Leading MATH Success

TIPS

for English Language Learners in Mathematics

Grades 7, 8, 9 Applied, 10 Applied

Grade 9 Applied

Unit 1: Measurement: 2-D and 3-D

Grade 9 Applied

Lesson Outline

BIG PICTURE:

English language learners will:

- create their personal word study notebook;
- begin to work productively in flexible student groupings.

Day	Lesson Title	Language Goals	Expectations
1	Fill It Up	• Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO) • Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO) • Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO) • Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)	MG2.04, NA2.01, NA2.03, NA2.04, NA2.08 CGE 5a
2	A Sweet Problem	• Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO) • Write in a variety of forms with teacher guidance. (ESLBO) • Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO) • Write in a variety of forms. (ESLBO)	MG2.04, MG2.05, NA2.01, NA2.02, NA2.03, NA2.04, NA2.08 CGE 3c
3	Diagnostic to Activate Prior Knowledge of Composite Figures	• Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO) • Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO) • Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO) • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO)	MG2.03 CGE 2c, 5a, 5b

ESLAO – Beginning communication in English builds on students' previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment

ESLBO – English in daily life expands students' essential English communication skills and introduces the language of classroom studies.

Unit 1: Day 1: Fill It Up (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Develop through investigation the formulas for the volume of a pyramid and cone, based on the volume of the corresponding prism or cylinder of the same radius and height.

Materials

- 3-D relational solids
- filler beads or sand
- baggies
- paper plates
- BLM 1.1.1, 1.1.2

Assessment Opportunities

Minds On ...

Small Groups → Exploration

Each group works with one set of solids. Orient students to the 3-D relational solids. Allow for exploration time.

Students establish their own criteria and rationale for sorting the various solids into groups.

If a sufficient number of sets is not available, consider sorting as a whole class and a carousel for Action.

Whole Class → Sharing

Each group shares its strategy.

Use this opportunity to review the terminology related to the geometric shapes.

Action!

Small Groups → Investigation

The groups compare the volumes of prisms (cylinder) and pyramids (cone) with congruent bases and equal heights (BLM 1.1.1).

Learning Skills (Teamwork)/Observation/Checklist: Observe how students interact as they investigate in their groups.

Prepare baggies of sand or beads and paper plates in advance of investigation.

Consolidate Debrief

Whole Class → Journal: Representing

Give the class the following journal prompt: Using words, pictures, numbers, and symbols, describe the relationships you discovered today.

Pairs → Pair/Share

Students complete BLM 1.1.2, alternating as indicated between writing and coaching.

Establish the criteria for journal writing and revisit at frequent intervals.
Refer to *Think Literacy, Mathematics 7–9*, Journal Writing – Developmental Stages, p. 92.

Differentiated Exploration Reflection

Home Activity or Further Classroom Consolidation

Use the 3-D relational sets and record as many paired relationships as you can. For example, the small, triangular prism is half the volume of the small, square-based prism.

The relationship between the cone and the hemisphere is examined in Day 2.

Unit 1: Day 1: Fill It Up

Grade 9 Applied

Terminology

volume
area
base
height
radius
cone
cylinder
pyramid
prism
geosolids
hypothesis

Language Goals

- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Demonstrate adaptations to school norms, key teacher expectations, and classroom routines.

Materials

Assessment Opportunities

Minds On…

Small Groups → Exploration
English-speaking students help English-language learners get involved in the sorting game by using key words and gestures to model the criteria for sorting the manipulatives.

Whole Class → Sharing
As each group shares their strategy, state the terms clearly as you write them on the board with diagrams. Have English language learners record the terms in their personal word study notebook accompanied by a picture of the geometric figures.

Begin a Word Wall for this unit.

Make It Comprehensible
Consider language levels and listening demands for English language learners as well as any cultural difference relating to classroom expectations.

Action!

Small Groups → Investigation
Group English language learners with English-speaking peers so that they can observe the steps involved with investigating the first shape before they repeat the steps to complete the investigation. The English-speaking students talk about what they are doing and learning.

Learning Skills (Collaboration)/Observation Checklist: Observe how students interact as they investigate in their groups.

Incorporate Identity
Build a positive classroom atmosphere by valuing different experiences and perspectives that all students bring.

Make It Language Rich
Encourage English language learners to access bilingual dictionaries and to consult with their group/partner to solve problems before they ask for assistance.

Consolidate Debrief

Whole Class → Journal: Representing
Write the journal prompt on the board connecting verbal, written, and pictorial representations of the same word.

Students put the new terms on the Word Wall.

Pairs → Pair/Share
Partner an English language learner with an English-speaking peer (and optimally, who also speaks their first language) so that they have opportunities to hear and see the terminology during oral and written aspects of the tasks.

Home Activity or Further Classroom Consolidation
English language learners share their work and new learning with parents/guardians in their first language.

Application
Exploration
Reflection

Unit 1: Day 2: A Sweet Problem (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Develop through investigation the formula for volume of a sphere based on the volume of a cylinder/cone.
- Consolidate volumes of prisms, pyramids, cylinders, cones, and spheres.
- Solve problems involving combinations of the figures using metric and imperial measure.

