



## Lesson Outline

### Big Picture

English language learners will:

- work productively in flexible student groupings;
- communicate in day-to-day classroom interactions;
- use manipulatives to develop and demonstrate concept understanding.

Day	Lesson Title	Language Goals *	Expectations
1	What Do Patterns Tell Us?	<ul style="list-style-type: none"> <li>• Begin to work with a partner on a common academic task (Stage 1).</li> <li>• Begin to understand teacher expectations and follow classroom routines (Stage 1).</li> <li>• Understand short, simple phrases and sentences, instructions, and brief oral notes in material with familiar vocabulary and context (Stage 2).</li> <li>• Identify main ideas and key information in text (Stage 2).</li> </ul>	8m56  CGE 2c, 3e
2	Different Representations of the Same Patterns	<ul style="list-style-type: none"> <li>• Recognize frequently used classroom vocabulary (Stage 1).</li> <li>• Begin to understand teacher expectations and follow classroom routines (Stage 1).</li> <li>• Begin to use language to explain, persuade, and negotiate (Stage 2).</li> <li>• Participate in directed group work (Stage 2).</li> </ul>	8m56, 8m57, 8m60, 8m78  CGE 3b, 5a
3	Finding the $n^{\text{th}}$ Term	<ul style="list-style-type: none"> <li>• Recognize frequently used classroom vocabulary (Stage 1).</li> <li>• Begin to understand teacher expectations and follow classroom routines (Stage 1).</li> <li>• Respond with increasing confidence to a variety of teaching strategies (Stage 2).</li> <li>• Expand academic vocabulary (Stage 2).</li> </ul>	8m57, 8m58, 8m60, 8m62, 8m63, 8m78  CGE 5b, 7j

### **\*English Language Learner Language Goals**

*The Ontario Curriculum Grades 1–8, English As a Second Language and English Literacy Development (A Resource Guide) 2001*

Stage 1 – Using English for Survival Purposes

Stage 2 – Using English in Supported and Familiar Activities and Contexts



**Math Learning Goals**

- Review patterning in real contexts, e.g., weather patterns, quilt patterns, patterns of behaviour, patterns in a number sequence or code.
- Develop an understanding that all patterns follow some order or rule and practice verbally expressing patterning rules.

**Materials**

- chart paper
- variety of everyday patterns
- variety of manipulatives
- BLM 2.1.1, 2.1.2

**Assessment Opportunities**

**Minds On... Small Groups → Graffiti**

Based on class size, set up three stations with different patterning examples at each station, e.g., atlases/maps (landforms, weather), artwork, pine cones, nautilus shells, bird migration patterns. Student groups at each station record all the patterns they discover in 1–2 minutes. Students rotate through all three stations.

Student groups summarize their findings and each group presents a brief summary to the class.

Students should be in heterogeneous groupings.

A recorder can be assigned in each group or all students may be involved in recording.

Encourage multiple representations of patterns.

**Action! Think/Pair/Share → Demonstration**

Using manipulatives, e.g., linking cubes, display the following patterns: 4, 8, 12, 16... and 1, 4, 7, 10.... Students determine a pattern and share with their partner.

In a class discussion students express the pattern in more than one way, e.g., the first pattern increases by 4 each term, or the pattern is 4 times the term number, the pattern is multiples of 4; the second pattern increases by 3 each term, the pattern is 3 times the term number subtract 2.

**Individual → Practice**

Students complete BLM 2.1.1, extending the pattern and expressing it in words.

**Content Expectations/Observation/Journal/Mental Note:** Circulate to assess for understanding of representing patterns.

**Consolidate Debrief Whole Class → Presentation**

Students represent the patterns visually and explain them.

**Home Activity or Further Classroom Consolidation**

*Exploration Reflection*

Find a pattern that you like. Record the pattern in your math journal in pictures and words.

Provide examples of patterns within the class.

**Terminology**

**Language Goals**

**Materials**

- Begin to work with a partner on a common academic task (Stage 1).
- Begin to understand teacher expectations and follow classroom routines (Stage 1).
- Understand short, simple phrases and sentences, instructions, and brief oral notes in material with familiar vocabulary and context (Stage 2).
- Identify main ideas and key information in text (Stage 2).

