Unit 5: Exponential and Logarithmic Functions MHF4U

Lesson Outline

Big Picture

Students will:

- develop the understanding that the logarithmic function is the inverse of the exponential function;
- simplify exponential and logarithmic expressions using exponent rules;
- identify features of the logarithmic function including rates of change;
- transform logarithmic functions;
- evaluate exponential and logarithmic expressions and equations;
- solve problems that can be modeled using exponential or logarithmic functions.

Day	Lesson Title	Math Learning Goals	Expectations
1–2	*(lessons not included)*	• Explore and describe key features of the graphs of exponential functions (domain, range, intercepts, increasing/decreasing intervals, asymptotes). • Define the logarithm of a number to be the inverse operation of exponentiation, and demonstrate understanding considering numerical and graphical examples. • Using technology, graph implicitly, logarithmic functions with different bases to consolidate properties of logarithmic functions and make connections between related logarithmic and exponential equations (e.g., graph $x = a^y$ using Winplot or Graphmatica or graph a reflection in $y = x$ using GSP®).	A1.1, 1.3, 2.1, 2.2
3	*(lesson not included)*	• Investigate the relationship between $y = 10^x$ and $y = b^x$ and how they relate to $y = \log x$ and $y = \log_b x$. • Make connections between related logarithmic and exponential equations. • Evaluate simple logarithmic expressions. • Approximate the logarithm of a number with respect to any base with technology. • Solve simple exponential equations, by rewriting them in logarithmic form.	A1.1, 1.2, 1.3
4–5	*(lessons not included)*	• Explore graphically and use numeric patterning to make connections between the laws of exponents and the laws of logarithms. • Explore the graphs of a variety of logarithmic and exponential expressions to develop the laws of logarithms. • Recognize equivalent algebraic expressions involving logs and exponents. • Use the laws of logarithms to simplify and evaluate logarithmic expressions.	A1.4, 3.1
6–7	*(lessons not included)*	• Solve problems involving average and instantaneous rates of change using numerical and graphical methods for exponential and logarithmic functions. • Solve problems that demonstrate the property of exponential functions that the instantaneous rate of change at a point of an exponential function is proportional to the value of the function at that point.	D1.4–1.9

Day	Lesson Title	Math Learning Goals	Expectations
8	*(lesson not included)*	• Pose and solve problems using given graphs or graphs generated with technology of logarithmic and exponential functions arising from real world applications.	A2.4
9	*(lesson not included)*	• Solve exponential equations by finding a common base. • Solve simple logarithmic equations. • Solve exponential equations by using logarithms.	A3.2, 3.3
10	Log or Rhythm *GSP® file*: Log Investigation	• Investigate the roles of the parameters *d* and *c* in functions of the form $y = \log_{10}(x-d)+c$ and the roles of the parameters *a* and *k* in functions of the form $y = a\log_{10}(kx)$. • Connect prior knowledge of transformations in order to graph logarithmic functions.	A2.3 CGE 2c
11	*(lesson not included)*	• Solve problems of exponential or logarithmic equations algebraically arising from real world applications.	A3.4
12	Jazz		
13	Summative Assessment		

75 min

Math Learning Goals

- Investigate the roles of the parameters d and c in functions of the form $y = \log_{10}(x - d) + c$ and the roles of the parameters *a* and *k* in functions of the form $y = a\log_{10}(kx)$.
- Connect prior knowledge of transformations in order to graph logarithmic functions.

Materials

- BLM 5.10.1–5.10.5.
- GSP® or graphing calculators

Assessment Opportunities

Minds On…

Groups of 5 → Activating Prior Knowledge

Provide each group with an envelope containing the squares of one type of function from BLM 5.10.1.

Students match the equation, the graph, and the written description of the transformation.

Each member of the group takes the three pieces of a match and finds members of the other groups that have the same transformation.

Groups summarize the similarities of the transformation notation regardless of the function type. One group member shares with the whole class.

Photocopy BLMs – different colour for each function, cut and placed in envelopes

Students weak in transformations should be given the quadratic transformations.

Note: the scales on the graphs are all 1:1 unless stated otherwise.

Action!