Materials

- 3-D relational solids
- filler beads
- BLM 1.2.1, 1.2.2
- computer/data projector

Assessment Opportunities

Minds On ...

Pairs → Think/Pair/Share/Discussion

Individually, students make hypotheses about the relative volumes of pairs of 3-D solids. In pairs, they compare and consolidate their lists.
Ask:

- Are there any other pairs where one is 3 times the other? 4 times? 2 times? [hemisphere × 3 = cylinder; small, triangular-based prisms × 4 = large, triangular prism; small cylinders × 2 = large cylinder;]
- Is there a series of shapes which all compare? [square and rectangular-based prisms (all 4 doubling each time)]
- Are there any shapes with equal volume? [hemisphere = cone]

Demonstrate selected examples, emphasizing the connection between the hemisphere and cone.

VolumeSphere.ppt.

Action!

Pairs → Investigation

Students complete BLM 1.2.1 using the 3-D relational solids. They read the problem and highlight important information needed to solve the problem.

Reasoning and Proving/Observation/Anecdotal: Listen to pairs discuss to determine their understanding of the relationships being investigated.

Whole Class → Presentation

To develop further understanding of the formula of the volume of a sphere, show the electronic presentation, Developing the Formula for the Volume of a Sphere, which formalizes the relationship between the volume of a sphere and a cone.

Consider using the volume of a sphere as equivalent to two cones until students are ready for the traditional formula.

Consolidate Debrief

Pairs → Practice

Students solve a variety of problems involving the volume of prisms, pyramids, and spheres (BLM 1.2.2).

Application
Differentiated
Exploration

Home Activity or Further Classroom Consolidation

Find pictures of buildings/structures in your community that are prisms or pyramids. Choose one of the pictures, estimate the dimensions, and present a complete solution for the volume of the structure.

This could be a pair or small-group activity.

Unit 1: Day 2: A Sweet Problem

Terminology

sphere
hemisphere
metric measure
imperial measure

Language Goals

- Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO)
- Write in a variety of forms with teacher guidance. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Write in a variety of forms. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Pairs → Think/Pair/Share/Discussion

Use contextual language and 3-D solids to assist English language learners to make a hypothesis about 3-D volumes.

Partner English language learners with peers who speak both English and their first language, if possible, or with a group of three so that they can listen to the other two.

English language learners record the new words in their personal word study notebook. Students put the new words on the Word Wall, including pictures and symbols.

Make It Language Rich
Encourage English language learners to access bilingual dictionaries.

Make Sure They Are Ready
Observe student interaction and involvement and intervene, as necessary.

Action!

Pairs → Investigation

Use the same pairs or groups of three as in Minds On.

Students highlight the important words and information in each problem to focus on essential mathematical terms needed to work through the investigation. Encourage them to refer to the Word Wall.

Model or have students model the problem, using the 3-D solids.

Reasoning and Proving/Observation/Anecdotal: Listen to pairs discuss to determine their understanding of the relationships being investigated.

Whole Class → Presentation

During the electronic presentation, point to key words and diagrams as you are speaking.

Consolidate Debrief

Pairs → Practice

Students work with first-language peers, if possible, so that they can discuss their understanding of the concepts as they work. The pair could also use the 3-D models and reference the Word Wall.

Application
Differentiated
Exploration

Home Activity or Further Classroom Consolidation

Model the Home Activity with exemplars of previous student work or teacher-prepared examples.

Make It Explicit
Post exemplars for students to use as models.

Unit 1: Day 3: Diagnostic to Activate Prior Knowledge of Composite Figures (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Activate students' prior knowledge of terminology related to identifying geometry shapes.
- Determine students' readiness to identify geometric figures in composition, and use appropriate calculations for perimeter and area.

Materials

- BLM 1.3.1, 1.3.2, 1.3.3
- placemat
- overhead projector

Assessment Opportunities

Minds On ...

Whole Class → Orientation

Outline the procedure for the day, including the purposes of each component of the lesson [**Minds On** – activate prior knowledge of shapes, **Action** – review their measurement skills, and **Consolidate** – demonstrate these skills in an activity]. Explain that assessment allows you to plan upcoming lessons according to their current levels of understanding and that the results do not influence their grade in the course.

Small Groups → Pass the Paper

Each group lists the geometric figures in the diagram (BLM 1.3.1). Circulate and provide direction and guidance, as necessary.

Pass the Paper is typically a timed activity, usually no more than 2 minutes.

Students pass one piece of paper around the group, writing a response each time they receive the paper. Responses should include rectangles, squares, triangles, trapezoids, circles (semi-circles) and may include adjectives, as in 'equilateral triangle.'

Learning Skill (Teamwork)/Observation/Mental Note: Observe students as they work. Encourage effective communication by referring to groupwork skills.

Action!

Pairs → Scale Drawings

Students answer the questions on BLM 1.3.2. Circulate to encourage discussion and to clarify information regarding the diagrams.

Whole Class → Discussion

Using an overhead of BLM 1.3.2, lead a discussion in which students share their answers.