**Assessment Opportunities**



**Minds On... Small Groups → Graffiti**

Provide clear, concise instructions demonstrating examples from one of the stations. English language learners should be spread out in various groups with English-speaking students. Assign the role of recorder to an English-speaking student.

When student groups summarize their findings to present to the class, groups demonstrate with pictures, diagrams, and models as well as with words.

**Incorporate Identity**

Include patterns that are represented in the culture of English language learners.

**Make It Language Rich**

If English language learners are comfortable, ask them to share additional background on patterns in their culture that were part of the stations.



**Action! Think/Pair/Share → Demonstration**

Pair English-speaking students with English language learners to promote discussion and sharing. Have students record patterns on chart paper for ease of sharing with the class.

**Individual → Practice**

Partner English language learners who speak the same first language, if possible, so that they can discuss their understanding of the patterns as they work on BLM 2.1.1.

**Make It Comprehensible**

Provide resource materials in the classroom for students to complete this activity.

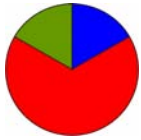


**Consolidate Debrief Whole Class → Presentations**

**Home Activity or Further Classroom Consolidation**

*Exploration Reflection*

Encourage English language learners to find patterns that might be common in their culture.



**Math Learning Goals**

- Examine (linear) patterns involving whole numbers presented in a variety of forms, e.g., as a numerical sequence, a graph, a chart, a physical model, in order to develop strategies for identifying patterns.

**Materials**

- a visual pattern
- BLM 2.2.1, 2.2.2, 2.2.3
- linking cubes
- rulers

**Assessment Opportunities**

**Minds On... Pair/Share → Patterning**

Model how to share a visual pattern, e.g., art, nautilus shell, in both words and pictures. Student A shares the pattern in words and pictures with Student B. Student B shares the pattern in words and pictures with Student A. Regroup pairs to form groups of four.

Student A in each pair will share Student B’s pattern with the group. Student B in each pair will share Student A’s pattern with the group.

Interesting visual patterns can be found by doing an online image search.

**Action! Small Groups → Investigation**

In heterogeneous groups, students rotate through the stations (BLM 2.2.1) They record their work on BLM 2.2.2. (The empty circle area on this BLM is used on Day 3.)

**Whole Class → Connecting**

Students share their findings and record any corrections on their worksheet. They label the four rectangular sections as: Numerical Model, Graphical Model, Patterning Rule, Concrete Model (BLM 2.2.2).

Lead students to the conclusion that all of these representations show the same pattern:

- What do you notice about the table of values and the concrete representation?
- What are the similarities? (i.e., they are all representations of the same pattern)

**Curriculum Expectations/Observation/Checklist:** Circulate to assess understanding that the representations all show the same pattern.

**Consolidate Debrief Whole Class → Four Corners**

Post charts in the four corners of the room labelled as: Graphical Model, Patterning Rule, Concrete Model, Numerical Model. Below each label, draw a rough diagram to aid visual learners.

Pose the question: For which model did you find it easiest to extend the pattern?

Students travel to the corner that represents their answer and discuss why they think that they found that method easier. One person from each corner shares the group’s findings.

**Home Activity or Further Classroom Consolidation**

Complete the practice questions.

Practice

Provide students with appropriate practice questions showing multiple ways of representing linear patterns.

**Terminology**

*linear pattern  
numerical model  
graphical model  
concrete model  
patterning rule*

**Language Goals**

- Recognize frequently used classroom vocabulary (Stage 1).
- Begin to understand teacher expectations and follow classroom routines (Stage 1).
- Begin to use language to explain, persuade, and negotiate (Stage 2).
- Participate in directed group work (Stage 2).

**Materials**

**Assessment Opportunities**



**Minds On...**

**Pair/Share → Patterning**

Pair English language learners with English-speaking students to communicate using English words and phrases. English language learners can repeat the words used by their English-speaking partner.

