Programming
Remediation and
Intervention for
Students in
Mathematics

PRISM North Western Ontario (PRISM-NWO)

Research Project

Final Report
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Lakehead District School Board

Northwest Catholic District School Board
Superior North Catholic District School Board
Northern Resource Alliance

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Abstract

Building on the premise that all students, but especially students at-risk, benefit significantly from highly qualified teachers, this project has focused on providing professional learning experiences for Northwestern Ontario grade seven teachers and studying the outcomes of these experiences. Mathematical knowledge for teaching, beliefs about teaching mathematics, and values about mathematics were examined using pre-tests and post-tests including standard and locally developed instruments to examine current capacity and to describe growth. Our situation is unique geographically in Ontario; some participants in our study were as much as a four-hour drive from one another. Recommendations for evaluating future professional development programs in Northwestern Ontario and elsewhere are also provided.

Introduction and Rationale

Based on the assumption that teachers' mathematical knowledge and beliefs impact their capacity for high quality teaching, and that the quality of classroom teaching effects all students but especially students deemed at risk (Expert Panel on Numeracy and Literacy Instruction for Students With Special Needs, 2005), this study examined various professional development opportunities and sought to document change in participating teachers. Effects were compared from the several different in-service components offered. Three instruments were used and compared in an attempt to probe for observable change in the relatively short time period. These were the Learning Mathematics for Teaching Middle School Content Knowledge for Teachers instrument (Hill, Schilling, & Ball, 2005) for measuring mathematics knowledge for teaching, the Nelson Attitudes and Practices for Teaching Math Survey (Ross, McDougall, Hogaboam-Grey, & LeSage, 2003) for measuring dimensions of reform based beliefs about the teaching of mathematics, and the Perceptions of Math instrument (Kajander, 2005) for examining beliefs and knowledge related to procedural and conceptual understanding of mathematics by teachers.

This study was a collaborative effort between a consortium of school boards lead by Lakehead Public Schools and a research team from the Faculty of Education of Lakehead University in Thunder Bay. Seventh grade teachers in a number of areas in the Northwestern region of Ontario were invited to participate in the study which began in May 2005 and ended in December 2005. Participating teachers were offered several in-service opportunities such as a professionally delivered mathematics reform program for teachers (Nelson PRIME Number and Operations training), as well as specific AQ (Additional Qualifications) courses. Pre-test and post-test data were collected using the various instruments to probe for differing effects of the treatments. Recommendations for future research and other programs are included.

Framework

It is well documented that teachers' content knowledge of mathematics is crucial for improving the quality of instruction in classrooms (Ambrose, 2004; An, Kulm & Wu, 2004; Hill & Ball, 2004; Ross et al, 2003; Stipek et al, 2001). Quality of teaching in turn is an important factor in student learning (Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs Kindergarten to Grade 6, 2005; Balfanz et al, 2006). In fact, teacher knowledge about teaching and learning has been cited as the most important predictor of student success (Greenwald, Hedges, & Laine, 1996). Thus, it is important to actively support the development of deep teacher knowledge in order to assist teachers in making an effective transition from the traditional classroom to the reform classroom, a transition that is still very much in progress or even stalled, according to 1999 TIMMS video data (Jacobs et al, 2006). Knowledgeable teachers are important for all students but crucial for those deemed "at risk" mathematically (Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs Kindergarten to Grade 6, 2005; Balfanz et al, 2006). The recent report by the Expert Panel on Student Success in Ontario reports that "mathematics learning strategies that benefit all students are a necessity for students at risk, and extra support may also be needed to close the gap" (2004, p. 37).

School boards can be faced with substantial costs in providing in-service opportunities to teachers. Hill and Ball (2004) cite a lack of measures of teachers' content knowledge as a difficulty in determining what features of professional development contribute most significantly to teacher learning. The type of knowledge held by teachers such as whether knowledge is conceptual as well as procedural (Ibid.), and strategies for supporting such learning in students (Rittle-Johnson & Koedinger, 2002) such as the extent to which teacher-student discourse probes deep conceptual understanding (Ross et al, 2003) may also play a role, and such knowledge may be harder to document on standardized tests.

Ma (1999) introduced the concept of "profound understanding of fundamental mathematics", and showed that teachers need a deep understanding of mathematical ideas that goes beyond a functional fluency. Ball (1990) had previously proposed a distinction between (a functional) knowledge of mathematics, and knowledge about mathematics. These are later further defined to be "knowledge of concepts, ideas and procedures and how they work" and "knowledge about 'doing mathematics' – for example, how one decides that a claim is true, a solution complete, or a representation accurate" (Hill, Schilling & Ball, 2005, p. 14). This framework formed the basis for the development of the knowledge-related section of the *Perceptions of Mathematics Instrument* used in the current study. Like Ball and her colleagues, our approach to studying content knowledge for teaching is grounded in a theory of instruction which begins with the fundamental understandings required by teachers to teach well (Ball & Bass, 2000; Hill, Schilling & Ball, 2004). We agree that

> "…teachers in Grades 7, 8 and 9 must have a thorough conceptual understanding of the subject. This understanding goes beyond what is required by completing Grade 12 mathematics" (Expert Panel on Student Success in Ontario, 2004, p. 79).

A certain conflict exists around the needs for students at risk. While the Standards (NCTM, 2000) recommend that all students have access to rich mathematical ideas, some schools may postpone instruction of higher order thinking skills until basic, low level skills have been mastered (Expert Panel on Student Success in Ontario, 2004). Yet low-achieving students may suffer most from a proficiency-driven curriculum (Ibid).

Hence, knowledge alone may not be sufficient for teachers to choose to teach differently from the ways in which they learned mathematics as children. Influencing teachers' beliefs and values may also be essential to changing teachers' classroom practices (Stipek et al, 2001). Ross et al (2003) argue that beliefs found in teachers' self-report surveys do relate to subsequent student achievement. Students may also be influenced by their teachers' beliefs, and evidence exists that if middle school students have a strong belief that mathematics is not valuable, they may resist spending time or effort on it (Schommer-Atkins, Duell & Hutter, 2005). Thus while content knowledge may be an important feature, clearly beliefs also play a role, and the opportunity for changing their beliefs is essential for teachers' development (Cooney et al, 1998).

Hill and Ball feel that teachers can deepen their mathematics knowledge for elementary school teaching in the context of a single professional development program, and that an important feature of successful programs is to foreground mathematical content (2004). However, they document that effects seem to vary from treatment to treatment. They state the need to probe more carefully into the content of professional development and to identify curricular variables associated with teachers' learning (Ibid.).

In recent in-depth case studies of five grade seven and eight teachers in Northwestern Ontario who were selected on the basis of showing strong reform-based beliefs based on results of a written survey, only three of these teachers actually taught their students in a reform based way when observed in their classrooms (Kajander, 1999; McDougall et al, 2000). These three teaches were observed to use techniques associated with reform-based learning such as students working together on rich open-ended problem solving tasks for which the problem solving itself (not just the answer) was valued. Students used concrete materials, worked in groups, and were called upon to explain and defend their thinking. However the other two teachers selected for study were not seen to teach in a reform based way as described above on the days they were observed. One teacher was very traditional throughout in all aspects of her approach. The other was an interesting mixture of styles. While on first glance the classroom activities *resembled* those of a reform-based environment, closer observation revealed something different. The students worked in groups, and were working on a relatively rich task. As part of the task, they had to make some decisions about the best representation of some data. In verbal interaction with the students, the teacher led them directly to the representation she wanted, thereby removing the decision from the students. Thus in her view of the task, it seemed to be the production of the *result* that counted, not the *thinkin*g that went into deciding about the choice of representation and method of solution. The outcome was being valued in this classroom over the thinking and problem solving process itself.

Such subtle differences are part of what makes improving the quality of teaching so difficult to measure and document. The latter teacher described claimed to believe in an approach to mathematics teaching consistent with the Standards (NCTM, 2000), and seemed to demonstrate that belief in her classroom layout and even choice of tasks. However, what she valued about *mathematics itself* may have influenced what aspects of the tasks she chose to assist student with. Her specific instructions to the students regarding the method to use seemed to indicate that what she wanted from the students was the *mathematical product.* These observations suggest that to this teacher, the product itself embodied the mathematics. Recent views of mathematics, on the other hand, would suggest that it is the *process of mathematical problem solving* that is the most important aspect and outcome of a rich task (NCTM, 2000; Whitely and Davis, 2003). Thus one's values about mathematics itself are suggested as an underlying component of how improvements in mathematics teaching are to be enacted in the classroom, and these may be separate from beliefs about reform style pedagogy.