Consolidate Debrief

Individual → Calculate Area

Students draw line segments on the composite figure on BLM 1.3.3. Point out that they must be able to calculate the area of each shape created.

Ask students to label their "place" on the placemat for reference.

Individual and Groups → Placemat

Describe the procedure for completing a placemat activity. Students individually respond to the question on BLM 1.3.3 for 5 minutes. The members of each group share their procedures, then complete the centre portion of the placemat, providing a model solution to the question. Designate a scribe for each group. Collect the placemats for assessment.

Curriculum Expectation/Placemat/Checkbric: Circulate and observe students as they complete a solution, recording individual strengths and needs.

The results of individual work allows you to differentiate appropriately for further instruction.

Whole Class → Discussion

Facilitate group sharing.

Application
Differentiated

Home Activity or Further Classroom Consolidation

Choose another example from daily life of a figure that is represented by more than one geometric shape. Include a sketch and estimate the dimensions for the figure used.

Unit 1: Day 3: Diagnostic to Activate Prior Knowledge of Composite Figures

Grade 9 Applied

Terminology

Language Goals

- Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Orientation

Explain the purpose of diagnostic assessments and assure students that they will not be graded. Encourage English language learners to look up the term in their bilingual dictionaries (first language/English).

Small Groups → Pass the Paper

Model the activity so that the steps are clear to the English language learners.

Make Sure They Are Ready
Additional vocabulary may be necessary, e.g., *pass, placemat*.

Action!

Pairs → Scale Drawings

Students highlight the important words and information to help them focus on what the problem is.

Pair English language learners with a first-language or English-speaking peer so that they will have opportunities to discuss the problems and how to arrive at solutions.

Whole Class → Discussion

To provide visual clues for the learners, record the students' responses on the board as they are discussed, connecting the verbal, written, and pictorial representations.

Consolidate Debrief

Individual → Calculate Area

Clearly describe the procedure for Area Challenge (BLM 1.3.3). Circulate, asking questions to help learners sort out what they know and how they can use their knowledge to solve the problem.

Make It Explicit
Post some of the placemats for future reference.

Individual and Groups → Placemat

Model how students are to complete their portion of the placemat. Encourage the use of pictures and words for clarity.

Curriculum Expectation/Placemat/Checkbric: Circulate and observe students as they complete a solution, recording individual strengths and needs.

Application
Differentiated

Home Activity or Further Classroom Consolidation

Explain the Home Activity, using an example of a composite figure found in the classroom.

Unit 4: Proportional Reasoning

Grade 9 Applied

Lesson Outline

BIG PICTURE:

English language learners will:

- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- begin to make short presentations.

Day	Lesson Title	Language Goals	Expectations
1	Ratio Carousel	• Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO) • Write in a variety of forms, with teacher guidance. (ESLBO) • Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO) • Write in a variety of forms. (ESLBO)	NA1.01, NA1.02, NA1.03, NA1.04, NA1.05 CGE 5a, 5e
2	Growing Dilemma	• Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO) • Write in a variety of forms, with teacher guidance. (ESLBO) • Read texts with familiar content or vocabulary, using a variety of reading strategies. (ESLBO) • Write in a variety of forms. (ESLBO)	LR1.03, LR2.02, LR2.03, MG2.02, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA2.02 CGE 3c, 4b, 5a, 5b
3	Pondering Proportions	• Find specific information in straight forward reference materials, with teacher guidance. (ESLAO) • Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO) • Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO) • Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)	NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, MG2.02 CGE 2c, 2d
4	I'd Rather Be Scaling	• Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO) • Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO) • Communicate orally, using accepted word order, common tenses, and other features of English grammar with some accuracy and consistency. (ESLBO) • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO)	NA1.01, NA1.02, NA1.03, NA1.04, NA1.05 CGE 2a, 2c, 5b

ESLAO – Beginning communication in English builds on students' previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment

ESLBO – English in daily life expands students' essential English communication skills and introduces the language of classroom studies.

Unit 4: Day 1: Ratio Carousel (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Investigate ratio as a tool for comparing quantities, both qualitative and quantitative.
- Estimate answers and devise and explain informal solutions (e.g., constant of proportionality, unit rate, equivalent ratios) in a variety of contexts (e.g., numerical, geometric, measurement, probability).

Materials

- 2 computers with GSP®4
- 60 colour tiles of 2 colours
- BLM 4.1.1, 4.1.2

Assessment Opportunities

Minds On ...

Groups of 3 → Graffiti

Use heterogeneous groupings. Prepare chart paper for each of the following terms: *ratio, rate, unit rate, equivalent ratios*. Each group uses a different-coloured marker, cycles through the chart paper stations (2 minutes per chart), and writes characteristics of the term.

Whole Class → Presentation

As a class, summarize the important points for each term students need to know for the carousel activity.

Middle Mania.gsp

When assigning groups, sort them by colour. This activity establishes the class's prior knowledge.

Action!

Groups of 3 → Carousel

Prepare sufficient sets of each of the three stations. Students use a pencil and calculator and record their findings on BLM 4.1.1.

Using the same groups as in the Minds On section, students rotate through the three stations. Direct the groups to move to the next station after 15 minutes.