**Incorporate Identity**

Include visual patterns found in different cultures



**Action!**

**Small Groups → Investigation**

The heterogeneous groups should include both English-speaking students and English language learners.

**Whole Class → Connecting**

As students share their findings they point to and show the different representations.

**Make It Explicit**

Post a well labelled example of each type: numerical model, graphical model, concrete model and patterning rule for students to use as a reference



**Consolidate Debrief**

**Whole Class → Four Corners**

Pose the Four Corners question in several different ways. Ask students to paraphrase the instruction for selecting the corner so that English language learners are clear on which corner they will choose.

Note which corners the English language learners have selected. This will be helpful for planning of future lessons.

**Engage the Senses**

Have concrete materials available at each station

*Concept Practice*

**Home Activity or Further Classroom Consolidation**

Reinforce the expectations for the practice questions with a written outline.



**Math Learning Goals**

- Determine, and represent algebraically the general term of a linear pattern ( $n^{\text{th}}$  term).
- Determine any term, given its term number, in a linear pattern represented graphically or algebraically.
- Check validity by substituting values.

**Materials**

- BLM 2.3.1, 2.3.2, 2.3.3
- linking cubes

**Minds On...**

**Whole Class → Four Corners**

Give each student a card. Students travel to the corner that corresponds to the representation on their card, e.g., A student with a card that has a graph goes to the graphical model representation corner. Students discuss “What is challenging about changing from one representation of a pattern to another?” Choose one person from each corner to share the group’s conclusions.

Pose the following scenario: Armando has a CD collection. He currently owns 2 CDs. Each week, he purchases a new CD for his collection. How could you represent this in a model? Students in each corner describe the scenario, using the model represented in their corner.

**Action!**

**Small Groups → Investigation**

With the class, model the results to the problem using two colours of linking cubes (2 red and 1 green for the first term, 2 red and 2 green for the second term, and so on). Discuss why the first term has 3 CDs in it. Students use linking cubes to build the concrete model of the pattern up to the 6th term and complete BLM 2.3.2 in groups.

Guide a class discussion about students’ findings (BLM 2.3.3).

**Representing/Oral Questions/Mental Note:** Observe students as they work on the small-group activity.

**Assessment Opportunities**

Cut BLM 2.3.1 into individual cards.

Collect the cards from students to use in a future activity.

Word Wall  
 • term number  
 • term value

**Consolidate Debrief**

**Whole Class → Algebraic Representation**

Ask:

- How can we think about the algebraic expression in another way? Decide what the  $n^{\text{th}}$  term represents (unknown term; a method to find any term; a “formula”).
  - How might you find the 12<sup>th</sup> term of the pattern?
  - Is it possible to find the 12<sup>th</sup> term without extending the table?
  - Find the 12<sup>th</sup> term. Can you use the same method to find the 100<sup>th</sup> term?
  - How can you determine if your  $n^{\text{th}}$  term is correct? (Substitute the term numbers in for  $n$  and the resulting answers should be the term values.)
- Students record this algebraic representation of the pattern in the circle on the placemat from Day 2 (BLM 2.2.2).

**Home Activity or Further Classroom Consolidation**

Complete the practice questions.

*Application  
 Exploration  
 Reflection*

Provide students with appropriate practice questions

**Terminology**

*term number  
term value  
variable  
substitution*

**Language Goals**

- Recognize frequently used classroom vocabulary (Stage 1).
- Begin to understand teacher expectations and follow classroom routines (Stage 1).
- Respond with increasing confidence to a variety of teaching strategies (Stage 2).
- Expand academic vocabulary (Stage 2).

**Materials**

**Assessment Opportunities**



**Minds On... Whole Class → Four Corners**

Pose the scenario on an overhead, using the board or a poster as well as orally. Note any students who are in corners that were not chosen the previous day and assist as necessary.

**Make Sure They're Ready**

Check on English language learners understanding that the various representations can be used for the same pattern by providing specific examples.



**Action! Small Groups → Investigation**

Reinforce the oral discussion of the problem by showing and pointing to the concrete model.

Discuss the various uses of the word “number” in the BLM 2.3.2 chart to assist with comprehension. Fill in the first row together to clarify each column.