Pairs → Exploration

Direct the students to the appropriate link for the GSP® activity.

Students follow the instructions and record their observations on the Student Observation Log (BLM 5.10.2).

Learning Skills/Teamwork/Checkbric: Circulate to assess how individual students stay on task and help each other complete the investigation.

Log Investigation.gsp

BLM 5.10.3 provides Alternate Graphing Calculator Investigation

Consolidate Debrief

Whole Class → Discussion

Lead the class in a discussion of their Student Observation Log sheet to review the effects of each of the parameters on the transformations of the logarithmic function. Post a summary on class wall.

Curriculum Expectations/Observations/Mental Note: Assess students' communication of transformations of functions orally, visually, and in writing, using precise mathematical vocabulary.

As a fun review and practice of understanding the various transformations, teach students the "tai chi" movements for each transformation (BLM 5.10.4).

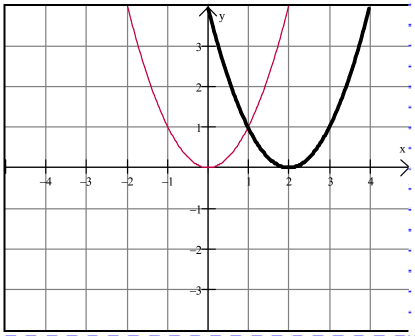
Exploration
Application

Home Activity or Further Classroom Consolidation

Complete Worksheet 5.10.5 CSI Math.

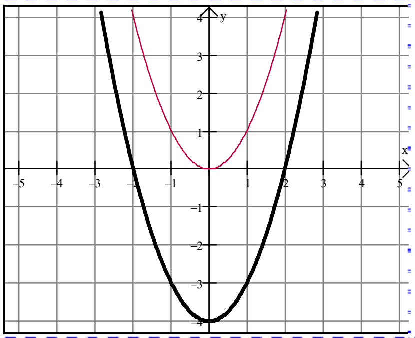
Collect next day for assessment.