Learning Skills (Teamwork)/Observation/Checklist: Observe and record students' collaboration skills.

Use different colours to distinguish each station set (one of each type). A group of students will complete one colour set of stations.

Use two sets of cards (BLM 4.1.2) and the GSP®4 file Midpoint Segments for station setup.

Consolidate Debrief

Whole Class → Summarizing

Lead a class discussion using guiding questions (BLM 4.1.1). Using information from the discussion, define *ratio, rate,* and *unit rate,* using examples from the activity.

The student notes should connect the prior knowledge from the graffiti with the knowledge from the carousel.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Find examples of ratio, rate, and unit rate in your environment to post on the bulletin board.

The bulletin board becomes a class portfolio showing that proportions are pervasive. Collect further examples in other lessons in the unit.

Unit 4: Day 1: Ratio Carousel

Grade 9 Applied

Terminology

ratio
rate
unit rate
equivalent ratios
midpoint

Language Goals

- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Write in a variety of forms. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Groups of 3 → Graffiti

Include English language learners in groups with English-speaking or first-language students so that they will have opportunities to listen to the discussion and write on the chart paper. Review the terms on the charts explicitly so English language learners are comfortable with participating in the activity.

Whole Class → Presentation

In summarizing, point to and highlight the appropriate graffiti chart and the key ideas listed to focus learners on what is important about each term.

Begin a Word Wall for this unit.

Additional vocabulary may be necessary, e.g., *carousel, graffiti*.

Make Sure They're Ready
Circulate to make sure that English language learners understand the problem at each station.

Action!

Groups of 3 → Carousel

Describe how a "carousel" works so that all students are clear on how the class will cycle through the stations.

Adjust the groups, if necessary, so that the English language learners are included and can participate in the carousel. Group members should use the manipulatives and talk as they complete the stations to help their peers understand the concepts.

Learning Skills (Teamwork)/Observation/Checklist: When assessing the English language learners' collaboration skills, look for growth.

Make It Language Rich
Leave the graffiti charts posted as a reference. Provide an opportunity for English language learners to access their bilingual dictionaries.

Consolidate Debrief

Whole Class → Summarizing

Have English language learners record the terms in their personal word study notebook.

Make It Explicit
Ensure that the purpose of each station is clear to the learner.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Share some prepared examples to help English language learners understand their assignment.

75 min

Math Learning Goals

- Investigate and determine what a ratio is using examples and non-examples of proportional and non-proportional situations (e.g., two ordered quantities that share a multiplicative relationship).
- Determine the characteristics of the graph of a proportional relationship.

Materials

- colour tiles (250)
- linking cubes (480)
- BLM 4.2.1, 4.2.2

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Lead a review of basic concepts needed for the investigation.

Students need to be familiar with the concepts of a square, perimeter, area, length of a diagonal, cube, volume, ratio, and lowest terms.

Discussion around the length of the diagonal must include mention of the Pythagorean theorem.

Pairs → Anticipation Guide

Students, individually, complete the Before column on the anticipation guide (BLM 4.2.1) and discuss their choices with their partner.

Action!

Pairs → Investigation

Distribute 16 colour tiles and 27 linking cubes to each pair.

Students work in pairs on the four investigations (BLM 4.2.2).

Circulate to prompt, clarify, and focus the students on the task.

Provide limited resources so that students will infer results for larger models.

Learning Skills (Collaboration)/Observation/Anecdotal: Observe the students' contributions to completing the task.

Consolidate Debrief

Pairs → Think/Pair/Share/Discussion

Students complete the After column of the anticipation guide and share their choices with their partner, providing reasons for their choices.

Adjacent pairs of students compare and discuss the results.

Whole Class → Note Making

Lead a discussion to bring out that the perimeter and diagonal investigation show proportional reasoning and the others do not. Students should be able to explain how first differences, the ratios, and the graphs can all show proportionality.

Concept Practice Reflection

Home Activity or Further Classroom Consolidation

- In your journal, write a personal example of proportional reasoning.
- Using the scenarios below, check for proportionality and justify your response.
  a) You are paid an hourly wage. If you work 3 times the number of hours, does your pay triple?
  b) Student council raffle tickets cost $0.50/each or 3 for $1. If you buy twice as many tickets, does your cost double?

Terminology

lowest term ratio
proportional reasoning
non-proportion

Language Goals

- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Read texts with familiar content or vocabulary, using a variety of reading strategies. (ESLBO)
- Write in a variety of forms. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Refer English language learners to their personal word study notebook so they can review terms from previous units: *square*, *perimeter*, *area*, *length of a diagonal*, *cube*, *volume*, *Pythagorean theorem*, *first differences*.

Define *lowest term ratio*, writing some numerical examples on the board.

Pairs → Anticipation Guide

English language learners can verbalize each statement in their own words before they respond on the worksheet.

Additional vocabulary may be necessary, e.g., *double*, *anticipation*.

Make Sure They're Ready
Circulate to make sure that English language learners understand each investigation.

Make It Language Rich
Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Action!

Pairs → Investigation

Ask students to highlight words that are giving them difficulty. Support them by using questions to help them sort out what they understand and what they need to do next.