Make frequent reference to the concrete model during the class discussion. Ask students to paraphrase explanations of others to help English language learners to process the discussion.

**Make It Engage the Senses**

Provide linking cubes and graph paper for students to build or draw the models as they progress to higher term numbers.

**Assess with Sensitivity**

Ask probing questions to determine the level of understanding; allow students to give one word or picture answers.



**Consolidate Debrief Whole Class → Algebraic Representation**

Post the questions asked on the board or use an overhead and underline the key words to help English language learners connect the words and meanings to mathematical understanding.

**Home Activity or Further Classroom Consolidation**

Partner English language learners with English-speaking students to discuss expectations for the practice questions. Clarify as needed.

*Application  
Exploration  
Reflection*



## Lesson Outline

### Big Picture

English language learners will:

- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- begin to make short presentations;
- share cultural differences with the class.

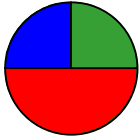
Day	Lesson Title	Language Goals *	Expectations
1	Pizza and Cake	<ul style="list-style-type: none"> <li>• Answer specific questions using single words or short phases (Stage 1).</li> <li>• Use learners' and bilingual dictionaries (Stage 1).</li> <li>• Recount familiar events stories and key information (Stage 2).</li> <li>• Organize information around a central idea using graphic organizers (Stage 2).</li> </ul>	8m13, 8m14, 8m15, 8m18  CGE 2c, 3c, 5a
2	Fraction Frenzy	<ul style="list-style-type: none"> <li>• Follow simple directions with support from visual clues (Stage 1).</li> <li>• Use learners' and bilingual dictionaries (Stage 1).</li> <li>• Begin to apply knowledge of basic writing conventions (Stage 1).</li> <li>• Respond to vocabulary, questions, and instructions in a familiar context (Stage 2).</li> <li>• Use reading strategies to assist in deriving meaning (Stage 2)</li> <li>• Write appropriate responses to written questions based on familiar academic content (Stage 2).</li> </ul>	8m18  CGE 3c, 4f
3	Parts Problems	<ul style="list-style-type: none"> <li>• Answer specific questions using single words or short phases (Stage 1).</li> <li>• Use short, patterned questions to seek information (Stage 1).</li> <li>• Demonstrate awareness of cultural differences and show pride in self and culture (Stage 1).</li> <li>• Recount familiar events stories and key information (Stage 2).</li> <li>• Ask questions (Stage 2).</li> <li>• Continue to use, take pride in, and respect the home language (Stage 2).</li> </ul>	8m19  CGE 3b, 5a, 5e

### **\*English Language Learner Language Goals**

*The Ontario Curriculum Grades 1–8, English As a Second Language and English Literacy Development (A Resource Guide) 2001*

Stage 1 – Using English for Survival Purposes

Stage 2 – Using English in Supported and Familiar Activities and Contexts

**Math Learning Goals**

- Activate and assess prior knowledge of fractions.
- Reason about fractions from a variety of perspectives and representations.

**Materials**

- geoboards
- fraction circles
- chart paper
- markers
- BLM 5.1.1, 5.1.2

**Minds On...****Small Groups → Exploration/Presentation**

Distribute fraction cards (BLM 5.1.1). Explain the task, including the presentation. Tell students that they are to use a variety of strategies and tools, including estimation, manipulatives, diagrams, anchors (of  $0$ ,  $\frac{1}{2}$ ,  $1$ ), and equivalent forms (decimals, percents) to complete the task and include in their presentation.

Students find other students who have cards of the same colour, arrange their group's fractions in order, and discuss their reasoning.

Two groups form a larger group to discuss the strategies and tools they used and plan and make a presentation.

**Curriculum Expectations/Observation/Anecdotal Notes:** Observe students' comfort and facility with fractions to determine what fraction experiences are needed in this unit.

**Assessment Opportunities**

Students may use a calculator to change each fraction to a decimal.

Students use their knowledge of multiples to determine common denominators.

**Action!****Small Groups → Modelling**

Set up multiple stations with the two activities (BLM 5.1.2).

