanding that the first differences are not the same and therefore the data is non-linear.

Curriculum Expectation/Demonstration/Checklist: Assess students' understanding that a constant second difference in a table of values determines that the relation is quadratic.

Students can confirm their predictions by selecting other sets of data from the experiments.

Groups of 4 → Investigation

Students build cubes of sides 1, 2, and 3, and record on a table of values the side length and volume. They calculate the volume for side lengths 4, 5, and 6, and put the data on a graph.

Ask:

- Is this linear or non-linear?
- Is the curve a parabola?
- How can you verify this?

Consolidate Debrief

Whole Class → Discussion

Discuss and summarize facts:

- The table of values associated with parabolas always has a common second difference.
- There are other curves that are not parabolas. These curves do not have common second differences.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete one task about parabolas:

- Determine if the situations are linear, quadratic, or neither, and provide reasons for your answers.

OR

- Place the parabolic picture that you brought to class on a grid and determine some of its points. Verify that it is or is not a parabola.

Provide a variety of situations and/or data that are linear, quadratic, or neither.

75 min

Mathematical Process Goals
- Use models and logic to infer and conclude.
- Present arguments in a logical and organized manner.
- Look for and use counter-examples.

Materials
- connecting cubes
- graph paper
- BLM 4.3.1(A)

Assessment Opportunities

Minds On…

Whole Class → Investigation
After students have completed their table of values ask them to predict what the graph will look like and explain their reasoning. Verify hypotheses by graphing. Coach students on the characteristics of an acceptable proof and model a complete logical argument.

Mathematical Process Focus: Reasoning and Proving

See TIPS4RM Mathematical Processes package pp. 3–4.

Action!

Pairs → Summarizing
Discuss why, if a relation is quadratic, it is not enough to say that the second differences are equal. Discuss the implication of equal second differences of zero.

Groups of 4 → Investigation
Students build cubes of sides 1, 2, and 3, and record in a table of values the side length and volume. They calculate the volume for side lengths 4, 5, and 6, and put the data on a graph.

Ask:
- Is this linear or non-linear?
- Is the curve a parabola?
- How can you verify this?

Provide coaching, as necessary, to get appropriate justification with each student response to the listed questions.

Possible guiding question:
- Why does checking first and second differences work?

Mathematical Process/Reasoning and Proving/Checklist: Assess students on how well they communicate their reasoning.

Consolidate Debrief

Whole Class → Discussion
Discuss and summarize facts:
- The table of values associated with parabolas always has a common non-zero second difference.
- There are other curves that are not parabolas. These curves do not have common non-zero second differences.

Discuss the validity of these two statements:
- Every quadratic relation is non-linear.
- Every non-linear relation is quadratic.

Provide an example of when the second statement is false.

Use student input to model an organized, logical written argument. Discuss the idea that an efficient way to prove a statement false is to find a counter-example.

Concept Practice

Home Activity or Further Classroom Consolidation
Determine if the situations are linear, quadratic, or neither, and provide reasons for your answers (worksheet 4.3.1(A)).

OR

Place the parabolic picture that you brought to class on a grid and determine some of its points. Verify that it is or is not a parabola.

Remind students that arguments need to be thorough, organized, and logical.

BLM 4.3.1(A): Let's Debate!

Job A: A starting salary of $40 000 with annual raises of $3 000.

Job B: A starting salary of $30 000 with a $500 raise after 1 year, an additional $1 000 after 2 years, then an extra $1 500 after 3 years, and so on.

Job C: A starting salary of $35 000 with a 2% increase each year.

1. Construct tables of values of the salaries for each job, showing the annual salary for each of the next six years. Extend the tables to show first and second differences.

2. Which of the jobs has a salary that grows linearly? Justify your choice.

3. Which of the jobs has a salary that grows quadratically? Justify your choice.

4. Which of the jobs has a salary whose growth is neither linear nor quadratic? Justify your choice.

5. Explain why you think it would be useful to know what type of a relation each is.

75 min

Math Learning Goals

- Determine the measures of the sides and angles of right-angled triangles using the primary trigonometric ratios and the Pythagorean relationship.

Materials

- BLM 2.4.1
- cardboard signs for sine, cosine, tangent, and Pythagorean relationship

Assessment Opportunities

Minds On…

Whole Class → Review

Discuss any issues regarding the research assignment.

Review the conventions for labelling triangles (opposite, adjacent, hypotenuse).

Review the ratios sine, cosine, and tangent, using the terms *opposite*, *adjacent*, and *hypotenuse*.

Pairs → Investigation

Draw a right-angled triangle on the board or overhead and provide the degrees of one of the acute angles and the length of one side.

Students investigate how they might use what they have learned previously to find one of the missing sides.

Circulate and ask leading questions, and listen to their dialogue to identify any misconceptions.

Ask: How did you know to use that particular ratio?

Pairs share their strategy for solving the problem with the rest of the class.

Provide further examples and demonstration, as required.

Curriculum Expectation/Oral Question/Anecdotal Note: Observe how students label the triangle and identify the ratio to determine the missing sides.

Word Wall
- ratio
- sine
- cosine
- tangent
- hypotenuse

Action!