Learning Skills (Collaboration)/Observation/Anecdotal: Observe the students' contributions to completing the task.

Consolidate Debrief

Pairs → Think/Pair/Share/Discussion

As they complete the anticipation guide, provide prompts to help them connect back to the investigation.

Whole Class → Note Making

Use a graphic organizer to help learners make the connections between first differences, ratios, graphs, and proportions.

Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

Clarify any questions students have about the assignment.

Students provide a concrete or pictorial example to demonstrate their understanding of proportion.

75 min

Math Learning Goals

- Explore and develop an understanding of proportions, estimate answers, and devise and explain informal solutions (e.g., constant of proportionality, unit rate) in a variety of contexts (e.g., numerical, geometric, measurement, probability, algebraic).
- Solve problems using the Pythagorean relationship to connect proportional reasoning to contexts.

Materials

- BLM 4.3.1, 4.3.2
- linking cubes
- colour tiles
- grid paper
- relational rods

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Lead a discussion in which students share the informal methods of solving proportions from the Home Activity. Students may need to be reminded about conversions between feet and inches. (Do this in the context of ratios.)

An alternate context may be more appropriate depending on the classroom environment.

Action!

Groups of 4 → Exploration

Form heterogeneous groups based on students' preferred learning style, using observations of their previous two days' work.

Students use concrete materials and at least two different informal methods for their exploration (BLM 4.3.1).

Tools and Strategies/Observation/Mental Note: Assess the selection of tools and computational strategies.

Direct members from struggling groups to visit and observe groups who are using successful strategies.

Consolidate Debrief

Whole Class → Discussion/Note Making

Members from different groups share their solutions. Ensure that a variety of solution strategies are shared.

Discuss which strategies were effective for the various types of problems.

Summarize the various methods including equivalent ratios, the constant of proportionality, and algebraic reasoning. The algebraic reasoning may have to be formally taught, using the questions on BLM 4.3.2.

Pythagorean theorem is required to complete BLM 4.3.2. Teachers may need to review this theorem.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

- Complete the questions on worksheet Television Dimensions.
- Complete Pythagorean theorem questions.

Select appropriate practice questions.

Unit 4: Day 3: Pondering Proportions

Terminology

feet
inches

Language Goals

- Find specific information in straight forward reference materials, with teacher guidance. (ESLAO)
- Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO)
- Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)
- Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Demonstrate equivalent measures using a yardstick and a 12-inch ruler. Write the corresponding equivalent ratios on the board for visual reference.

Additional vocabulary may be necessary, e.g., *optimal, television, HDTV*.

Make Sure They're Ready

Model the problem by drawing a television on the board and having students demonstrate the optimal distance away from the television.

Action!

Groups of 4 → Exploration

Model ways of using manipulatives for solving problems. English language learners restate other group members' comments, asking questions or adding ideas of their own.

Tools and Strategies/Observation/Mental Note: Assess the selection of tools and computational strategies focusing on growth in their choices.

Consolidate Debrief

Whole Class → Discussion/Note Making

Encourage English language learners to participate in the sharing of their group's solutions by modelling part of the solution or by presenting the statement in a visual format.

Make It Language Rich

Leave the conversion ratio work on the board so the calculations can be used as a model. Encourage English language learners to access their bilingual dictionaries.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

English language learners start the worksheet 4.3.2 with a partner to ensure they understand the contexts.

75 min

Math Learning Goals

- Investigate a variety of methods for solving problems using proportions (e.g., scaling/tables, drawings, constant of proportionality, unit rate, cross products).
- Solve problems involving ratios, rates, and directly proportional relationships in a variety of contexts.
- Use estimation and proportional reasoning to determine the population size based on a random sample.

Materials

- photos of crowds
- grid paper
- 2 colours of number cubes
- overhead transparencies
- BLM 4.4.1, 4.4.2

Assessment Opportunities

Minds On ...

Pairs → Peer Coaching

Students compare solutions and help each other with practice questions from the home activity.

Whole Class → Discussion

Show an aerial photograph of a large crowd. Students suggest ways the number of people could be counted.

"Will the African Elephant Become Extinct in Your Lifetime?" *Impact Math: Data Management and Probability*; p. 25, crowd photos on newspaper websites (e.g., outdoor rock concerts, parades).

Action!

Pairs → Estimating

Differentiate this activity by providing different photographs with varying density and distribution of people and pairing students who are at similar mathematical development levels. Students draw a grid on the photograph or use an overhead overlay. The grid should have six rows and six columns. Use one coloured number cube to randomly choose a row and the other number cube for the column.

Pairs of students choose three grid squares by rolling the number cubes. They count the number of people in one of the grid squares and estimate the total number of people using a proportion. Repeat the process using the total for all three grid squares. Pairs compare their results with other pairs who used the same photograph.

Grid paper in a variety of scales can be created using Graphpap software. This freeware program can be downloaded from: http://pharm.kuleuven.be/pharbio/gpaper.htm

Connecting/Oral Questions/Anecdotal: Observe students as they discuss and compare their results with other groups.