This framework suggests then that a plurality of aspects form prerequisites for successful improved teaching in mathematics. While evidence exists that both deep teacher knowledge of mathematics as well as beliefs about the nature of how students learn are important, beliefs and values about mathematics itself may also play a role. Examining one of these aspects without the others may lead to mixed results in terms of success of professional development initiatives.

Balfanz et al (2006) report on an extensive in-service support program for teachers of middle school students in high poverty areas, which provided teachers with up to36 hours of professional development per year. Looking at the magnitude of student achievement gains over a four year period, they recommend a richer and stronger curriculum, extensive professional development and teacher support, and a whole-school reform model as important factors in improving student achievement. In-classroom coaching was included in the Balfanz model.

Based on the recommendations provided in this framework by Hill and Ball (2004), Balfanz (2006), and others, the current study attempted to provide participating teachers with multiple opportunities for professional learning, and to measure changes in their knowledge, beliefs, and values about mathematics. Various professional development opportunities were provided to teachers, including extensive training based on mathematical content understanding, on-line courses, and participation in Professional Learning Groups. It should be noted however, that the whole school reform model, and the in-class coaching components recommended in the Balfanz et al (2006) model were not included.

It is important for school boards and education ministries to have access to such research regarding the effectiveness of various types of in-service learning opportunities in order to make appropriate decisions about training and to effectively allocate funds. It is also relevant to probe the usefulness and efficiency of different measures in documenting such change. While the ideal method of documentation might involve case study research of post-treatment behavior of teachers in their classrooms, such research is time consuming and expensive. Examining the effectiveness of different measures in documenting teacher change is important for school boards who may need to justify the expenditures required for in-service opportunities for their teachers, and need such information to choose and evaluate the effectiveness of professional development programs.

Goals of Study

A fundamental goal of the study was to improve opportunities for students deemed at-risk by building on teacher capacity for teaching mathematics. Significant in-service learning opportunities were provided for participating teachers, focusing on conceptual understanding of fundamental mathematics, appropriate use of manipulatives, use of representations, and differentiated instruction. The main focus of the training was on understanding related to the strand of number and operations, or Number Sense and Numeration as it is called in the Ontario curriculum. As well, some teachers in the study participated in one or more Additional Qualifications courses in mathematics.

The following research questions guided the study:

1. Are changes in mathematics knowledge and beliefs of intermediate teachers measurable after a relatively short (eight month) professional development experience?
2. Which measures of teacher knowledge and beliefs are most useful for identifying teacher change under these circumstances?
3. Are there discernable differences in measurable results between varying types of professional development experiences and courses for teachers?

Based on the framework, the assumptions of the study were that teacher mathematics knowledge and beliefs significantly affect classroom practice and student learning, particularly learning for students deemed ‘at risk’ mathematically.

Methodology

The study involved a sample of 40 volunteering grade seven teachers. About half of the teachers lived in a small city and had received some in-service training already from their local school board related to mathematics reform before the current project began. Some of these teachers had also been involved with Professional Leaning Groups which had been funded by the board to meet for a half day a month over the previous year. The rest of the study sample was composed of teachers from smaller towns, up to four or five hours drive away from any other urban center. Thus in-service opportunities for these teachers had previously been more limited. Grade seven teachers in these locations were offered the opportunity to participate in the project but were not required to do so.

Prior to the beginning of treatment, three measures were administered to participating teachers in a written pre-test format during a separate meeting. These were:

1. Middle School Form A (Learning Mathematics for Teaching, 2005)
2. Nelson Attitudes and Practices for Teaching Math survey (Ross et all, 2003)
3. The Perceptions of Math (POM) Survey (Kajander, 2005)
   This instrument is described further in the Appendix.

All three sub-sections of the Learning Mathematics for Teaching measure were scored at the pre-test, namely Number and Operations, Algebra, and Geometry. All ten dimensions of the Nelson survey were also scored at the pre-test, but only the dimensions “Manipulatives and Technology” and “Attitude and Comfort with Mathematics” were used at the post-test. The Perceptions of Mathematics (POM) Survey yielded scores in four areas, namely Procedural Knowledge, Conceptual Knowledge, Procedural Values, and Conceptual Values. Values are defined as beliefs about the importance of learning mathematics procedurally and conceptually. It is possible to have high scores in both areas; that is, to believe in the importance of both types of knowing, however a shift towards reform based beliefs might be thought of as including an increase in Conceptual Values (CV), and possibly also a decrease in Procedural Values (PV). The knowledge scores on this instrument were teachers’ own demonstrated understandings in these areas. For example, providing the correct answer to a subtraction of negative integers question was scored as Procedural Knowledge (PK). When teachers were asked to explain how or why the procedure worked or give an example or model, it was scored as Conceptual Knowledge. Providing a rule such as “because two negatives make a positive” did not score any points for Conceptual Knowledge (CK); rather, teachers were required to provide an explanation or model as to *why* this makes sense. Knowledge questions were based strongly on Ma’s interview questions in her landmark (1999) study. The Perceptions of Math (POM) instrument is described in detail elsewhere (Kajander, 2005). All instruments are also described further in the Appendix to this report.

Treatment began with each group of teachers receiving two full days of training provided by professional mathematics in-service trainers from Nelson Canada, on the PRIME Number and Operations strand. Teachers from the city were grouped in one cluster (group A), and regional teachers were grouped together in a second cluster (group B). This training focused on the Number and Operations content and emphasized use of manipulatives, conceptual understanding of methods, student generated algorithms, differentiated instruction, and other aspects of mathematical content knowledge and reform based practice. All participants received this initial training which took place during May 2005. A third day of PRIME training was offered to all participating teachers in the fall of 2005.

Eight teachers in group A also participated in one or two AQ (“Additional Qualifications”) courses in mathematics. The 18 teachers in Group B had no further training other than the Nelson training.

In September of 2005, teachers in group A were also invited to join Professional Learning Groups (PLG’s) organized by their board for grade 7 and 8 teachers. One half day of orientation was provided for these teachers to meet together receive some introduction, and meet their group members. Beginning in October, each PLG member subsequently received one half day of release time for a monthly meeting of their group. Hence these teachers in group A would have also participated in approximately two PLG meetings by the post-test data collection in late November. All three pre-test instruments were re-administered during this final session at the end of November devoted to post-test data collection, which was to involve all teachers in the study. In fact not all teachers attended the final session which was unfortunately rather close to reporting deadlines, so complete data is only available for the 30 teachers who completed both pre-test and post-test.

Results and Discussion

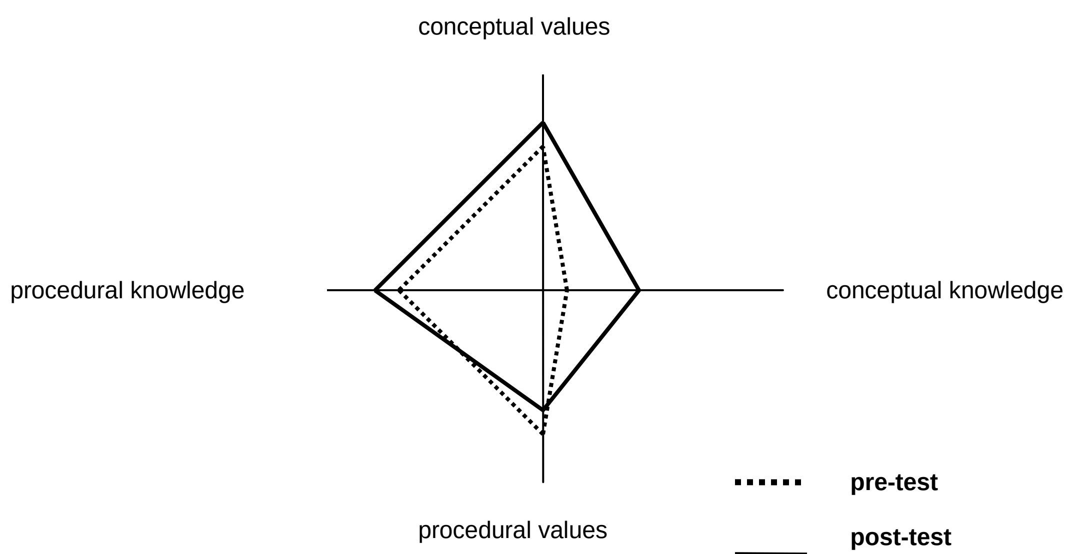
<u>Mathematics Knowledge of Teachers</u>

Two instruments were used to probe for change in teacher knowledge. These were the Perceptions of Mathematics (POM) instrument and the Learning Mathematics for Teaching (LMT) middle school instrument. Results from each will be described separately, followed later by discussion of relationships among instruments.