Whole Class → Guided Instruction

Using questions 1–4, guide students to determine whether they would use sine, cosine, tangent ratios, or the Pythagorean theorem to solve for the unknown side or indicated angle (BLM 2.4.1). Start at the reference angle on the diagram and draw arrows to the two other pieces of information stated in the problem. One of the pieces will be unknown. Label the sides as opposite, adjacent, or hypotenuse and decide which is the appropriate ratio needed to solve the problem.

As students complete questions 1–4, summarize the correct solution(s). Students then complete questions 5–8 individually.

Note: Some of the questions can use more than one method.

Students could use a mnemonic device or make up a sentence to help them to remember the primary trigonometric ratios, e.g., SOHCAHTOA

Consolidate Debrief

Pairs → Share Solutions

Pairs share their solutions for questions 5–8; identify incongruent solutions; and make corrections, as required.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete the practice questions.

Provide students with appropriate practice questions.

75 min

Mathematical Process Goals

- Determine appropriate strategies to solve problems involving the primary trigonometric ratios and the Pythagorean relationship by reflecting on the relative given data.
- Assess the reasonableness of solutions.
- Verify solutions to problems, using different methods.

Materials

- BLM 2.4.1(A)
- cardboard signs for sine, cosine, tangent, and Pythagorean relationship

Assessment Opportunities

Minds On…

Whole Class → Review

Discuss any issues regarding the research assignment.

Review the conventions for labelling triangles (opposite, adjacent, hypotenuse).

Review the ratios sine, cosine, and tangent, using the terms *opposite*, *adjacent*, and *hypotenuse*.

Pairs → Investigation

Draw a right-angled triangle giving the degrees of one of the acute angles and the length of one side.

Students consider how they might use what they have learned previously to find a particular missing side.

Circulate and listen to their dialogue to identify any misconceptions. Ask:

- What measures are you given?
- For what measure are you solving?
- How did you know to use that particular ratio?

Pairs share their strategy for solving the problem with the rest of the class.

Mathematical Process/Communicating/Checklist: Observe students as they communicate their solutions, noting correct use of mathematical terminology.

Mathematical Process Focus: Reflecting

See TIPS4RM Mathematical Processes package p. 5

Action!

Whole Class → Guided Instruction

Draw a right-angled triangle with one of the acute angles and the length of one side different from **Minds On**. Model how to use the reflective process to consider and decide on appropriate strategies for finding all missing measures.

Coach students with prompts such as: In considering how to start, I note that… and this leads me to think that using… ratio would be appropriate. So now I would… I am going to check my answer by….

Show how the Pythagorean relationship and different trigonometric ratios can be used to verify or provide alternate strategies. When discussing reasonableness of answers, show that the longest side in a right-angled triangle is across from the right angle, the shortest side is across from the smallest angle, etc.

Using questions 1–4, guide students to determine whether they would use sine, cosine, or tangent ratios, or the Pythagorean theorem to solve for the unknown side or indicated angle (BLM 2.4.1(A)). Start at the reference angle and draw arrows to the two other pieces of information stated in the problem. One of the pieces will be unknown. Label the sides as opposite, adjacent, or hypotenuse. Decide which is the appropriate ratio needed to solve the problem.

As students complete questions 1–4, summarize the correct solution(s), stressing how they know which ratio to use and how they know their answer is reasonable. Students then complete questions 5–9 individually.

Possible guiding questions:

- How is this similar to a problem we have done before?
- What information are we given?
- What is another way we can solve this problem?
- How do we know our answer is reasonable?

Consolidate Debrief

Pairs → Share Solutions

Pairs share/compare their solutions for questions, identify non-matching solutions, and correct the solutions, as required.

Concept Practice

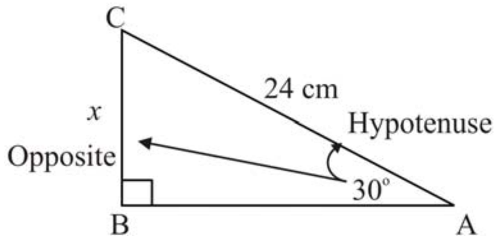
Home Activity or Further Classroom Consolidation

Complete practice questions.

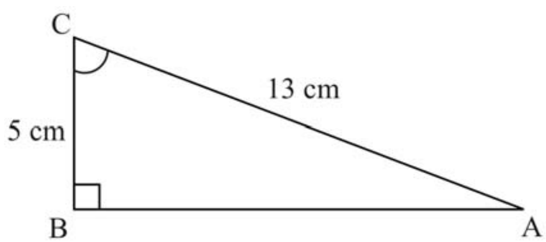
Assign BLM 2.4.1(A) question 9.

BLM 2.4.1(A): What's My Triangle?

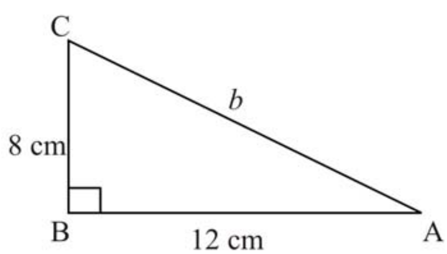
1. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find *x.* How did you know which one to use? Solve for *x*. How do you know your answer is reasonable?



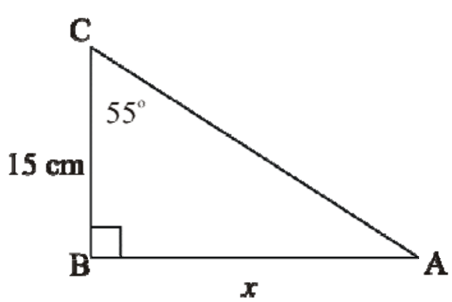
2. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find $\angle C$. How did you know which one to use? Solve for $\angle C$. How do you know your answer is reasonable?



3. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find *b*. How did you know which one to use? Solve for *b.* How do you know your answer is reasonable?

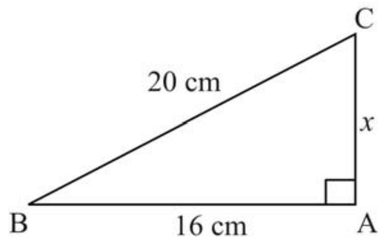


4. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find *x*. How did you know which one to use? Solve for *x.* How do you know your answer is reasonable?

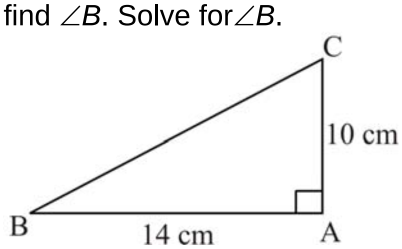


BLM 2.4.1(A): What's My Triangle? (continued)

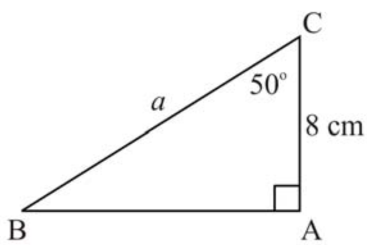
5. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find *x*. Solve for *x.*



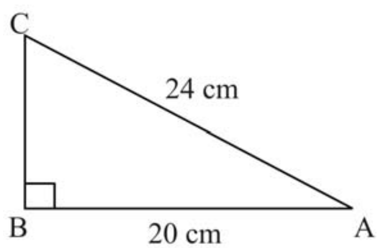
6. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find ∠*B*. Solve for∠*B*.



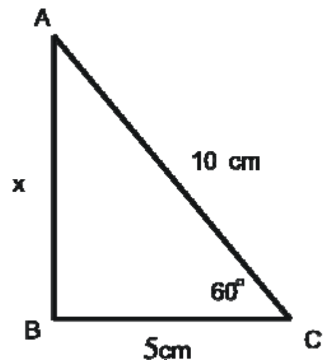
7. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find *a*. Solve for *a*.



8. Decide whether to use the sine, cosine, or tangent ratio, or the Pythagorean relationship to find ∠*C*. Solve for ∠*C*.



BLM 2.4.1(A): What's My Triangle? (continued)



9. a) Solve for *x*.

b) Is your answer reasonable? How do you know?

c) Verify your solution using a different technique.

d) List other techniques that could have been used to solve for *x*.

e) Which method do you think is the best? Why?

Math Learning Goals

- Solve problems that arise from realistic situations that can be modelled by two linear relations.
- Determine graphically the point of intersection of two linear equations.
- Interpret the story told by a graph, i.e., point of intersection, what occurs before and after the intersection point.

75 min

Materials
- BLM 4.8.1

Assessment Opportunities

Minds On…

Whole Class → Presentation

Two or three students share their graphs and paragraphs from the Day 7 Home Activity.

Action!

Whole Class → Demonstration

Discuss methods of graphing linear relations and merit of each.

Demonstrate graphing lines using slope and *y*-intercepts, using $y = \frac{2}{5}x + 1$ and using intercept/intercept using $3x - 4y - 12 = 0$.

Individual → Practice

Students practise graphing systems of linear equations, finding the point of intersection, and explaining the meaning of what they found (BLM 4.8.1).

Select appropriate linear system problems from the textbook.

Curriculum Expectation/Observation/Checklist: Observe students' accuracy in graphing straight lines and identifying the point of intersection.

Consolidate Debrief

Pairs → Discussion

Students compare their results with a partner (BLM 4.8.1).

As a class, discuss the methods of the graphing the meaning of the point of intersection (BLM 4.8.1). Note special cases 5 and 6. Summarize the possibilities for points of intersection of two straight lines (1 point, i.e., different lines $\nparallel$; no points, i.e., parallel lines $\parallel$; infinite points, i.e., same line).

Application
Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

Complete the practice questions.

Provide students with appropriate practice questions to prepare for unit assessment.

Unit 4: Day 8: Solve the Problem Using a Graph (A)

Grade 10 Applied

75 min

Mathematical Process Goals
- Select appropriate tools and computational strategies to graph a system of two linear equations.
- Determine the point of intersection graphically.

Materials
- BLM 4.8.1(A)
- graphing technology

Assessment Opportunities

Minds On…

Whole Class → Presentation
Two or three students share their graphs and paragraphs from the Home Activity.

Action!

Whole Class → Demonstration
Review methods of graphing linear relations and the merits of each. What if the system of equations is: $y = 20x + 150$ and $y = 35x + 105$? Demonstrate how to use graphing technology and the window and zoom features to identify the point of intersection (3, 210).

Ask: What if the system of equations is: $y = 2.1x + 0.45$ and $y = 3x – 1.35$? Demonstrate how to use graphing technology and the window and zoom features to identify the point of intersection (2, 4.65).

Clarify how to efficiently use different tools and computational strategies to find the point of intersection of two linear relations, and how to identify when one choice is a more efficient than another.