Students work at one of the stations for half the time, then switch stations. They prepare their solutions on chart paper for a whole-class discussion.

Students review the factors of composite numbers, as they reduce fractions.

**Consolidate Debrief****Whole Class → Discussion**

Use the chart paper solutions to consolidate understanding:

- Equal fraction pieces (same area) can have different shapes.
- Equal fractions can be expressed in different ways.
- Fractions can be expressed with common denominators for addition.

- $\frac{n}{n} = 1$

- Fractions can be reduced when numerator and denominator share a common factor greater than 1.

Discuss how to use common denominators and benchmarks ( $0$ ,  $\frac{1}{2}$ , and  $1$ ) when comparing fractions.

**Home Activity or Further Classroom Consolidation**

Make a mind map of things you remember about fractions. Include:

- terminology, e.g., proper, improper
- how to add and subtract fractions using symbols
- how to represent fractions on a number line

*Reflection*

See *Think Literacy: Mathematics* pp. 76–81, Graphic Organizers.

**Terminology**

*numerator  
common  
denominator  
equivalent forms:  
decimals,  
percents  
composite  
numbers*

**Language Goals**

- Answer specific questions using single words or short phrases (Stage 1).
- Use learners’ and bilingual dictionaries (Stage 1).
- Recount familiar events stories and key information (Stage 2).
- Organize information around a central idea using graphic organizers (Stage 2).

**Materials**

**Assessment Opportunities**

**Minds On...**

**Small Group → Exploration/Presentation**

Demonstrate the use of geoboards and fraction circles if this is the first opportunity that English language learners have had to use them.

English language learners record key vocabulary in their personal vocabulary lists with examples.

Define the terminology explicitly to help English language learners with the language that will be used in this unit.

Group English language learners with English-speaking students to facilitate use of content vocabulary.

Additional vocabulary may be necessary, e.g., proper, improper, benchmarks

**Make Sure They’re Ready**

Observe English language learners to determine if any difficulty is due to the language.

**Action!**

**Small Groups → Modelling**

Group English language learners with peers who speak the same first language, if possible.

**Incorporate Identity**

Create a poster with key terminology written in the various first languages represented in your class.

**Make It Language Rich**

English language learners record key vocabulary using the English word, the words in their first language and a visual.

**Consolidate Debrief**

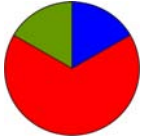
**Whole Class → Discussion**

Ask students to record the key ideas captured on the chart paper in their notes. English language learners can copy words and phrases to make notes.

*Reflection*

**Home Activity or Further Classroom Consolidation**

English language learners can create their mind map in both their first language and English to assist them with the graffiti activity for Day 2.



**Math Learning Goals**

- Assess for prior learning of fractions.

**Materials**

- BLM 5.2.1
- pattern blocks
- fraction circles
- geoboards

**Minds On...**

**Small Groups → Pass It On!**

Post graffiti sheets in different locations of the room with the following titles:

- 1) Show different ways to find  $2\frac{2}{3} + 1\frac{1}{2}$
- 2) Show different ways to find  $2\frac{2}{3} - 1\frac{1}{2}$
- 3) List fraction words and meanings.
- 4) Show some fractions on a number line. (Include the number lines.)

Students can take their mind maps from the Home Activity as they move in groups to different locations. Circulate to answer/pose questions. Leave sheets posted during assessment for prior learning.

**Assessment Opportunities**

*Think Literacy: Cross-Curricular Approaches, Grades 7–12, p. 66, Graffiti*

Check that sheets contain correct information. Use “think aloud” to share the class’ collective knowledge of fractions, as shown on the graffiti sheets.

**Action!**

**Individual → Diagnostic**

Review instructions (BLM 5.2.1). Students complete the worksheet.

**Curriculum Expectations/Paper-Pencil Assessment/Rubric:** Assess students’ knowledge and understanding of fractions and use the information to plan future instruction, e.g., differentiated instruction.

**Consolidate Debrief**

**Individual → Reflection**

Students reflect on their answers to question 8.