5.10.1: Log or Rhythm



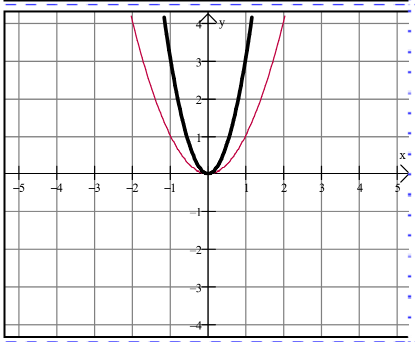
$f(x) = (x-2)^2$

Horizontal Translation
Right _____



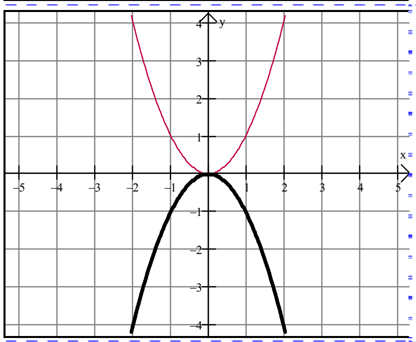
$y = x^2 - 4$

Vertical Translation
Down _____



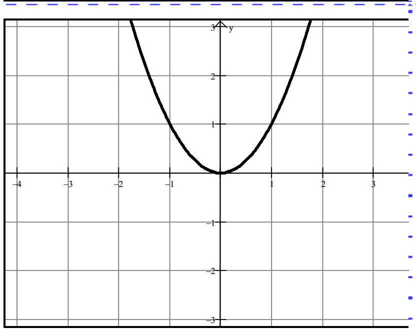
$f(x) = 3x^2$

Vertical Stretch
Factor _____
from the x-axis



$y = -x^2$

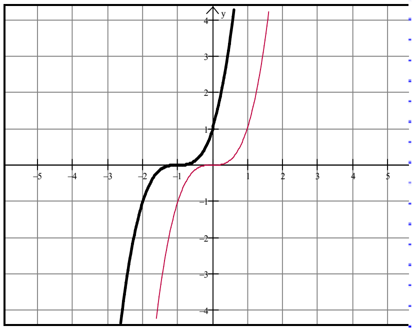
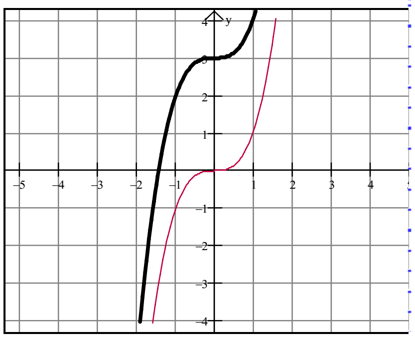
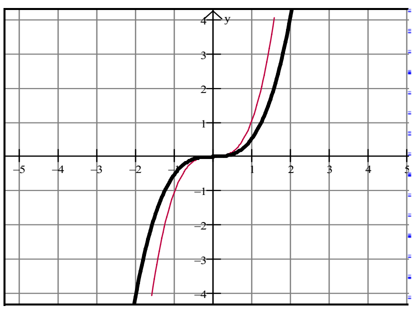
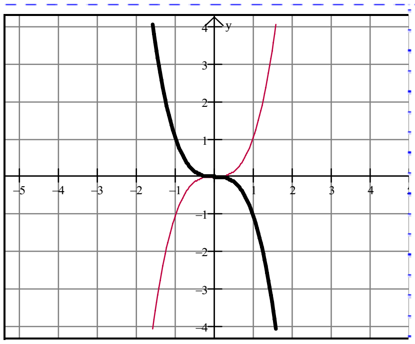
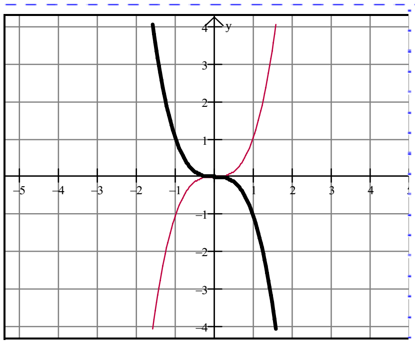
Vertical Reflection
in the x-axis



$f(x) = (-x)^2$

Horizontal Reflection
in the y-axis

5.10.1: Log or Rhythm (continued)

	$y=(x+1)^3$	Horizontal Translation Left _____
	$f(x)=x^3+3$	Vertical Translation Up _____
	$\frac{1}{2}x^3$	Vertical Compression Factor _____ to the x-axis
	$-x^3$	Vertical Reflection in the x-axis
	$(-x)^3$	Horizontal Reflection in the y-axis