Individual → Practice
Students practise graphing systems of linear equations, finding the point of intersection, and explaining the meaning of what they found (BLM 4.8.1(A)).

Mathematical Process/Selecting Tools and Computational Strategies/ Checklist: Observe student effectiveness in selecting and using the tools and computational strategies.

Possible guiding questions:
- How did the strategy you chose contribute to your solving of the problem?
- What other method did you consider using? Explain why you chose not to use it.

Consolidate Debrief

Pairs → Discussion
Students compare their strategies and results for finding the point of intersection, with a partner (BLM 4.8.1(A)).

As a class, discuss the strategies used to graph the different systems in order to calculate the point of intersection and why one strategy may be more efficient than another (BLM 4.8.1(A)). Note special cases in question 5 (parallel lines) and question 6 (the same line), and use of a graphing calculator for 7 and 8.

Summarize the solution possibilities for points of intersection of two straight lines (1 point, i.e., different lines ╱; no points, i.e., parallel lines || ; infinite points, i.e., same line /).

Home Activity or Further Classroom Consolidation
Complete the practice questions.

Application
Concept Practice

Mathematical Process Focus: Selecting Tools and Computational Strategies

See TIPS4RM Mathematical Processes package pp. 6–7.

Have graphing technology available for student use.

Prompt students to change the window for question 8.

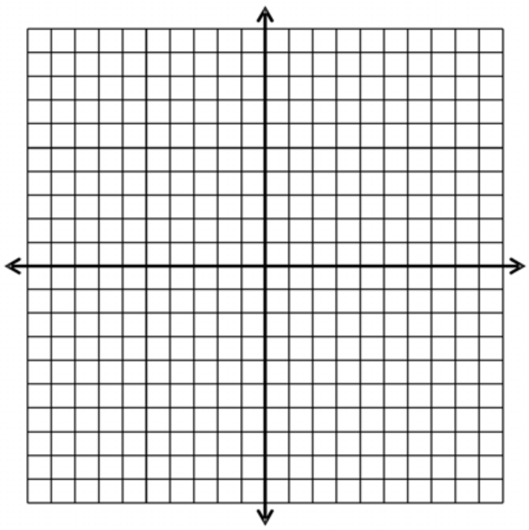
If questions provided require technology to graph efficiently, provide website information where students can access this technology. GSP®4 also has built in graphing technology.

4.8.1(A): Pairs of Equations

Consider each pair of equations and decide on the best method to graph them in order to calculate the point of intersection. Label your graphs. State what the point of intersection means for each question.