Perceptions of Mathematics (POM) Instrument
The Perceptions of Mathematics (POM) instrument was originally designed to assess knowledge and beliefs of pre-service junior intermediate teachers. One of the purposes of the measure was to portray these attributes on a two dimensional graph, in order to motivate teachers to improve upon their (often low) conceptual understanding to support their (often high) beliefs in the value of such learning. Figure 1 shows the mean scores of two samples of preservice teachers at the beginning and end of their mathematics methods course, averaged from 2004-2005 (N=114) and 2005-2006 (N=131), shown on the *Profile* graph. This data was obtained from the companion CRYSTAL project at Lakehead University which informed the choice of instruments for the current study. A shift upwards and to the right on the *Profile* represents a more reform-based conception of mathematics, encompassing deeper beliefs in the value of conceptual learning coupled with deeper conceptual understanding on the part of the teacher.

Figure 1. POM *Profile* of Pre-service Teachers

(Note: Profile graph is for illustrative purposes only and is not perfectly to scale).



Knowledge and beliefs are assessed separately in the POM instrument. Procedural and Conceptual Knowledge will be discussed first.

Conceptual and Procedural Knowledge
Procedural Knowledge (PK) is defined as teachers' ability to follow procedures to generate correct answers, while Conceptual Knowledge (CK) is defined as their ability to model, explain, justify or give examples to explain how and why these same methods work (Kajander, 2005). Based on the study with pre-service teachers, slight modifications were made to the POM instrument for use in the current study. The POM survey was administered to teachers using a paper and pencil format and was scored by a research assistant, after triangulating with another research assistant and the researcher until consistency was reached. The Appendix shows the POM instrument as used in the current study.

Descriptive statistics as shown in Table 1 indicate that the mean of Conceptual Knowledge increased slightly from the pre-test (M=6.95, SD=1.84) to the post-test (M=6.99, SD=2.21). All values are shown out of a maximum of 10. The shape of the distribution for the Conceptual Knowledge post-test looks more negatively skewed than the pre-test as depicted in Figures 2 and 3.

Table 1.
Descriptive Statistics Conceptual and Procedural Knowledge

	N	Minimum	Maximum	Mean	Std. Deviation
Post-test PK	30	4.4	10.0	8.556	1.5754
Post-test CK	30	1.4	10.0	6.998	2.2140
Pre-test PK	30	6.7	10.0	9.000	1.0254
Pre-test CK	30	2.9	10.0	6.952	1.8468
Valid N (listwise)	30				

Fig 2 **Pre-test Conceptual Knowledge**

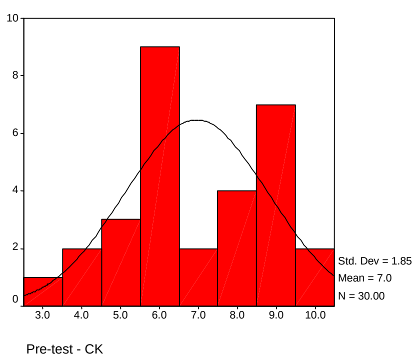
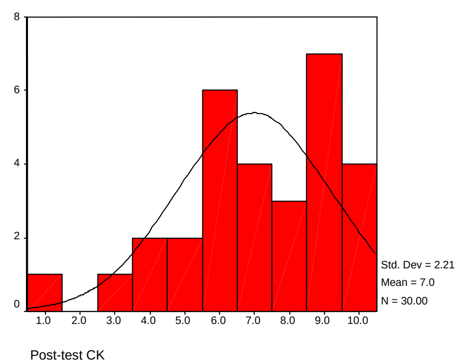


Fig 3 **Post-test Conceptual Knowledge**



The Descriptive statistics as shown in Table 1 also indicate that the mean of Procedural Knowledge decreased slightly from the pre-test (M=9.0, SD=1.02) to the post-test (M=8.55, SD=1.57). The shape of the distribution for the Procedural Knowledge post-test looks slightly more negatively skewed than the pre-test as shown in Figures 4 and 5.

Fig 4 Procedural Knowledge Pre-test

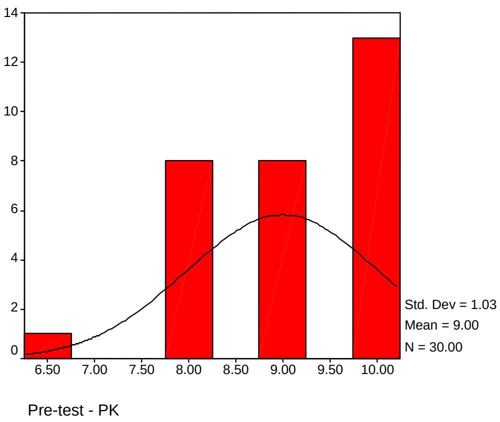
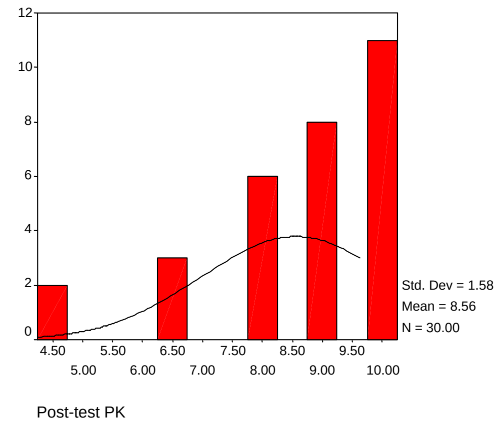


Fig 5 Procedural Knowledge Post-test



In order to assess if there were significant differences in these knowledge variables between the pre and post tests, a repeated measures t-test was performed as shown in Table 2. The repeated measures t-test suggests that there is not significant improvement in Conceptual Knowledge due to the intervention, t (29) = .168, p > .05, nor is there significant change in Procedural Knowledge, t (29) = -1.588, p > .05.

Table 2.
Repeated Measures t-test Conceptual and Procedural Knowledge

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	Post-test PK Pre-test PK	-.444	1.5330	.2799	-1.017	.128	-1.588	29	.123
Pair 2	Post-test CK Pre-test CK	.046	1.4882	.2717	-.510	.601	.168	29	.868

Some teachers complained about the noise level due to participants chatting at the post-test, which was conducted by the board personnel. A more formal environment might have contributed to the results. As well, ten of the original sample of 40 teachers were absent at the post-test, thus reducing the sample size to 30. This issue impacted all the measures. However, as will be seen in the Discussion to follow, other factors may also have been relevant.

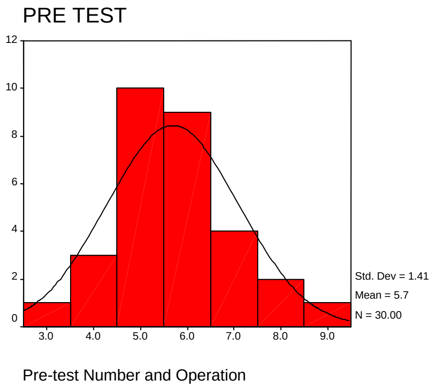
Content Knowledge for Teaching – LMT Instrument

The Learning Mathematics for Teaching Project has developed a number of measures of teachers' content knowledge. For the current study, the newly released Middle School Form A was used. This form yielded scores in three separate strands of mathematics, namely number and operations, algebra, and geometry. It should be noted that the Nelson training in the treatment consisted exclusively of the Number and Operations strand.