5.10.1: Log or Rhythm **(continued)**

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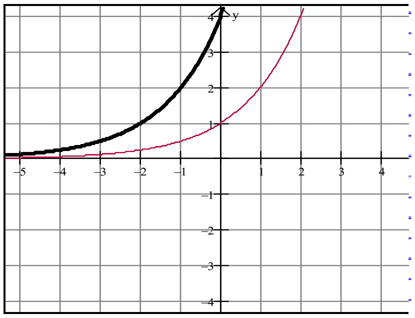
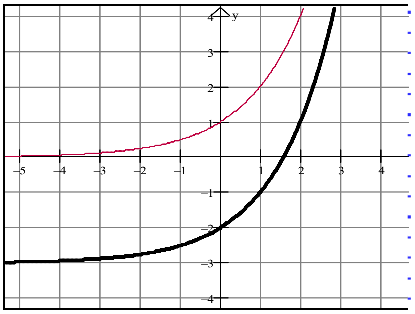
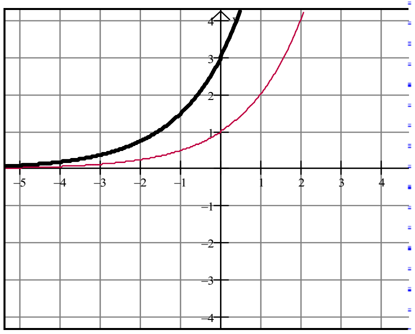
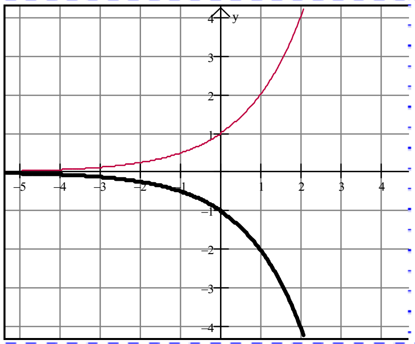
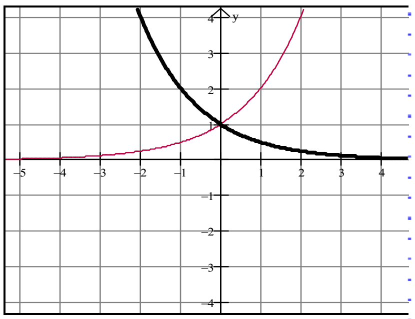
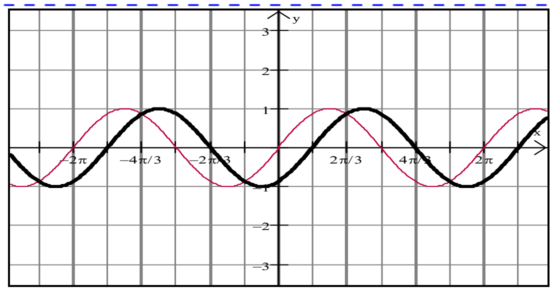
	$f(x) = 2^{x+2}$	Horizontal Translation Left ______
	$y = 2^x - 3$	Vertical Translation Down ______
	$f(x) = 3(2^x)$	Vertical Stretch Factor ______ from the x-axis
	$y = -2^x$	Vertical Reflection in the x-axis
	$f(x) = 2^{-x}$	Horizontal Reflection in the y-axis

5.10.1: Log or Rhythm (continued)

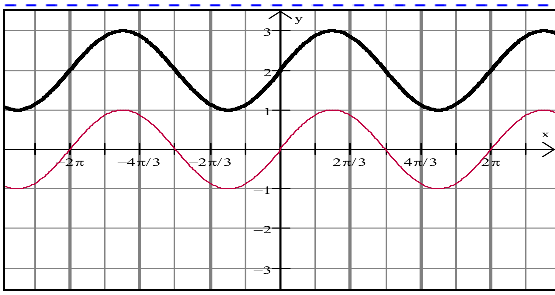
✂



Each interval on the horizontal axis represents $\frac{\pi}{3}$

$y = \sin\left(x - \frac{\pi}{3}\right)$

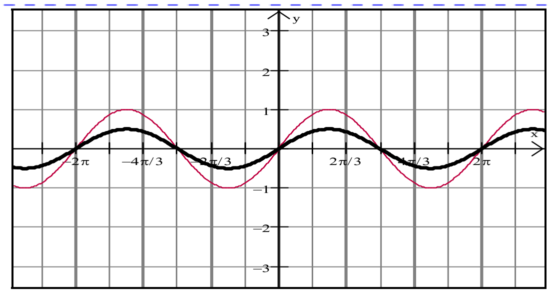
Horizontal Translation Right _____



Each interval on the horizontal axis represents $\frac{\pi}{3}$

$f(x) = \sin(x) + 2$

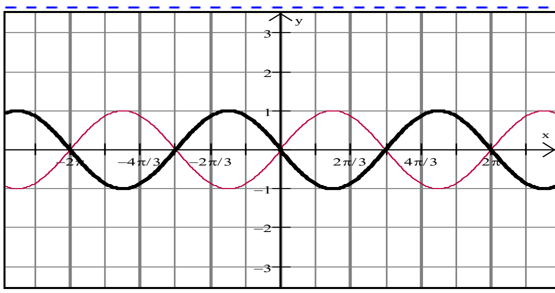
Vertical Translation Up _____



Each interval on the horizontal axis represents $\frac{\pi}{3}$

$y = \frac{1}{2}\sin(x)$

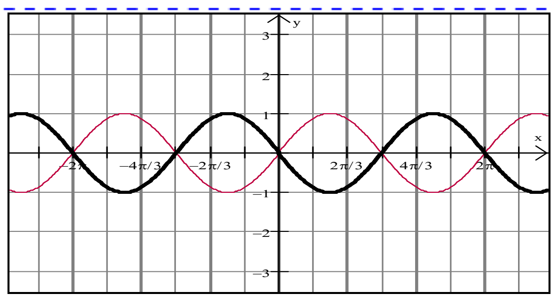
Vertical Compression Factor _____ to the x-axis