**Home Activity or Further Classroom Consolidation**

*Reflection Application*

Create stories for situations that can be modelled by the expression:  $6 \times \frac{2}{3}$ .

**Terminology**

*less than*  
*more than*

**Language Goals**

- Follow simple directions with support from visual clues (Stage 1).
- Use learners’ and bilingual dictionaries (Stage 1).
- Begin to apply knowledge of basic writing conventions (Stage 1).
- Respond to vocabulary, questions, and instructions in a familiar context (Stage 2).
- Use reading strategies to assist in deriving meaning (Stage 2).
- Write appropriate responses to written questions based on familiar academic content (Stage 2).

**Materials**

**Minds On...**

**Small Groups → Pass It On**

English language learners add their first language words and meanings to the grafitti sheet labelled “Fraction Words and Meanings,” so that English language learners have fraction words and meanings in their own language as well as in English during the diagnostic assessment.

**Action!**

**Individual → Diagnostic**

When reviewing the instructions, act out each of the symbols.

Write out *one-quarter*, *less than one-quarter*, *more than one-quarter* using numbers and symbols for question 1, to clarify the meaning of the words.

**Consolidate  
Debrief**

**Individual → Reflection**

Describe and post the questions for clarity, demonstrate that students need to choose only one of them to answer.

English language learners reflect on their question in their first language and translate key words and phrases into English to share their reflections.

*Reflection*  
*Application*

**Home Activity or Further Classroom Consolidation**

English language learners can create their stories in their first language and translate key ideas into English to write complete, creative stories.

**Assessment  
Opportunities**

**Incorporate Identity**

Refer to the poster with key terminology written in the various first languages created in a previous class.

**Make It Language Rich**

Encourage English language learners to use their bilingual dictionaries and to ask clarifying questions based on language difficulties.

**Make It Explicit**

Define *less than* and *more than* clearly.

**Assess with Sensitivity**

Allow English language learners to demonstrate their understanding of the concepts in alternative ways, if language is a barrier.



**Math Learning Goals**

- Use manipulatives and symbols to represent the multiplication of a whole number by a fractional quantity.
- Calculate the product of a whole number and a fractional quantity.

**Materials**

- fraction circles
- pattern blocks
- cube links
- graph paper
- BLM 5.3.1

**Assessment Opportunities**

**Minds On... Whole Class → Sharing**

Students share responses to the previous day’s Home Activity. Record and post samples of their responses. Encourage the students to ask each other questions about their stories, if they don’t understand.

**Action! Small Groups → Connecting**

Students reflect on the posted stories and choose one that matches the numerical problem, determine the solution, and explain their reasoning. Challenge students who successfully complete the solution to represent the problem using a different manipulative.

**Curriculum Expectations/Observation/Anecdotal Notes:** Circulate, asking each group reflective questions. Determine if each student can state the representation for one whole – every other representation depends on this.

**Whole Class → Instruction**

Demonstrate  $6 \times 4$  by putting 6 identical objects in each of 4 bags and also 4 identical objects in each of 6 bags. The total is 24 objects in both cases. Discuss why this is so.

Model  $6 \times \frac{2}{3}$ . Explain that, while demonstrating  $6 \times 4$  you had to use four “somethings,” and now you will need “two-thirds of something.” The “something” is always a whole, in this case 6, and  $\frac{2}{3}$  is just a bit more than half of the whole.

Represent one whole with one hexagonal pattern block piece. Students reproduce the shape (or cover it using overhead pieces) using three identical rhombus pieces. The rhombus piece is one-third of the whole and two rhombus pieces are two-thirds of a whole. Demonstrate  $6 \times \frac{2}{3}$  by putting two rhombus pieces into each of six bags. Take them all out and count how many one-thirds there are to get twelve-thirds, i.e.,  $\frac{12}{3}$ . So,  $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = \frac{12}{3}$ .

Write the symbols for the solution and discuss why the answer is  $\frac{12}{3}$  and why this is simplified to 4. Demonstrate that the solution is the same if triangles are used instead of the rhombus.