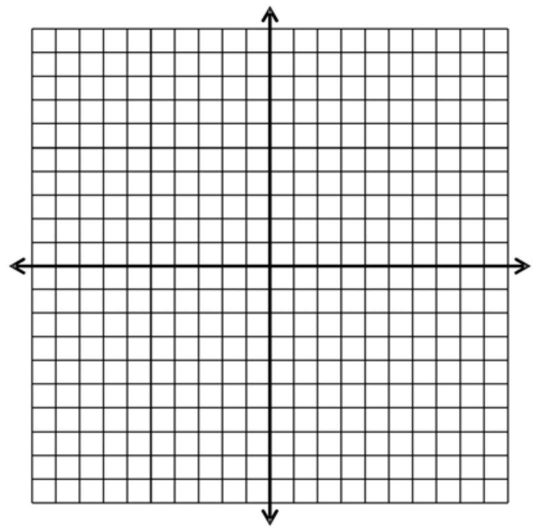
1. $y = -x - 7$

   $y = x - 3$



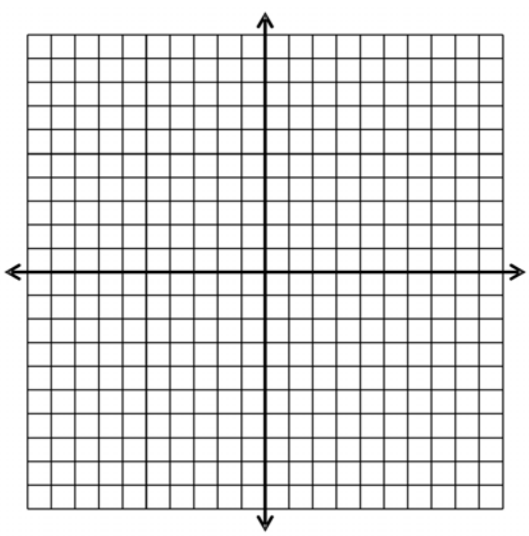
2. $y = 2x - 10$

   $y = -x - 4$



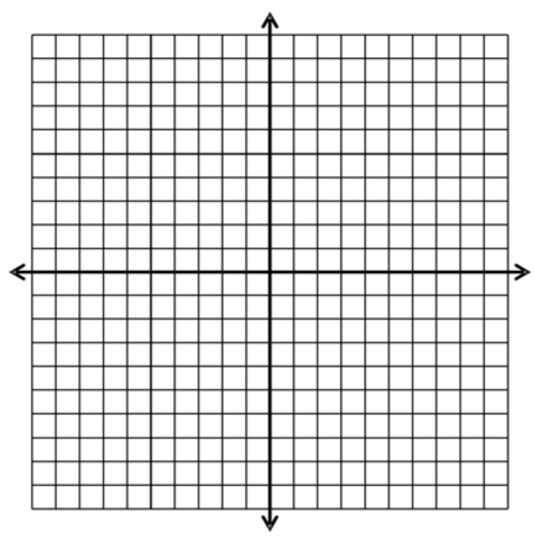
3. $2x - 3y - 6 = 0$

   $x + 3y - 3 = 0$



4. $y = \frac{1}{3}x - 5$

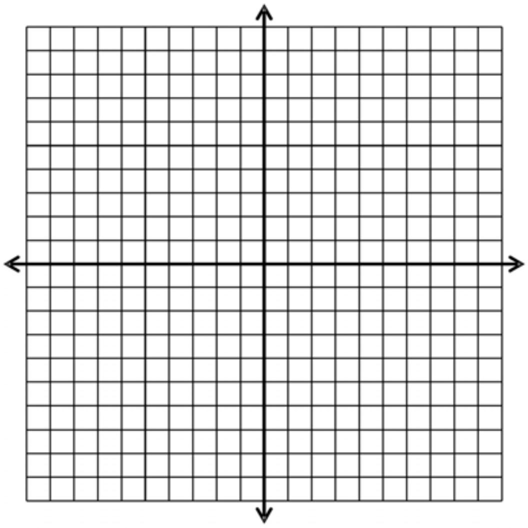
   $y = -\frac{2}{3}x - 2$



4.8.1(A): Pairs of Equations **(continued)**

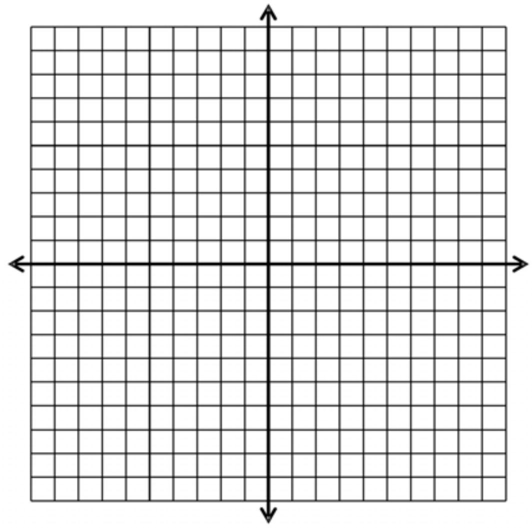
5. $y = 3x + 2$

$2y - 6x - 12 = 0$



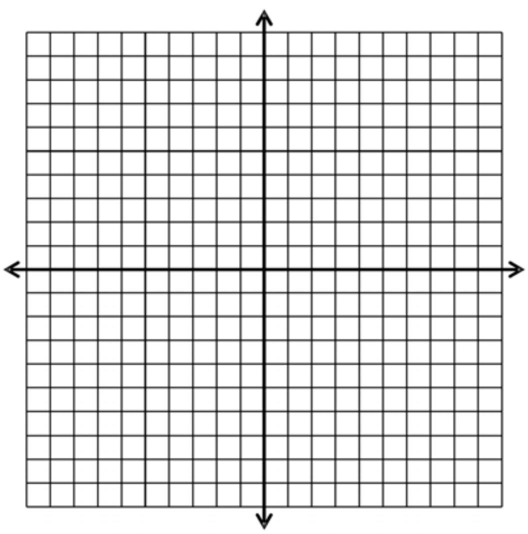
6. $y = -\frac{1}{2}x + 3$

$5x + 10y = 30$



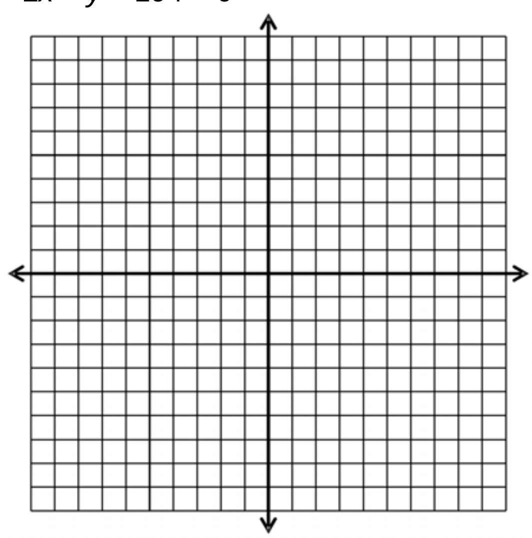
7. $y = 0.3x + 1.2$

$y = -x + 6.4$



8. $\frac{x}{2} - y - 61 = 0$

$2x + y - 194 = 0$



Unit 7: Day 2: Multiply a Binomial by a Binomial Using a Variety of Methods

Grade 10 Applied

75 min

Math Learning Goals

- Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial, using the chart method, and the distributive property.

Materials

- BLM 7.2.1, 7.2.2
- algebra tiles
- graphing calculators

Assessment Opportunities

Minds On…

Whole Class → Discussion

Students expand $(x + 1)(x + 1)$ with algebra tiles.

Discuss the meaning of repeated multiplication, i.e., $(x + 1)(x + 1) = (x + 1)^2$.

Individual → Practice

Students practise multiplication of a binomial with positives only, using BLM 7.2.1 Part A and algebra tiles.

Representation with algebra tiles is best for expressions with positive terms only.

Action!

Whole Class → Guided Instruction

Model the different combinations of multiplication, i.e., monomial × monomial and monomial × binomial with positive and negative terms, using the chart method.