Pairs → Scaling

Distribute BLM 4.4.1. Assign each pair of students one of the following enlargements (12 × 10 double both vertically and horizontally, 6 × 10 double only vertically, 12 × 5 double only horizontally, 12 × 15 double horizontally and triple vertically, 18 × 15 triple horizontally and vertically). Pairs of students make an enlarged figure on grid paper and write their scaling instructions below it.

Consolidate the aerial photo activity before beginning the scaling activity.

Consolidate Debrief

Whole Class → Discussion

Discuss the impact of sample size on the accuracy of the estimate.

- What limitations does this method have?
- Which of your enlarged figures are distorted? Why?

Discuss what constitutes a scale diagram, the constant of proportionality, and work through examples on how to use them.

Students record definitions and examples in their notes.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

- Complete the worksheet, More Scaling Problems.
- Complete the practice problems.
- Bring in examples of scale diagrams and aerial photos for the bulletin board.
- Bring in grocery flyers with prices of snacks and drinks for tomorrow's activity.

The teacher provides appropriate problems.

Terminology

proportions
scale
population size
random sample

Language Goals

- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Communicate orally, using accepted word order, common tenses, and other features of English grammar with some accuracy and consistency. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Pairs → Peer Coaching
As pairs share their responses, give positive feedback, using appropriate prompts and praise.

Action!

Pairs → Estimating
English language learners should be paired with peers who are at similar mathematical development levels.

Connecting/Oral Questions/Anecdotal: Observe students as they discuss and compare their results with other groups.

Pairs → Scaling
Use prompts to help learners understand their specific instructions for scaling.

Consolidate Debrief

Whole Class → Discussion
Post all the scale diagrams and sort to provide a visual summary of the key concepts, allowing students to see what scaling instructions lead to a proportional relationship.

Home Activity or Further Classroom Consolidation
Show examples of scale diagrams, aerial photographs, and appropriate grocery flyers so that learners will understand what they are to find.

Additional vocabulary may be necessary, e.g., *aerial photograph, density, distribution, limitations, distortions*.

Make It Language Rich
Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Make It Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.

Unit 7: Algebraic Models

Grade 9 Applied

Lesson Outline

BIG PICTURE:

English language learners will:

- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- continue to make short presentations;
- begin to participate in full class discussions.

Day	Lesson Title	Language Goals	Expectations
1	Linear and Non-linear Investigations (Part 1)	• Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO) • Write in a variety of forms, with teacher guidance. (ESLBO) • Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO) • Write in a variety of forms. (ESLBO)	NA2.07, LR3.02, LR3.04, LR4.03, LR4.05 CGE 5a, 7i
2	Linear and Non-linear Investigations (Part 2)	• Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO) • Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO) • Understand and use some subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO) • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO)	NA2.07, LR3.02, LR3.04, LR4.03, LR4.05 CGE 5a, 7i
3	Building Models	• Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO) • Participate in conversations on familiar topics in some social situations. (ESLAO) • Read texts with familiar content or vocabulary related to classroom studies. (ESLBO) • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO)	NA2.01, NA2.05, LR4.03, LR4.04, LR4.05 CGE 5a, 7i
4	Simplifying Algebraic Models	• Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO) • Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO) • Participate in conversations on familiar topics in some social situations. (ESLAO) • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO)	NA2.01, NA2.05, NA2.06, LR4.03, LR4.04, LR4.05 CGE 2a, 2b

ESLAO – Beginning communication in English builds on students' previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment

ESLBO – English in daily life expands students' essential English communication skills and introduces the language of classroom studies.

Unit 7: Day 1: Linear and Non-Linear Investigations (Part 1) (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Investigate linear and non-linear relationships.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials

- BLM 7.1.1, 7.1.2
- BLM 7.1.3 (Teacher)
- see BLM 7.1.3 for additional materials

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Explain what the students will be doing at each station.

Review terminology: linear and non-linear; rate of change and initial value (refer to Word Wall).

Action!

Small Groups → Carousel of Activities

Learning Skill (Teamwork)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' contributions to the group as they complete the activities.

Arrange the four stations by placing the appropriate materials and one colour-coded copy of the experiment (BLM 7.1.2) at each station.

Students complete each of the four experiments and record their answers on BLM 7.1.1 (You will need four copies per group).

Consolidate Debrief

Whole Class → Connecting

After students have completed all four of the experiments, help them make the connection between the first differences and the type of relationship (linear and non-linear). If students have not finished all four of the experiments, allocate more time the next day and make connections then. (See Day 2 for guiding questions.)

See Answers to Experiments (BLM 7.1.3).

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Complete the following journal entry:

Sally was not in class today. She doesn't know how to use differences to determine if a relationship is linear or non-linear. Use words, pictures, and symbols to explain it to her.

Unit 7: Day 1: Linear and Non-Linear Investigations (Part 1)

Grade 9 Applied

Terminology

linear,
non-linear
rate of change
initial value
mathematical models:
numerical,
graphical,
algebraic

Language Goals

- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)
- Write in a variety of forms. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Model how to complete the record sheet and demonstrate how to use the manipulative for each investigation.