Each interval on the horizontal axis represents $\frac{\pi}{3}$

$f(x) = -\sin(x)$

Vertical Reflection in the x-axis



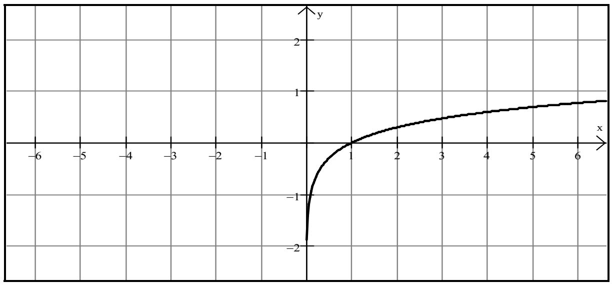
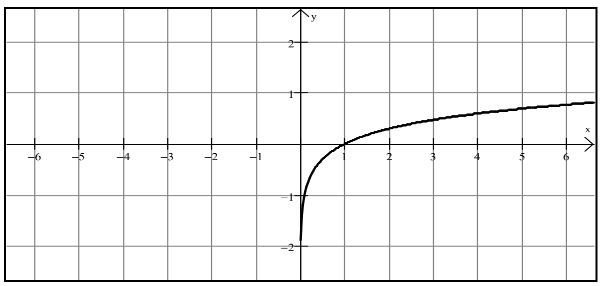
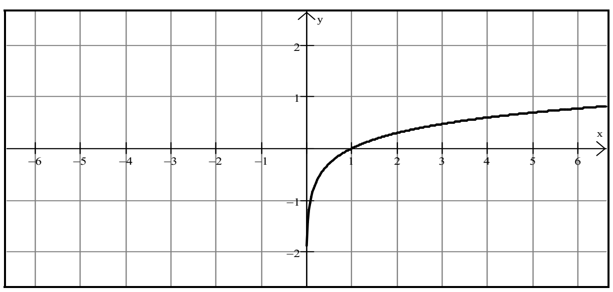
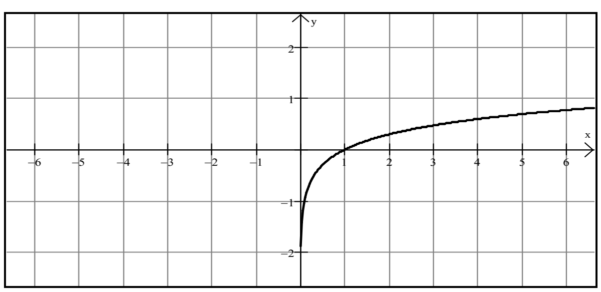
Each interval on the horizontal axis represents $\frac{\pi}{3}$