Connect the use of algebra tiles to the "chart method."

Model the chart method for multiplication of a binominal × binomial. Students practise this method, BLM 7.2.1 Part B.

This representation can be used for binomials with positive and negative terms.

Use algebra tiles to show each of these and recall the distributive property.

$x(x + 3)$

$2(x + 3)$

Multiply $(x+2)(x+3)$ using tiles, and show "double distributive," (the algebraic model).

Model the steps involved in the algebraic manipulation of multiplying a binomial by a binomial. Connect to distributive property by calling it "double distributive."

Using the Distributive Property

Lead students to understand the connection between the chart method and the algebraic method of multiplying binomials.

$(x + 4)(x – 3)$

$= x (x – 3) + 4(x – 3)$

$= x^2 – 3x + 4x – 12$

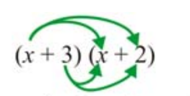
$= x^2 + x – 12$

	x	$+4$
x	x^2	$4x$
$–3$	$–3x$	$–12$

	x	$+3$
x	x^2	$3x$
$+2$	$2x$	6

Students practise this method, using BLM 7.2.1 Part C.

Learning Skills/Work Habits/Observation/Checklist: Assess how well students stay on task and complete assigned questions.



$= x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

Use visual aid to show double distributive

Consolidate Debrief

Individual → Journal

In your journal, write a note to a friend who missed today's class. Summarize the three methods of multiplying binomials that you worked with. Use words, diagrams, and symbols in your explanation.

Reflection

Home Activity or Further Classroom Consolidation

Solve the problems by multiplying the binomials.

Provide chart template 7.2.2 and algebra tile template 7.1.3, as needed.

Unit 7: Day 2: Multiply A Binomial By A Binomial Using A Variety of Methods (A)

Grade 10 Applied

75 min

Mathematical Process Goals

- Make connections between finding the product of two binomials and multiplying two 2-digit numbers.
- Make connections between the chart method and the distributive property.

Materials

- BLM 7.2.1(A), 7.2.2(A), 7.2.3(A)
- algebra tiles
- base ten materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Compare the result of multiplying 11 × 11 using base 10 material so that students understand why the result of $(x + 1)(x + 1)$ is not $x^2 + 1^2$. Also connect $11 \times 11 = 11^2$ with $(x + 1)(x + 1) = (x + 1)^2$.

Individual → Practice

Students practise multiplication of a binomial with positives only using algebra tiles and base 10 materials (BLM 7.2.1(A) Part A).

Mathematical Process Focus: Connecting

See TIPS4RM Mathematical Processes package p. 8

Refer to TIPS4RM Unit 7 Day 1 for base 10 material set up.

Action!

Whole Class → Guided Instruction

Guide students to see the connection among algebra tiles, the chart method for multiplication of a binominal × binomial, and multiplying two 2-digit numbers. Students include this connection in Part B of BLM 7.2.1A.

Using the Distributive Property

Demonstrate how a 2-digit number can be expressed as a two-term expression then lead students through the multiplication of two 2-digit numbers using the chart method and then the algebraic method for multiplying two binomials. Emphasize the connections between the two methods.

To multiply 13 × 12:

	10	+3
10	100	30
+2	20	6

13 × 12 =
(10 + 3)(10 + 2)
= 100 + 20 + 30 + 6
= 156

To multiply 14 × 7

	10	+4
10	100	40
–3	–30	–12

14 × 7 =
(10 + 4)(10 – 3)
= 100 – 30 + 40 – 12
= 98

Consolidate Debrief

Individual → Journal

In your journal, write a note to a friend who missed today's class. Summarize the three methods of multiplying binomials that you worked with and show the connection to multiplying two 2-digit numbers. Use words, diagrams, and symbols in your explanation.

Possible guiding questions:

- What connection do you see between a problem you did previously and today's problem?
- What connections do you see between using algebra tiles, an area model, a chart method, or base ten materials to multiply?
- When could this procedure be used in daily life?

Mathematical Process/Connecting/Checklist: Assess how well the students communicate their understanding of how the concepts are connected.

Concept Practice

Home Activity or Further Classroom Consolidation

Solve the problems by multiplying the binomials, including multiplication of two 2-digit numbers, using the chart method on worksheet 7.2.2(A)

See BLM 7.2.3 (A) for answers

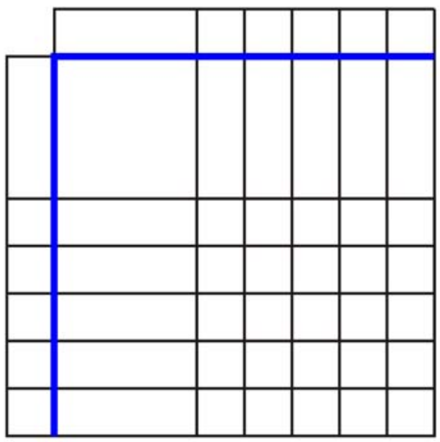
7.2.1(A): Multiply a Binomial by a Binomial

Name:

Part A

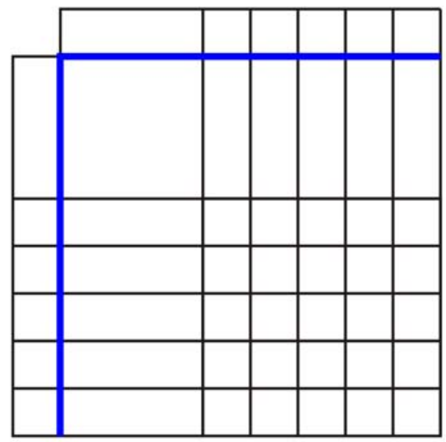
Use algebra tiles to multiply and simplify each of the following binomial products. Include an equivalent area model diagram for each to show the multiplication of two 2-digit numbers, with $x = 10$.