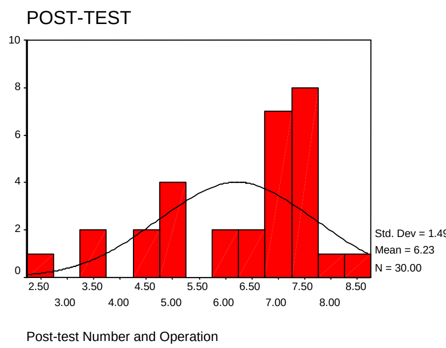
Descriptive statistics, as shown in Table 3, indicate that the mean of Number and Operation score for the pre-test (M=5.7, SD=1.41) increased at the post-test (M=6.2, SD=1.48).

(Note: These scores have all been rescaled to be shown out of 10, for the sake of consistency with the POM instrument and ease of comparison). The shape of the distribution for the Number and Operations post-test looks more negatively skewed than the pre-test as shown in Figures 6 and 7.

<u>Figure 6</u>. Number and Operations Pre-test



<u>Figure 7</u>. Number and Operations Post-test



<u>Table 3.</u>
Descriptive Statistics: Learning Math for Teaching

	N	Minimum	Maximum	Mean	Std. Deviation
Number and Operation post-test	30	2.3	8.6	6.227	1.4869
Algebra post-test	30	3.5	9.4	6.710	1.4007
Geometry post-test	30	4.7	10.0	7.533	1.7433
Number and Operation pre-	30	3.2	9.1	5.695	1.4147
Algebra pre-test	30	3.2	8.7	6.548	1.4529
Geometry pre-test	30	4.7	10.0	7.333	1.7420
Valid N (listwise)	30				

In order to assess if there were significant differences in Number and Operation scores between the pre and post tests, a repeated measures t-test was performed as shown in Table 4. The repeated measures t-test suggests that there is a significant improvement in Number and Operations mathematics knowledge as a result of the intervention, t (29) =2.28, $p < .05$. However, there were not significant differences between the pre and post test for scores in Algebra, t (29) =0.97, ns and Geometry, t (29) =0.83, ns. Since changes were only measurable in the strand in which full group training was provided, it seems likely that the change was in fact due to the treatment, which does appear to have had a significant effect of teachers' knowledge of concepts of number and operations.

Table 4.
Repeated Measures t-test - Learning Math for Teaching

Paired Samples Test

		Paired Differences					t	df	Sig. (2-taile)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviati	Std. Erro Mean	Lower	Upper			
Pair 1	Post-test Number and Operations Pre-test number and Operations	.533	1.2762	.2330	.056	1.009	2.286	29	.030
Pair 2	Post-test Algebra Pre-test Algebra	.161	.9114	.1664	-.179	.502	.969	29	.340
Pair 3	Post-test Geometry Pre-test Geometry	.200	1.3235	.2416	-.294	.694	.828	29	.415

Summary of Knowledge Changes

Although slight shifts were seen in the descriptive statistics for the Procedural and Conceptual Knowledge variables, the best measure of change in mathematics knowledge for the full group of teachers was the Number and Operations strand of the Learning Mathematics for Teaching measure, which did show significant growth. The fact that the other two strands (Algebra and Geometry) did not show significant growth validates the change being from the PRIME treatment, in which all formal training was based on Number and Operations. It should be noted that the POM instrument used to assess Procedural and Conceptual Knowledge focused on areas of number and operations as well as algebra and geometry. The strand specificity of the training may explain why no significant overall change was measurable using the POM instrument, just as change was not noticeable in the other strand measures of LMT. Further training in the strands of algebra and geometry may have produced measurable change in these areas as well. The LMT measure seems to have correctly pinpointed the strand in which treatment occurred and indicates that significant positive change took place.

Beliefs Related to Mathematics Teaching and Learning

Two measures were also used to examine beliefs. These were the values portion of the POM instrument, and the Nelson beliefs survey. Both are shown in the Appendix.

Perceptions of Mathematics (POM) Instrument

The POM instrument produces scores on Procedural Values (teacher's belief in the importance of students learning accurate methods to efficiently generate answers) and Conceptual Values (teacher's belief in the importance of students really understanding why the methods work, and learning to explain and connect ideas). Reliability of the survey questions using Cronbach's Alpha was established at the pre-test at r = .70 for Procedural Values and r = .78 for Conceptual Values.

Descriptive statistics as shown in Table 5 indicate that the mean of Conceptual Values for the pre-test (M=8.4, SD=0.84) increased by the post-test (M=8.7, SD=0.99). The shape of the distribution for the Conceptual Value post-test looks more negatively skewed than the pre-test as depicted in Figures 8 and 9.

Table 5.
Conceptual and Procedural Values

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Post-test PV	30	1.8	7.6	4.163	1.5558
Post-test Cv	30	5.8	9.8	8.742	.9938
Pre-test PV	30	3.3	8.0	5.003	1.1725
Pre-test CV	30	6.2	9.8	8.383	.8420
Valid N (listwise)	30				

Fig 8. Pre-test Conceptual Values

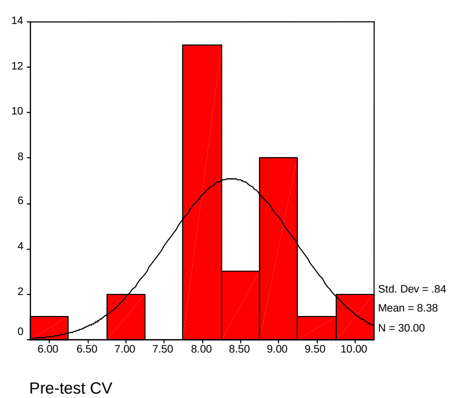
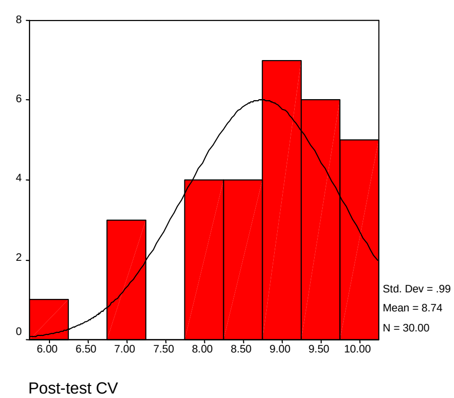


Fig 9. Post-test Conceptual Values



While Conceptual Values rose significantly, Procedural Values decreased. Descriptive statistics as shown in Table 5 indicate that the mean of Procedural Values for the pre-test (M=5.0, SD=1.17) is higher than the mean of Procedural Values for the post-test (M=4.16, SD=1.55). The shape of the distribution for the Procedural Value pre-test looks more positively skewed than the post-test as depicted in Figures 10 and 11.

Fig 10. **Pre-test Procedural Values**

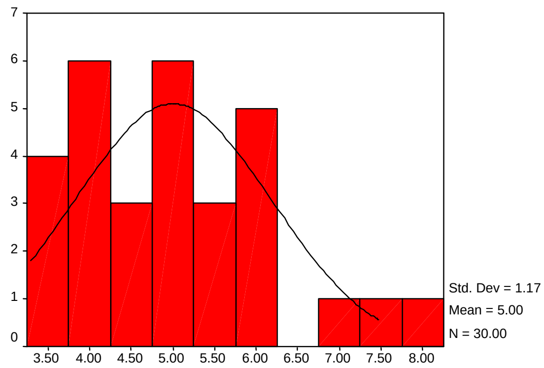
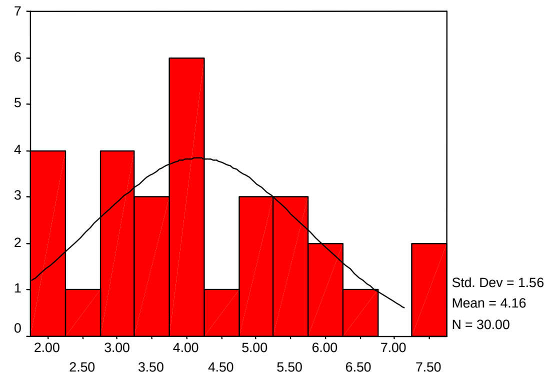