English language learners can refer to their personal word study notebooks and the Word Wall for terminology.

Begin a Word Wall for this unit.

Additional vocabulary may be necessary, e.g., *figure, "burning the candle at both ends*."

Make Sure They're Ready
Circulate to make sure that English language learners understand each investigation.

Action!

Small Groups → Carousel of Activities

Arrange groups so that the English language learners work with English-speaking students who can model using the manipulatives and assist with the vocabulary for performing the investigation and recording the results.

Learning Skill (Teamwork)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' collaboration skills focusing on the English language learners' progress and accountability to the group.

Make It Language Rich
Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Make It Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.

Consolidate Debrief

Whole Class → Connecting

Use an overhead of a completed record sheet to highlight the connections among the first differences, the graph, and the algebraic model. Summarize the connections using a graphic organizer or a diagram.

Home Activity or Further Classroom Consolidation

Clarify any questions students have about the assignment.

Application
Concept Practice

Unit 7: Day 2: Linear and Non-Linear Investigations (Part 2) (TIPS4RM) Grade 9 Applied

75 min

Math Learning Goals

- Investigate linear and non-linear relationships through investigation.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials

- graph paper
- BLM 7.2.1
- BLM 7.2.2 (Teacher)

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Summarize how to identify whether a relationship is linear or non-linear using first differences. (BLM 7.2.1)

Action!

Small Groups → Carousel of Activities

Students continue to complete the experiments if not completed from Day 1.

Learning Skill (Initiative)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' initiative as they work in their groups.

Consolidate Debrief

Whole Class → Connecting

Use the following guiding questions:

- Which experiments had a linear relationship? (Take up equations using the graph, and identify the rate of change and the initial value.)
- Identify the rate of change and initial value for each linear relation. Write the equation for each relation.
- How can you use the table of values to predict if a relationship will be linear or non-linear? (Emphasize that the *x* values are increasing by 1, and that the differences are all the same.)

Discuss how changing the conditions of the experiments affects the graph (linear only).

Discuss with the students whether or not it makes sense to join the points on the graph based on whether the relationship is discrete or continuous.

As an alternate approach to taking up the activities, have students present their answers to the activities.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Graph the relationships from worksheet 7.2.1 and identify the rate of change and the initial value for the linear relationships. Write the equation for each relation.

Solutions to BLM 7.2.1 are provided on BLM 7.2.2.

Unit 7: Day 2: Linear and Non-linear Investigations (Part 2)

Terminology

Language Goals

- Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Model the process on the board, connecting verbal, written and pictorial representations of the words.

Additional vocabulary may be necessary, e.g., *changing conditions, multiple representations.*

Make It Language Rich

Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Action!

Small Groups → Carousel of Activities

Circulate to make sure that learners are recording all the representations for the investigations.

Learning Skill (Initiative)/Observation/Checkbric and **Curriculum Expectations/Investigation/Rubric:** Observe and record students' initiative as they work in their groups. Observe how they seek clarification from their group.

Make it Explicit

Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.

Consolidate Debrief

Whole Class → Connecting

English language learners can participate in the discussion by restating other students' comments, asking questions, or adding their own ideas.

English language learners may wish to share explanations using an overhead or by drawing/writing on the board.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Have learners check their solutions to the first questions on BLM 7.2.1 to verify that they understand the Home Activity.

75 min

Math Learning Goals

- Use multiple representations (physical, numerical, algebraic).
- Develop an understanding that simplification is necessary to determine if two algebraic expressions are equivalent.

Materials

- computer/data projector
- BLM 7.3.1, 7.3.2
- algebra tiles
- pattern blocks

Assessment Opportunities

Minds On ...

Whole Class → Presentation

Use the electronic presentation Patterns to introduce number patterns and terminology.

Patterns.ppt

Pattern recognition is a valuable skill. Discuss how it is used in different careers/disciplines, e.g., music, sports, art, history.

Pairs → Discussion

Students complete BLM 7.3.1 in pairs, comparing and refining responses.

Students work with another pair to compare/refine their responses to the problems.

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior knowledge.

Pair more capable students with a partner who needs support.

Action!

Whole Class → Setting Context

Introduce the task (BLM 7.3.2) and establish a purpose for finding a pattern (e.g., Frieda may want to know how many chairs she needs for 32 tables or how many tables she needs for 108 people).

Small Groups → Guided Exploration

Students complete BLM 7.3.2.

Consolidate Debrief

Whole Class → Discussion

Debrief the Feeding Frenzy activity to determine that students can build an algebraic model from a number pattern and that students recognize that there may be more than one correct algebraic model. Compare the equivalent models and simplify them to demonstrate that they are the same. Discuss and compare the patterns in both parts.

Home Activity or Further Classroom Consolidation

Concept Practice
Making Connections

- Use algebra tiles to show that the given three expressions are equivalent:
  (i) $2 + 4n$ (ii) $1 + 2n + 2n + 1$ (iii) $6 + (n - 1)(4)$
- Journal entry: Jason thinks that both of the Feeding Frenzy examples show a linear relationship between the number of tables and number of chairs. What evidence can you offer to support his claim?
- Practise your skills with algebraic expressions.