1. $y = (x + 1)(x + 3) =$ ________________



11×13 area model diagram:

2. $y = (x + 2)(x + 3) =$ ________________



12×13 area model diagram:

Part B

Use the chart method to multiply and simplify each of the following binomial products, Complete an equivalent chart for each to show the multiplication of two 2-digit numbers, with $x = 10$.

1. $y = (x + 1)(x + 3) =$ __________________

11×13

	10	1
10		
3		

2. $y = (x + 2)(x +3) =$ __________________

12×13

	10	2
10		
3		

7.2.1(A): Multiply a Binomial by a Binomial (continued)

3. $y = (x + 2)(x - 1) =$ ____________________

4. $y = (x - 2)(x + 3) =$ ___________________

12×9

	10	2
10		
–1		

8×13

	10	–2
10		
3		

5. $y = (x - 1)(x - 1) =$ ____________________

6. $y = (x - 1)\,(x - 2) =$ ___________________

____ × ____

____ × ____

7.2.2(A): Multiply a Binomial by a Binomial

Part C

Multiply and simplify the two binomials, using the chart method and the distributive property.

1. $(x + 4)(x - 3)$

	x	+4
x		
–3		

2. $(x - 3)(x - 3)$

	x	–3
x		
–3		

3. $(x + 2)^2$

	x	+2
x		
+2		

4. $(x + 2)(x - 1)$

	x	+2
x		
–1		

5. $(x - 2)\ (x + 1)$

	x	–2
x		
+1		

6. $(x - 1)^2$

	x	–1
x		
–1		

7. $(x - 1)(x - 2)$

	x	–1
x		
–2		

8. $(x - 3)(x - 4)$

	x	–3
x		
– 4		

7.2.3(A): Multiply a Binomial by a Binomial (Teacher)

Answers to Part B

1. $y = x^2 + 4x + 3$

	x	1
x	x^2	x
3	$3x$	3

2. $y = x^2 + 5x + 6$

	x	2
x	x^2	$2x$
3	$3x$	6

3. $y = x^2 + x - 2$

	x	2
x	x^2	$2x$
-1	$-x$	-2

4. $y = x^2 + x - 6$

	x	-2
x	x^2	$-2x$
3	$3x$	-6

5. $y = x^2 - 2x + 1$

	x	-1
x	x^2	$-x$
-1	$-x$	1

6. $y = x^2 - 3x + 2$

	x	-1
x	x^2	$-x$
-2	$-2x$	2

Answers to Part C

1. $x^2 + x - 12$
2. $x^2 - 6x + 9$
3. $x^2 + 4x + 4$
4. $x^2 + x - 2$
5. $x^2 - x - 2$
6. $x^2 - 2x + 1$
7. $x^2 - 3x + 2$
8. $x^2 - 7x + 12$

75 min

Math Learning Goals

- Investigate the method of factoring a difference of two squares using patterning strategies and diagrams.
- Use the technique of factoring the difference of two squares to determine the x-intercepts of a quadratic relation.

Materials

- graphing calculators
- BLM 7.8.1

Assessment Opportunities

Minds On…

Individual → Reflection

Students reflect on the connection between $x^2 - 36$, $y = x^2 - 36$, and the corresponding graph. They fold a paper into thirds and write the headings "**I Think, I Wonder, I Know**" in the columns. Students complete the first and second columns, and share their reflection with a partner.

Math Process/Communicating/Observation/Anecdotal Note: Assess students' use of mathematical language related to quadratic relations.

Action!

Pairs → Investigation

Students use a graphing calculator to identify the intercepts of quadratic relations of the form $y = x^2 - a^2$ and connect the x-intercepts to the factors (BLM 7.8.1).

Circulate to clear any misconceptions and to guide pairs, as needed.

Whole Class → Guided Instruction

Activate students' prior knowledge by factoring the relation $y = x^2 + 7x + 12$.

$3 \times 4 = 12$, $3 + 4 = 7$. Therefore $y = x^2 + 7x + 12$ can be expressed in factored form as follows: $y = (x + 3)(x + 4)$

Ask how $y = x^2 - 49$ could be written as a trinomial. [Answer: $y = x^2 + 0x - 49$]

Model the process: $(+7)(-7) = -49$ and $(+7) + (-7) = 0$

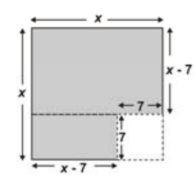
Therefore, $y = x^2 - 49$ can be expressed in factored form as $y = (x + 7)(x - 7)$.

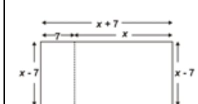
Reinforce the fact that the bx term is $0x$ and thus is not written in the expression. (Zero times x is zero.)

Explain why this type of quadratic is called a "difference of perfect squares," illustrating both algebraically and pictorially.

Students practise solving problems involving factoring a difference of squares.

The shaded area is $x^2 - 49$. You can see the two squares and the shaded area is the difference between their areas. When we factor, we want to find length and width of the rectangle having this area.