Fig 11. **Post-test Procedural Values**



In order to determine if there were significant differences in Conceptual and Procedural Values between the pre and post-tests, a repeated measures t-test was performed as shown in Table 6. The repeated measures t-test suggests that there is a significant increase in Conceptual Values after the intervention, t (29) =3.018, $p < .05$ The repeated measures t-test also suggests that there is a significant effect in Procedural Values. The effect shows a decrease in Procedural Values, t (29) = -2.75, $p < .05$

Table 6.
Repeated Measures t-test - Conceptual and Procedural Values

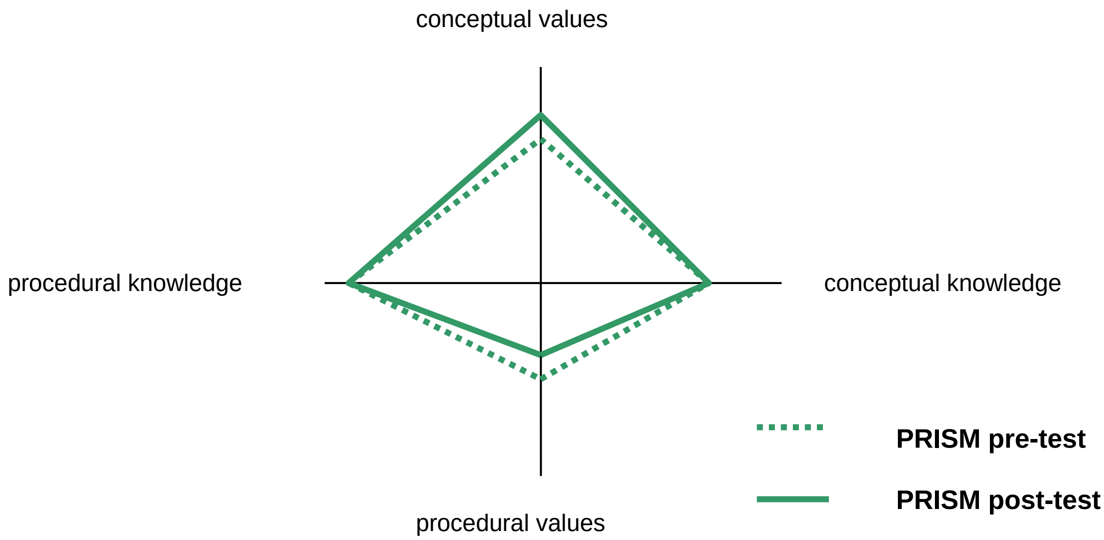
Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviati	Std. Errc Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	Post-test PV Pre-test PV	-.841	1.6696	.3048	-1.464	-.217	-2.758	29	.010
Pair 2	Post-test CV Pre-test CV	.359	.6507	.1188	.116	.602	3.018	29	.005

In summary, both beliefs variables in the POM instrument showed significant change. Conceptual Values rose significantly, while Procedural Values dropped significantly. Both of these changes could be indicative of a shift towards a more reform oriented conception of mathematics teaching and learning, as described in the *Standards* (NCTM, 2000) and elsewhere. These changes show that this group of teachers may have moved further towards a conception of mathematics that included wanting their students to deeply understand mathematical ideas in order to be able to explain and model mathematical concepts, as well as to be able to make connections. These teachers appear to be becoming less concerned with students being able to accurately and efficiently follow set procedures to generate correct answers.

Figure 10 shows the *Profile* graph shown previously (cf. Figure 1 with results for pre-service teachers) with pre-test and post-test results for the POM instrument with the current group of teachers. Comparing with Figure 1, these teachers are seen to have both deeper knowledge and deeper conceptual values than the pre-service teachers which are being studied in the companion CRYSTAL research project also at Lakehead University. However, it should be noted that a slightly different version of the POM instrument was used in the two studies, so statistical comparison is not possible. Nevertheless it seems likely that the inservice teachers in the current study have moved along a trajectory towards a reform oriented conception of mathematics teaching and learning.

Figure 12. The POM Profile for PRISM Teachers.



Nelson Beliefs Survey

A relatively short survey of 20 Likert style questions is used to generate the Nelson "Ten Dimensions" of Teacher Performance. Some dimensions, such as "Use of Manipulatives and Technology" have as few as two survey questions used to generate the score, so reliability was not investigated by us for this instrument.

The Nelson survey produces scores on ten variables related to beliefs. Initial analysis at the pre-test indicated that the variables of Manipulative and Technology (Dimension 7) and Attitude and Comfort with Mathematics (Dimension 10) were the only ones that seemed related to the other measures used at the pre-test. Hence only these two variables were examined at the post-test. Descriptive statistics as shown in Table 7 indicate that the mean of Manipulatives and Technology for the pre-test (M=5.16, SD=0.77) increased slightly by the post-test (M=5.21, SD=1.06). The mean of Attitude and Comfort for the pre-test (M=4.78, SD=0.60) also increased slightly by the post-test (M=5.02, SD=1.06).

Table 7.
Descriptive Statistics - Nelson Beliefs (Manipulatives and Attitude)

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Post-test Manipulatives	30	1.00	6.00	5.2167	1.06418
Post-test Attitude	30	2.20	6.00	5.0267	.76426
Pre-test Attitude	30	3.60	6.00	4.7867	.60100
Pre-test Manipulatives	30	3.50	6.00	5.1667	.76939
Valid N (listwise)	30				

In order to assess if there were significant differences in these variables between the pre and post tests, repeated measures t-tests were performed as shown in Table 8. The repeated measures t-test suggest that there are not significant differences between the pre- and post-test for Manipulatives and Attitude as a result of the intervention, t (29) =0.23, ns and t (29) =1.45, ns.

Table 8.
Repeated Measures t-test - Nelson Survey

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed
					95% Confidence Interval of the Difference				
		Mean	Std. Deviatio	Std. Error Mean	Lower	Upper			
Pair 1	Post-test Manipulatives - Pre-test Manipulatives	.0500	1.16966	.21355	-.3868	.4868	.234	29	.817
Pair 2	Post-test Attitude - Pre-test Attitude	.2400	.86367	.15768	-.0825	.5625	1.522	29	.139

Summary of Beliefs Results

The Perceptions of Math (POM) instrument was effective in demonstrating overall changes in beliefs. Conceptual Values rose significantly, while Procedural Values decreased significantly. This may be seen as a shift towards a more reform based conception of mathematics teaching and learning.

The Nelson survey dimensions examined showed slight positive changes, but these were not significant. A larger sample or a treatment which was focused more on pedagogical ideas might have produced clearer results, but it did not contribute to results in the current study.

Summary of Teacher Change and Usefulness of Instruments

Teachers in the study showed an increase in their understanding of Number and Operations concepts which were likely as a result of the study Number and Operations training. Since this is the strand that was involved in the PRIME training, it can be concluded that this training was effective. Algebra and Geometry PRIME training was not provided to the whole group, which might explain why neither of these LMT scales showed significant increase, nor did the POM knowledge variables which are based on all of number and operations, algebra and geometry. Using the strand specific LMT form seems the best way to assess strand specific training and show positive change. These teachers changed significantly but only in the strand addressed by the training received by the whole group.

A shift in beliefs towards valuing conceptual learning more, and valuing procedural learning less, was shown by the POM instrument. Thus it is likely that the teachers continued to move towards valuing reform based practices more highly. The Nelson Dimensions studied did not show significant change, but descriptive statistics showed slight positive increases.

Relationship Among Variables and Instruments

A significant negative correlation at the 0.01 level (2-tailed) existed between post-test Procedural Values and post-test Conceptual Values (see Table 9). This indicates that teachers who tended to value procedural learning more (often associated more with traditional mathematics classrooms) tended to value conceptual learning less. Conceptual learning may be thought of as more typical of values in a reform based classroom, where deep understanding and problem solving are often emphasized more than procedural fluency.

Table 9.
Procedural and Conceptual Values.