$y = \sin(-x)$

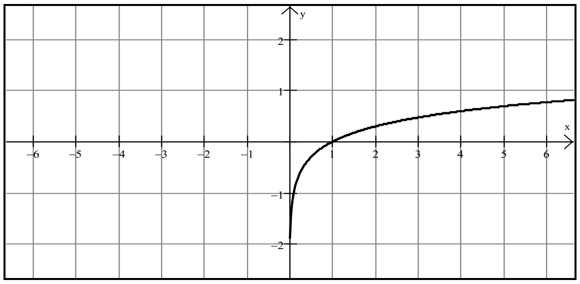
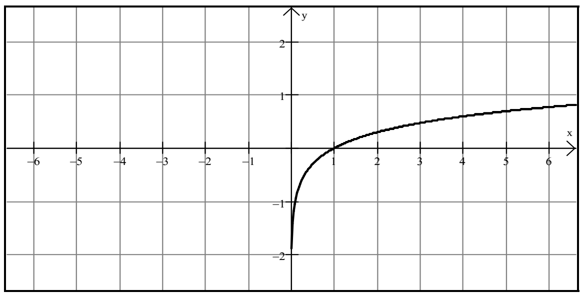
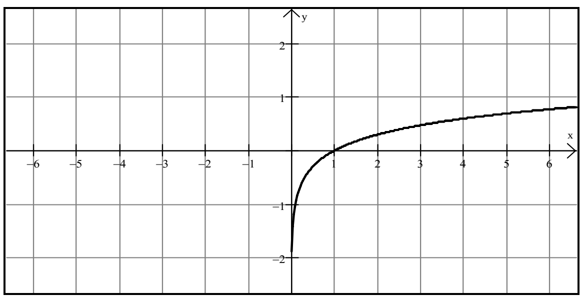
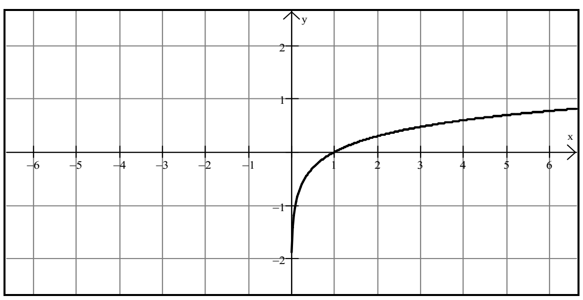
Horizontal Reflection in the y-axis

5.10.2: Student Observation Log

As you work through the investigations, use the chart below to record your observations. Record sketches of the transformed graphs and give a brief but accurate description in words of how each parameter affects the graph of $y = \log_{10}x$.

	Graphs	Descriptions
$y = \log_{10} x + c$		
$y = \log_{10} x - c$		
$y = \log_{10}\left(x + d\right)$		
$y = \log_{10}\left(x - d\right)$		

5.10.2: Student Observation Log (continued)

	Graphs	Descriptions
$y = a\log_{10} x$		
$y = -\log_{10} x$		
$y = \log_{10}(kx)$		
$y = \log_{10}(-x)$		

Special Notes:

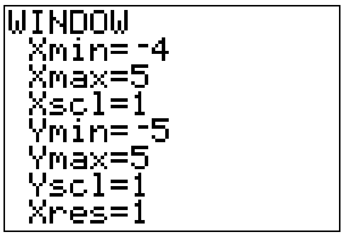
5.10.3: Alternate Graphing Calculator Investigation

Investigation of Transformations of the Logarithmic Function

In this investigation you will explore the roles of the parameters c, and d in the function $y=\log_{10}(x-d)+c$ and of the parameters a and k in $y=a\log_{10}(kx)$. You will record your observations on the Student Observation Log sheet.

Investigation 1: $y=\log_{10}(x)+c$

Change the WINDOW settings to the settings shown in the screen shot below.



Enter the function $y=\log_{10}(x)$ into the equation editor as y_1 and the following three functions in y_2, one at a time, to compare and contrast their graphs:

$f(x)=\log_{10}(x)+3$

$f(x)=\log_{10}(x)-2$

$f(x)=\log_{10}(x)+1.5$

Describe the effect that the parameter 'c' has on the transformations of $y=\log_{10}(x)+c$:

- what happens when 'c' is positive?
- what happens when 'c' is negative?
- record your graph and observations on the Student Observation Log.

Investigation 2: $y=\log_{10}(x-d)$

Enter the function $y=\log_{10}(x)$ into the equation editor as y_1 and the following three functions in y_2, one at a time, to compare and contrast their graphs:

$f(x)=\log_{10}(x+3)$

$f(x)=\log_{10}(x-2)$

$f(x)=\log_{10}(x+1.5)$

Describe the effect that the parameter 'd' has on the transformations of $y=\log_{10}(x-d)$:

- what happens when 'd' is positive?
- what happens when 'd' is negative?
- record your graph and observations on the Student Observation Log.

5.10.3: Alternative Graphing Calculator Investigation (continued)

Investigation 3: $y = a\log_{10}(x)$

Enter the function $y = \log_{10}(x)$ into the equation editor as y_1 and the following three functions in y_2, one at a time, to compare and contrast their graphs:

$f(x) = 3\log_{10}(x)$

$f(x) = -2\log_{10}(x)$

$f(x) = \frac{1}{2}\log_{10}(x)$

$f(x) = -0.5\log_{10}(x)$

Describe the effect that the parameter '$\boldsymbol{a}$' has on the transformations of $y = a\log_{10}(x)$:

- what happens when '$\boldsymbol{a}$' is positive?
- what happens when '$\boldsymbol{a}$' is negative?
- record your graph and observations on the Student Observation Log.