The length is $x + 7$ and width is $x - 7$.

Consolidate Debrief

Whole Class → Discussion

Students complete the third column in their "**I Think, I Wonder, I Know**" chart.

Review factoring a difference of squares and its connection to the graph, as needed.

Practice

Home Activity or Further Classroom Consolidation

Practise factoring and connecting the factors to the graph.

75 min

Mathematical Process Goals

- Create graphical, algebraic, and geometric representations to show a difference of two squares.
- Connect and compare graphical, algebraic, and geometric representations of the difference of squares.

Materials

- graphing calculators
- BLM 7.8.2(A), 7.8.3(A)
- area cutouts
- overhead projector

Assessment Opportunities

Minds On…

Individual → Sketching

Students create a sketch of their predicted graphical representation of $y = x^2 - 49$ and share their sketch with an elbow partner.

Action!

Pairs → Investigation

Students use a graphing calculator to identify the intercepts of quadratic relations of the form $y = x^2 - a^2$ and connect the x-intercepts to the factors (TIPS4RM BLM 7.8.1).

Circulate to clear any misconceptions and to guide pairs, as needed.

Whole Class → Guided Instruction/Discussion

Activate students' prior knowledge by factoring the relation $y = x^2 + 7x + 12$.

$3 \times 4 = 12$, $3 + 4 = 7$. Therefore $y = x^2 + 7x + 12$ can be expressed in factored form as follows: $y = (x + 3)(x + 4)$

Ask how $y = x^2 - 49$ could be written as a trinomial. [Answer: $y = x^2 + 0x - 49$]

Model the process: $(+7)(-7) = -49$ and $(+7) + (-7) = 0$

Therefore, $y = x^2 - 49$ can be expressed in factored form as $y = (x + 7)(x - 7)$.

Reinforce the fact that the bx term is $0x$ and thus is not written in the expression. (Zero times x is zero.)

Challenge students to explain why this type of quadratic is called a "difference of perfect squares," illustrating both algebraically and pictorially.

Students practise solving problems involving factoring a difference of squares.

Once the factored form of $x^2 - 49$ is developed, illustrate showing an area model. Use overhead cutouts and physically move the pieces so that students see how both illustrations represent the same area. Ask students to come up with expressions for the length, width, and resulting area.

Mathematical Process Focus: Representing

See TIPS4RM Mathematical Processes package p. 9

Possible guiding questions:

- Why are the area models of $x^2 - 49$ and $(x + 7)(x - 7)$ equivalent expressions?
- What clues does the factored form give us about the graph of $y = x^2 - 49$?
- What does each representation show?

Consolidate Debrief

Whole Class → Discussion

Use BLM 7.8.2(A) to provide a visual organizer, illustrating the various representations of $y = x^2 - 49$ discussed.

Pairs → Practice

Students complete copies of BLM 7.8.3(A) for: $y = x^2 - 4$, $y = x^2 - 9$, and $y = x^2 - 16$.

Mathematical Process/Representing/Checklist: Observe students as they complete the charts noting their comfort level using different representations.

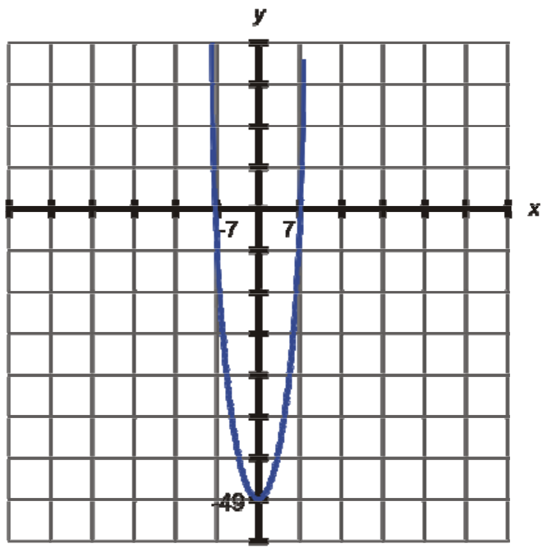
Practice

Home Activity or Further Classroom Consolidation

Complete copies of worksheet 7.8.3(A) for $y = x^2 - 25$, and $y = x^2 - 36$.

7.8.2(A): Representations of $y = x^2 - a^2$ (Teacher)

Graphical Form

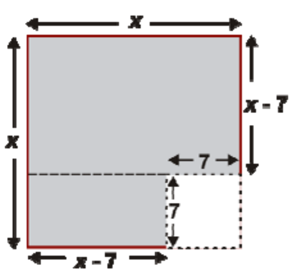


Factored Form

$$y = x^2 + 0x - 49$$
$$= (x + 7)(x - 7)$$

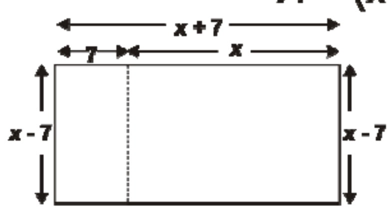
$$y = x^2 - 49$$

Area Models



$A = x^2 - 49$

$A = (x + 7)(x - 7)$



Non-examples

$y = x^2 + 49$

$y = x^2 + 6x - 49$

7.8.3(A): Representations of $y = x^2 - a^2$

Name:

Complete a Frayer Model for each of the given equations.