Instruments	Correlation	Post-test PV	Post-test CV
Post-test PV	Pearson Correlation Sig. (2-tailed)	1	-.510** .004
Post-test CV	Pearson Correlation Sig. (2-tailed)	-.510** .004	1

** Correlation is significant at the 0.01 level (2-tailed).

Procedural Values showed no relationship with any of the three LMT subscales at the post-test. This implies that the degree to which a teacher values procedural fluency in their classroom is not related to their own understanding of the mathematics. However, Conceptual Values were significantly related to all three LMT knowledge strands at the post-test, and highly significantly related to Number and Operation, as shown in Table 10. This result might imply that teachers who had achieved deeper mathematical knowledge themselves also valued deep understanding for their students more.

Table 10.
Relationship of Conceptual Values and Strand-Specific Knowledge

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Post-test LMT Number Operation & Post-test CV	30	.620	.000
Pair 2	Post-test LMT Algebra & Post-test CV	30	.603	.000
Pair 3	Post-test LMT Geometry & Post-test CV	30	.536	.002

Relationships were also evident between the POM knowledge measures and the LMT scores. Procedural Knowledge as well as Conceptual Knowledge were both highly significantly related to all three strand-specific LMT measures at the post-test, as shown in Table 11. This result serves to strengthen validity of the POM Knowledge scales.

Table 11.
Relationship of Post-test Knowledge Variables

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Post-test LMT Number Operation & Post-test PK	30	.718	.000
Pair 2	Post-test LMT Algebra & Post-test PK	30	.753	.000
Pair 3	Post-test LMT Geometry & Post-test PK	30	.624	.000
Pair 4	Post-test LMT Number Operation & Post-test CK	30	.802	.000
Pair 5	Post-test LMT Algebra & Post-test CK	30	.863	.000
Pair 6	Post-test LMT Geometry & Post-test CK	30	.623	.000

More detailed examination of individual items on the LMT measures indicates a need for a certain degree of procedural fluency in many items, as well as a more conceptual understanding in others. The authors themselves refer to types of questions on the LMT instrument that require "common knowledge of content" and "specialized knowledge of content" (Hill et al, 2005, p.16). Such understandings are described as needs for teachers because they "must compute, make correct mathematical statements, and solve problems" as well as "building or examining alternative representations, providing explanations, and evaluating alternate student methods" (Ibid.). The above notions seem highly aligned with the concepts of procedural knowledge and conceptual understanding (Ball, 2005). Hill et al suggest that:

> " …common and specialized mathematical knowledge are related yet not completely equivalent; the possibility exists that individuals might have well-developed common knowledge yet lack specific kinds of knowledge needed to teach." (2004, p.24).

The pre-service teachers described earlier would certainly be examples of this phenomenon; their Conceptual Knowledge scores were very low compared to their Procedural Knowledge scores at the beginning of their methods courses, and this result was consistent over two years (See Figure 1 previously). The in-service teachers in the current study however, demonstrated much deeper conceptual understanding relative to their procedural understanding at the time of this study. It is possible (and indeed something we would hope to be possible) that the act of caring and reflective teaching would contribute to deepening such knowledge, and this study provides preliminary evidence for this conjecture. Without question, teaching mathematics requires more and different understanding than just a "good" mathematical background. Hence the POM instrument may have potential for teasing apart these ideas for further study.

In order to further explore the degree of linear relationships among the dependent variables measured in the study by the three instruments, the change between the pre and post test was computed for each dependent variable. A Pearson Product Moment correlation matrix was created to identify the relationship between dependent variables at .05 level of significance (2 tailed). The results also revealed a significant positive correlation between change in Conceptual Knowledge and change in Procedural Knowledge at r = 0.417 as shown in Table 12.

<u>Table 12.</u>
Correlations to Show the Relationship among the Dependent Variables

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
CHANGENO	Equal variances assumed	.348	.560	.654	28	.519	.3192	.4884	-.6812	1.3195
	Equal variances not assumed			.680	23.597	.503	.3192	.4694	-.6505	1.2889
CHANGALG	Equal variances assumed	8.046	.008	-2.214	28	.035	-.7177	.3242	-1.3818	-5.36E-02
	Equal variances not assumed			-1.892	13.317	.080	-.7177	.3794	-1.5354	9.998E-02
CHANGGEO	Equal variances assumed	.635	.432	-1.091	28	.284	-.5455	.4998	-1.5692	.4783
	Equal variances not assumed			-.961	14.405	.353	-.5455	.5678	-1.7600	.6691

Significant negative correlation between change in Conceptual Values and change in LMT Operations was r = -0.432. Thus the teachers that changed the most in terms of Conceptual Values (CV) were not the ones who increased their knowledge the most in the training as per LMT. A number of explanations are possible for this result. It is possible that teachers whose CV was already relatively high were the ones who were more motivated and hence benefited most from the training, and thus their LMT knowledge changed more. Alternately, teachers whose knowledge was already high (and hence changed little) may have been most influenced by the training in terms of beliefs about teaching and learning. Further study will be needed to examine this delicate relationship in more detail, and determine if one aspect is indeed a prerequisite to the other.

Positive significant correlation between change in Nelson Manipulatives and change in Nelson Attitude r = 0.524. Positive significant correlation between Nelson Manipulatives and change in LMT Operations r = 0.513. Positive significant correlation between change in Nelson Attitude and change in LMT Operation r = 0.368. Hence the teachers that increased their knowledge of Operations more also changed more in their beliefs about manipulatives, technology and their own comfort with mathematics. These were not however the teachers whose Conceptual Values increased the most, and it was Conceptual Values (rather than the Nelson measures) that showed significant change in the group of teachers taken as a whole. Again, further study is needed to examine these relationships further. Examination of the actual beliefs measures (see Appendix) does indicate that they are measuring different aspects of beliefs. The Nelson measures are more related to teachers' beliefs about teaching methods; the POM measures are more about fundamental conceptions of mathematics itself. It could be conjectured that fundamental change in beliefs about mathematics *itself* must precede changes in beliefs about its teaching. If such a conjecture proves valid, then the current study has created significant effect on teachers' fundamental values and beliefs about mathematics, and perhaps the slight (but not significant) changes seen on the Nelson instrument are indicative of a ripe potential for change in actual classroom teaching behavior. Follow up study of these teachers in the companion CRYSTAL project would be most interesting on this point and the ideas mentioned previously as requiring further investigation.

Comparison of Cohorts and Treatments

Comparing Local and Regional Teachers

Lakehead Public Schools has invested significantly in teacher professional development particularly at the elementary level in recent years. It has provided workshops, for example, in dynamic geometry software, as well as Destination Math software which was purchased to support the work of intermediate teachers. A Mathematics Sub-committee has produced various documents to support teachers, such as a grade 7 to 10 continuum of mathematics expectations. Current work relates to the grade 8 to 9 transition, and work is in place in the companion CRYSTAL project to develop a diagnostic to assist teaches and parents. As well, the Board has consistently funded teachers to attend regional conferences for teachers of mathematics such as the NOAME (Northwestern Association of Mathematics Educators) conferences.

Providing professional development to regional teachers however is not so straightforward. Sending teachers from the region to events such as the NWOAME conference require at least one overnight stay, as well as at least one travel day as many are driving four to five hours each way. These factors cause the cost of sending a teacher to such an event to be double or more likely triple. Obviously if boards are funded for professional development on a per-teacher basis, this distance factor means that only half to one third of the teachers from the region can attend such events compared to more centrally located teachers. Other workshops and training opportunities also require travel to Thunder Bay, or possibly even Southern Ontario. The PRIME training for the current project, for example, required regional teachers to travel to Thunder Bay and stay overnight each time. These difficulties may also have contributed to the fallout from pre-test to post-test in the current project.

Given our geography, it is not a surprise that local teachers scored at least somewhat higher on all pre-test knowledge instruments (all three LMT subscales as well as both PK and CK of POM) used in the study, and showed a more reform-based belief system (lower PV and higher CV as well as higher Nelson scores related to Use of Manipulative and Attitude and Comfort with Math) than regional teachers at the pre-test, as shown in Table 13. While data was not collected based on years of teaching experience (it was felt that given the relatively small numbers, teachers might feel such information would make then identifiable), looking around the room on the training days for both groups A and B (local and regional teachers respectively) showed no obvious differences in the age mixes of the groups. It seems reasonable to conclude that local teachers have simply had more professional development opportunities prior to the start of the study, and that these opportunities do make a difference.