Investigation 4: $y = \log_{10}(kx)$

Enter the function $y = \log_{10}(x)$ into the equation editor as y_1 and the following three functions in y_2, one at a time, to compare and contrast their graphs:

$f(x) = \log_{10}(3x)$

$f(x) = \log_{10}(-2x)$

$f(x) = \log_{10}(x)$

$f(x) = \log_{10}(-0.5x)$

Describe the effect that the parameter '$\boldsymbol{k}$' has on the transformations of $y = \log_{10}(kx)$:

- what happens when '$\boldsymbol{k}$' is positive?
- what happens when '$\boldsymbol{k}$' is negative?
- record your graph and observations on the Student Observation Log.

5.10.4: Math "Tai Chi"

Write all of the transformations on the board. Each time you point to a new transformation, students must display the appropriate movement. As the students get better at recognizing each one, a natural development is an elimination game. This makes a great review before a test!

Transformation	Description	Movement
$f(x)+d$	Vertical Translation UP	Slowly raise arms, palms UP
$f(x)-d$	Vertical Translation Down	Slowly lower arms, palms DOWN
$f(x-c)$	Horizontal Translation Right	Slowly motion arms to the RIGHT
$f(x+c)$	Horizontal Translation Left	Slowly motion arms to the LEFT
$af(x),\ a>0$	Vertical Stretch	Left arm up, right arm down (pulling an elastic apart vertically)
$af(x),\ a<0$	Vertical Compression	Start with arms apart, and bring them together (opposite of V. Stretch)
$f(kx),\ k>0$	Horizontal Stretch	Slowly pull arms apart horizontally (pulling an elastic apart horizontally)
$f(kx),\ k<0$	Horizontal Compression	Slowly bring palms together (opposite of H. Stretch)
$-f(x)$	Reflection in the x-axis	Hold arms in upright goal post position and then flip them down at the elbow to point to the floor.
$f(-x)$	Reflection in the y-axis	Hold arms in the upright goal position and then cross them in front to make an X.

5.10.5: CSI Math

The drama class is writing a murder mystery, and they need some props. The case hinges on the time of death which can be determined by Newton's Law of Cooling. But, they would like to be as accurate as possible, so they have come to Professor Wiggin's math class for a graph that will help them solve the problem.

Your task is to create a graph of the cooling body temperature on chart paper so that it is large enough for an audience to see.

Here is some information to help you complete this task.

A coroner uses a formula derived from Newton's Law of Cooling, a general cooling principle, to calculate the elapsed time since a person has died. The formula is:

$$t = -23\log(T - RT) + 33, \text{ where}$$

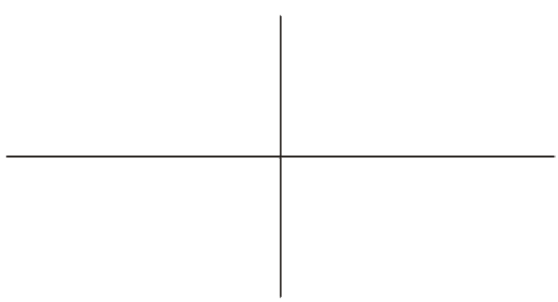
- t is the time elapsed in hours since death
- RT is the constant room temperature
- T is the body's measured temperature (°F)

A more accurate estimate of the time of death is found by taking two readings and averaging the calculated time.

Use your knowledge of the logarithmic function to describe the transformations that occur in the equation $t = -23\log(T - RT) + 33$.

•
•
•
•

Now, using your descriptions of the transformations, draw a 'rough' sketch of the graph, and estimate the window settings that will allow an optimal view of the time in question. Check your sketch using a graphing calculator.



Prepare a full size accurate graph on chart paper for the drama class to use.

GSP® file Log Investigation