Provide appropriate practice questions.

Unit 7: Day 3: Building Models

Terminology

patterns
algebraic expression
models:
- *numerical*
- *algebraic*
- *graphical*

Language Goals

- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Participate in conversations on familiar topics in some social situations. (ESLAO)
- Read texts with familiar content or vocabulary related to classroom studies. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

Assessment Opportunities

Minds On…

Whole Class → Presentation

Reinforce the information during the electronic presentation using words and pointing to or circling key ideas on the screen.

Pairs → Discussion

Use questions to help English language learners sort out what they understand.

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior knowledge.

Additional vocabulary may be necessary, e.g., *frenzy, catering.*

Make It Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.

Action!

Whole Class → Setting Context

Model how to complete the first portion of the worksheet using visual aids.

Small Groups → Guided Exploration

Arrange groups so English language learners work with English-speaking peers to help them structure their explanations.

Consolidate Debrief

Whole Class → Discussion

During the discussion have learners explain their thinking using the manipulatives as part of their explanation.

To increase English language learners' comfort in participating, have them present the group's statement visually, e.g., with diagrams or manipulatives.

Home Activity or Further Classroom Consolidation

Clarify any questions students have about the assignment.

Concept Practice

Unit 7: Day 4: Simplifying Algebraic Models (TIPS4RM)

Grade 9 Applied

75 min

Math Learning Goals

- Use multiple representations (physical, numerical, algebraic).
- Simplify algebraic expressions.

Materials

- overhead projector
- algebra tiles
- computer/data projector
- BLM 7.4.1, 7.4.3 (Teacher)
- BLM 7.4.2, 7.4.4

Assessment Opportunities

Minds On ...

Small Groups → Practice

Students found a relationship between the term number and the expression for the term (Day 3). Provide each group with a set of answer cards. Hold up a cue card with an expression and students match it with one of their set.

Students explain their choices.

Clear up any misunderstandings.

Whole Class → Practice

Do several examples like the following:

- What is an algebraic expression for "3 more than double a number"? (Answer: $3 + 2x$)
- Use algebra tiles to create a model for the expression. (Answer: Use the overhead to show 2 *x*-tiles and 3 one-tiles.)

Collecting Terms.ppt,

Algebraic Equations.ppt

The activity Cue Cards is designed to give students practice in creating algebraic expressions from words.

Set a date with students for a proficiency test on simplifying algebraic expressions.

Action!

Whole Class → Presentation

Use electronic presentations Collecting Terms Using Algebra Tiles, and Expanding and Simplifying Algebraic Expressions.

Pairs → Practice

Students complete BLM 7.4.2.

Learning Skill (Work Habits)/Observation/Rating Scale: Observe and record how students work to complete the assignment.

Consolidate Debrief

Whole Class → Connecting

Use Debrief Notes to consolidate learning and make connections between numerical, algebraic, and graphical models (BLM 7.4.3).

Application

Home Activity or Further Classroom Consolidation

Complete worksheet 7.4.4 We're All Correct!

How can you determine if two expressions are equivalent?

Students may need additional practice questions.

Unit 7: Day 4: Simplifying Algebraic Models

Grade 9 Applied

Terminology

sum
product
quotient
increased
decreased
difference
times

Language Goals

- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Participate in conversations on familiar topics in some social situations. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

- toothpicks

Assessment Opportunities

Minds On…

Small Groups → Practice

Explain the meanings of homophones using a math context. English language learners record the terms in their personal word study notebook.

Show the cue card and read it slowly, more than once, orally emphasizing and pointing to the key words. When an incorrect response is given, read the cue card again, pointing to the math symbols on the correct answer card. Post the cue cards with the answer for reference.

Whole Class → Practice

Model a few questions using visual aids and oral clues.

Additional vocabulary may be necessary, e.g., *cue cards, double*.

Make Sure They're Ready
Circulate to make sure that English language learners understand the terminology.

Action!

Whole Class → Presentation

Pairs → Practice

There are two parts to this practice: simplifying the algebraic expression and translating the algebraic expression into a word statement. Circulate to determine the portion of the practice with which students may be having difficulty, and use appropriate prompts to guide their learning.

To determine that students understand there are multiple ways to state an algebraic expression, have them share different word statements, e.g., $x + 26$ is twenty-six more than a number, the sum of a number and twenty-six, a number increased by twenty-six,…

Learning Skill (Work Habits)/Observation/Rating Scale: Observe and record how students work to complete the assignment, noting their growth.

Make It Language Rich
Post the cue cards as reference. Provide an opportunity for English language learners to access their bilingual dictionaries.

Engage the Senses
Have students demonstrate different toothpick models to each other to clarify the solution.

Make It Explicit
For translation reference, post signal words, e.g., addition - *add, sum, total, plus, increased by*…; product - *multiply, times, double, triple*… .

Consolidate Debrief

Whole Class → Connecting

To describe the process that transforms $3(x - 1)$ into $3x - 3$, learners use arrows or write the steps in numerical form on the board.

Application

Home Activity or Further Classroom Consolidation

Students can model their thinking of the solution with the toothpicks.