Table 13:
Pre-test Mean Scores by Sub-group

Measure	Local (A)	Regional (B)
Numb & Op	5.837	5.448
Algebra	6.570	6.510
Geometry	7.439	7.152
PK	9.298	8.485
CK	7.256	6.429
PV	4.953	5.091
CV	8.463	8.245
Manips	5.211	5.091
Attitude	4.832	4.709

(All scores are shown out of 10).

Local teachers remained at least somewhat higher in knowledge and more reform-oriented in beliefs that teachers from the region according to all strands of the LMT measure and the POM measure at the post-test as shown in Table 14. As discussed before, the Nelson results did not show significant change. Mean scores improved overall by approximately the same amount for the significant measures of the study, namely LMT Operations and POM CV and PV (for which a drop is considered an improvement), for both groups. Hence while improvement is shown for both groups, it is relative to participants' pre-test scores.

Table 14:
Post-test Mean Scores by Sub-group

Measure	Local (A)	Regional (B)
Numb & Op	6.388	5.950
Algebra	6.893	6.393
Geometry	7.930	6.848
PK	8.713	8.283
CK	7.065	6.883
PV	3.625	5.091
CV	8.856	8.545
Manips	5.263	5.136
Attitude	5.021	5.063

(All scores are shown out of 10).

Hence we have provided evidence that incoming knowledge and beliefs may be important to consider in predicting the outcomes of any professional development programs. Training is a developmental process, rather than a "quick fix" intended to bring everyone up to the same level by the end. The outcomes seem to depend on participants' initial capacities, and the process is developmental.

Effects of Varying Treatment: The AQ Courses

Anecdotal evidence was strong in support of the professional development opportunities offered. For example, one participant noted that her practice in the classroom had been positively influenced:

> "I think that I did gain many good things from the PRISM project. I think that the AQ course was my favourite because it allowed the most time to devote to thinking about math. I also was able to have lengthy conversations and work collaboratively with a wider group of people. What do I need to improve my practice with students at risk with math? I feel that I do an adequate job supporting them in terms of simplifying the concepts or addressing the essential outcomes. I try to run a parallel program with modifications where they are supported by either an educational assistant, or a model or explicit examples. I feel that I need to continue to refine those skills. I know that when I am not running a parallel program I am not as successful. For those students, if they do not have an adult to work with them it can look a lot like back-line masters time. They really need someone to work with since often they do not have the reading skills to work independently."

Pre-test measures indicated that the teachers choosing to take one or two AQ courses as part of this study were slightly weaker initially on the instruments used in the study. This made significant positive outcomes harder to identify due to this treatment. However, when *change* of the group taking one or two AQ courses was compared with change for the non AQ group, some differences did emerge. The analysis shows that the change in Algebra knowledge as measured by LMT was significantly greater for the AQ group than the non AQ group (p<.05) as seen in Table 15.

Table 15.
Change in Knowledge Due to AQ Courses

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
CHANGENO	Equal variances assumed	.348	.560	.654	28	.519	.3192	.4884	-.6812	1.3195
	Equal variances not assumed			.680	23.597	.503	.3192	.4694	-.6505	1.2889
CHANGALG	Equal variances assumed	8.046	.008	-2.214	28	.035	-.7177	.3242	-1.3818	-5.36E-02
	Equal variances not assumed			-1.892	13.317	.080	-.7177	.3794	-1.5354	9.998E-02
CHANGGEO	Equal variances assumed	.635	.432	-1.091	28	.284	-.5455	.4998	-1.5692	.4783
	Equal variances not assumed			-.961	14.405	.353	-.5455	.5678	-1.7600	.6691

While a larger change in CV is not seen for the AQ group (significant change was seen overall in CV as was previously discussed), participants in the AQ courses did change (drop) significantly more in PV as is seen in Table 16, (p<.05). In other words, the AQ courses helped teachers "let go" more of the need to insist on procedural fluency as being highly important for students.

Table 16.
Change in Procedural Values Due to AQ Courses

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
CHANGECV	Equal variances assumed	2.628	.116	-.473	28	.640	-.1181	.2499	-.6300	.3938
	Equal variances not assumed			-.533	27.732	.598	-.1181	.2215	-.5721	.3359
CHANGEPV	Equal variances assumed	3.979	.056	3.430	28	.002	1.8527	.5402	.7462	2.9593
	Equal variances not assumed			3.018	14.390	.009	1.8527	.6139	.5394	3.1661

The aim of the AQ courses was to increase teacher content knowledge of mathematics through practical classroom applications. Content focus included all strands, but with particular emphasis on number, patterning and algebra, and geometry. It appears that these courses have been successful both in content knowledge support as well as in giving teachers permission to “let go” of more traditional values.

Conclusions and Recommendations

This study provides evidence that significant changes in both knowledge of mathematics for teaching, and beliefs about mathematics itself and how it should be understood, are possible in even a relatively short (eight month) time period which included strand specific mathematics professional development. This is an important result for those in a position to be making decisions about professional development funding opportunities for teachers.

It should be noted that the teachers in this study volunteered to participate, with no incentive other than receiving the provided training. Certainly being absent from their classrooms for the training and data collection days of the study would have contributed to their overall workload, hence it can be concluded that there was significant buy-in on the part of the teachers in the current study. This may have been an important factor in terms of the success of the study, and school boards wishing to provide a similar experience for teachers should be aware of the factor of participant support which Balfanz et al (2006) cite as an important feature for success. Preliminary evidence from the companion CRYSTAL project of which Group A teachers in the current study were a subset, indicates that the teachers involved in the PRISM study may have been a relatively strong sample of teachers even at the pre-test. Since our evidence also shows that growth is relative, it must be understood that initial participant capacity is a predictor of resultant capacity. Teachers will improve, but relative to their initial levels of understanding and beliefs.

While the treatment did include extensive in-service training grounded in mathematical understanding as recommended by Hill and Ball (2004), classroom coaching as recommended by Balfanz et al (2006) did not form part of the model nor was whole-school reform involved. It is possible that inclusion of these aspects would have contributed to even stronger results. This also may partly explain why changes were seen in mathematical knowledge for teaching, as well as in beliefs about mathematics itself, but that changes were not significantly apparent in more pedagogically related beliefs scales.

The instruments chosen in the study were never-the-less successful at documenting significant change. The strand-specific Learning Mathematics for Teaching instrument and the beliefs portion of the Perception of Mathematics measure were particularly useful in the current study. Since the Learning Mathematics for Teaching measures have been broadly researched and tested, the current study also provides validation for the locally developed Perceptions of Math instrument. It can be concluded that the training was most successful at deepening content specific mathematics understanding, as well as values about mathematics itself and ways it might be understood. Pedagogically-related beliefs according to the Nelson survey were less effected, and may require other interventions such as classroom-based coaching or a whole school reform model to be more significantly influenced.

Teachers living in a larger center (Thunder Bay) began the study with slightly deeper knowledge and more reform-oriented understandings of mathematics. This was not entirely unexpected; this group of teachers has had more opportunity for professional development in general prior to the study, and this study provides evidence that such opportunities do matter.

Investigation of development due to the AQ courses was less clear, partly because the teachers taking these courses were slightly weaker initially as a group. However, improvement was seen in algebra knowledge in particular. As well, the AQ courses contributed to teachers’ valuing procedural learning less; in other words they became more willing to down-grade the importance of procedural learning after the experiences in the AQ courses.

Classroom mathematics teaching is a complex process requiring many skills. Teachers make many day to day classroom decisions that have significant effects on all students, and especially students at risk. Teachers must decide what the most important ideas in the curriculum are and how to manage the available time to bring these to the fore. Possibly they make the decision to follow a textbook, or less often they decide to create their own activities or choose them from other sources. Teachers choose the classroom tasks, inherently influencing the kind of mathematical thinking that will be emphasized. In some classrooms, tasks might be hands-on and exploratory in nature, with students being aware that their thinking and problem solving, and abilities to discuss and defend these, are valued in this classroom. In other classrooms, emphasis might be on basic skills, proficiency and accuracy, with practice at following provided methods seen as the best way for students to succeed. Sometimes manipulatives are seen as objects to get started with, but that students should learn to operate eventually without. Sometimes they are seen as fundamental mathematical objects that students are encouraged to reason with at all times, even on assessment activities. Assessment itself may promote different conceptions of mathematics; for example, do any provided problem solving activities simply lead to a final test in which procedural proficiency is the primary source of evaluation, or are other rich problem-solving tasks to be used for assessment? Such decisions push all teachers to deeply contemplate and enact their own values about and understandings of mathematics and the aspects they feel are most crucial. All teachers make every effort to behave in the best interest of their students. This research provides evidence that teachers can significantly deepen their own mathematical understanding and shift their beliefs about mathematics itself towards a more reform-oriented conception based on professional development, and makes it clear that treatment for all teachers, in all strands of the elementary mathematics curriculum, is a necessary and crucial first step towards effecting mathematics education reform at the classroom level.

We also believe strongly that the current research must inform teacher training practices, and that specific course work based on developing conceptual understanding of fundamental mathematical ideas must be included in initial teacher training programs in addition to current methods courses. This would most likely have to be done in a special mathematical content course for teaching taken prior to the teacher training year, if a one year format is to be retained in Ontario. This agrees with the recommendations of the Expert Panel of Student Success on Ontario, which include the following:

> "It is time for a comprehensive review of the minimum mathematics requirements for a junior-intermediate teaching certificate. Faculties of education should provide leadership by prescribing minimum requirements that respond to the needs of today's students, including those at risk. The faculties of education and mathematics departments of all Ontario universities should then collaborate to develop strategies for ensuring that these minimum requirements are being met." (2004, p. 79).

It is unlikely that such recommendations will actually come to fruition without support from both the Ministry of Education and the Ontario College of Teachers.

In conclusion, evidence has been provided that the short term professional development opportunities offered in the current study were relatively successful. However, this study also provides evidence that increased knowledge in one strand of mathematics does not immediately transfer to other areas of mathematics in a measurable way, and hence training needs to be offered across all mathematics strands, as well as more widely available to all teachers. A larger scale implementation of the current model applied more broadly would clearly be helpful in terms of building on teacher capacity to provide quality teaching to all students. Further research may be required to make specific recommendations about what is required for the translation to actual reform based classroom teaching practice, which is also a crucial factor for students at risk in intermediate mathematics. Such investments are important if the goal of widespread mathematics education reform is to be attained in Ontario.

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Tables and Figures

Tables

Figures

Appendix: Perceptions of Math (POM) Instrument

Code: ____________________　　　　　Date: ________.

"Perceptions of Mathematics" (POM) Survey
Procedural and Conceptual Knowledge and Values Scale for Mathematics

<u>Values Questions</u>
Please answer the questions by circling the response, where 0 is low or poor or disagree, and 3 is high or positive or agree. Please do not add other responses such as "not sure" – choose the closest response to your feeling.

Getting the correct answer is a very important part of mathematics learning.	0	1	2	3
It is important to me to really understand how and why math procedures work.	0	1	2	3
It is important for everyone to be able to accurately do basic math calculations such as addition or multiplication, without a calculator.	0	1	2	3
Everyone needs to deeply understand how and why math procedures work if they are going to make effective use of them.	0	1	2	3
It is important to be able to recall math facts such as addition facts or times tables quickly and accurately.	0	1	2	3
There are often several correct ways to express the solution to a math question.	0	1	2	3
It is the teacher's job to teach each new math method to the students before they have to use it.	0	1	2	3
There are often several correct ways to get an answer.	0	1	2	3
It is important to practice on many familiar shorter math questions in school than tackle a bigger problem.	0	1	2	3
It enriches student understanding to have to think about different ways to solve the same problem.	0	1	2	3
Usually there is only one right or best method to get an answer.	0	1	2	3
Children learn better by exploring problems in which they are not taught the method in advance.	0	1	2	3
I learn math best if someone directly teaches me the methods.	0	1	2	3
It is important to have to think through a variety of math methods and strategies.	0	1	2	3
There is usually one best way to write the solution to a math question.	0	1	2	3
I learn math best if I can explore and investigate on my own or in small groups to develop understanding.	0	1	2	3
Most people learn math best if they are taught the methods directly.	0	1	2	3

Most people learn math best if they explore individually or in small groups to develop understanding of the ideas.	0	1	2	3
When I'm learning math I just want to know "how to do it".	0	1	2	3
When I'm learning math I really want to know "how" and "why" the methods and ideas work.	0	1	2	3
It is important to regularly practice math facts such as times tables to develop speed and accuracy.	0	1	2	3
It is important to develop connections between ideas by working on multi step problems.	0	1	2	3
Mathematical fluency and calculation skills are highly important in mathematics.	0	1	2	3
Mental math is enhanced by a deep understanding of how numbers are composed.	0	1	2	3
It is important to encourage children to practice precise calculations such as those with percentages, fractions and long division to develop proficiency without a calculator.	0	1	2	3
Calculators are important as an investigation and learning tool.	0	1	2	3
Mental math is enhanced by repeatedly practicing set methods such as addition with regrouping to achieve speed and fluency.	0	1	2	3
Problem solving skills in mathematics are highly important to develop.	0	1	2	3
Calculators shouldn't be used too much because they can lessen opportunities to practice computational skills.	0	1	2	3
Estimation skills to judge the reasonableness of answers are highly important.	0	1	2	3

2. Mathematics Questions

PART A:

Answer the questions showing your steps as needed to illustrate the method you used.

1. 1.6 x 3

2. 5 – (-3)

3. 1 ¾ ÷ ½

PART B

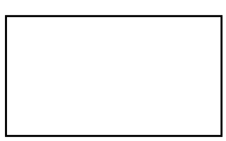
4. Find and state the pattern rule that relates n and the result.

n	result
1	4
2	9
3	14
4	19

5. For the rectangle below, calculate

a) the perimeter

b) the area



3 cm

5 cm

6 Change 7 020 mm to more appropriate units.

7. Expand (x + 2)(x + 4) and simplify.

PART C

8. The calculation for 3 x 1.6 could also correctly be done by calculating (circle all that are mathematically correct)

   a) 1 x 3 + 0.6 x 3

   b) 3 x (8/5)

   c) 0.3 x 16

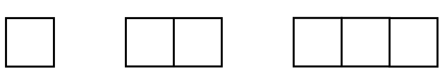
   d) 3 x (16/10)

9 Give a concrete example or model or suitable explanation to illustrate the operation 5 – (-3).

10. Circle any of the following that are examples of 1 ¾ ÷ ½ (circle all that are appropriate)

   a) There are 1 ¾ pizzas left. If these are to be shared equally by 2 children, how much pizza does each child get?
   b) There is 1 ¾ m of string on a spool. If you use half of it to tie a parcel, how many meters of string did you use?
   c) There is 1 ¾ m of string on a spool. If packages take ½ m of string each to tie up, how many packages can you tie up?
   d) I don't know if any are correct.

11. For the number of lines or "toothpicks" in the pattern below, where n is the number of squares, child A writes 1 + 3n and child B writes 4 + 3(n-1).



Who is correct? (circle)
   a) child A
   b) child B
   c) both child A and B are correct
   d) I'm not sure if either are correct

12. A student solves the following subtraction problem as follows.

```
 341
-215
----
  -4
  30
 100
 126
```

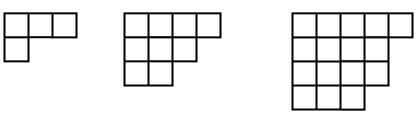
Will this method work in general?

a) yes
b) no
c) only in certain restricted cases
d) I'm not sure

13. Draw a diagram (for example with algebra tiles) to illustrate why (x + 2)(x + 3) is equivalent to x^2 + 5x + 6.

14. A 10 cm x 10 cm x 10 cm cube (such as the large 'thousands' cube in a set of base 10 blocks) is used to represent the number 1. Based on this understanding of 1, draw a diagram with base 10 blocks showing the number 0.012

15. State 3 different forms of an algebraic pattern rule for the diagrams below, using n as the frame number, which you would mark as correct if they were submitted by a student. (The diagrams show the first 3 terms of the pattern).



1 2 3

a)
b)
c)

16. Is it true that as the perimeter of a rectangle increases, so does the area? Explain.

[For scoring instructions contact ann.kajander@lakeheadu.ca]