Note if anyone thought that  $6 \times \frac{2}{3}$  should turn out to be  $\frac{6 \times 2}{6 \times 3}$ , i.e.,  $\frac{12}{18}$ . If they did, have them reduce  $\frac{12}{18}$  to get  $\frac{2}{3}$ . Ask if multiplying  $6 \times \frac{2}{3}$  should get the same result as  $1 \times \frac{2}{3}$ ? If they accept that  $\frac{12}{18}$  doesn’t make sense, show that  $6 \times \frac{2}{3}$  is the same as  $\frac{6}{1} \times \frac{2}{3}$ . Now ask how they might work that out.

**Consolidate Debrief Whole Class → Discussion**

As students present and explain their representations highlight a variety of representations. Compare these questions:  $5 \times 3$ ,  $5 \times \frac{3}{8}$ ,  $5 \text{ cm} \times 3 \text{ cm}$ .

Ask What is the same and what is different when you calculate answers using just the symbols? Summarize student discoveries on multiplying a whole number by a fractional part. Include observations on reducing fractions and changing forms (proper to improper and vice versa).

Students complete BLM 5.3.1.

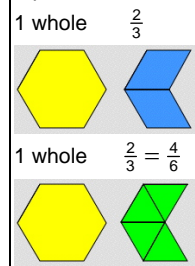
**Home Activity or Further Classroom Consolidation**

*Application* Create and solve five questions that involve a whole number multiplied by a fractional part.

**Example responses**

- 6 bottles are each  $\frac{2}{3}$  filled with water. How many full bottles of water are there in total?
- Jay walked  $\frac{2}{3}$  of a kilometre. Keri walked 6 times as far. How far did Keri walk?

Sample representations:



If students use triangles, then  $6 \times \frac{2}{3} = 6 \times \frac{4}{6}$ , confirmed by  $\frac{12}{3} = 4 = \frac{24}{6}$ .

**Terminology**

**Language Goals**

- Answer specific questions using single words or short phrases (Stage 1).
- Use short, patterned questions to seek information (Stage 1).
- Demonstrate awareness of cultural differences and show pride in self and culture (Stage 1).
- Recount familiar events stories and key information (Stage 2).
- Ask questions (Stage 2).
- Continue to use, take pride in, and respect the home language (Stage 2).

**Materials**

**Minds On...**

**Whole Class → Sharing**

Encourage English language learners to share their stories and ask questions. Post some of their stories so that all students have an opportunity to see, share, and respect the cultural diversity in your class.

**Assessment Opportunities**

Additional vocabulary may be necessary, e.g., whole and part (as related to fractions.)

**Action!**

**Small Groups → Connecting**

Group English language learners with English-speaking students so that they can hear the story in English in a small-group setting. Students work together using the manipulatives and English-speaking students assist English language learners with the language to explain their reasoning, as necessary.

**Whole Class → Instruction**

Demonstrate using the manipulatives and add gestures for emphasis. Ask students to repeat or paraphrase the instruction or demonstration to reinforce comprehension.

**Make It Language Rich**

Assist English language learners with the appropriate vocabulary to express their understanding, as necessary.

**Engage the Senses**

Provide enough manipulatives so that English language learners can make sense of the whole class instruction using their own manipulatives.

**Consolidate Debrief**

**Whole Class → Discussion**

Record and summarize the key ideas for the discussion. English language learners can copy them into their notes.

*Application*

**Home Activity or Further Classroom Consolidation**

Allow English language learners to create some questions in their first language but encourage them to create at least one or two questions in English.

## Lesson Outline

### Big Picture

English language learners will:

- continue with their own personal vocabulary lists;
- work productively in flexible student groupings;
- communicate in day-to-day classroom interactions;
- use graphic organizers.

Day	Lesson Title	Language Goals *	Expectations
1	Size It Up	<ul style="list-style-type: none"> <li>• Work with a partner on a shared academic task (Stage 1).</li> <li>• Begin to apply knowledge of basic writing conventions (Stage 1).</li> <li>• Participate in directed group work (Stage 2).</li> <li>• Organize information around a central idea using graphic organizers (Stage 2).</li> </ul>	8m26, 8m27, 8m33, 8m68, 8m70  CGE 4b, 5a, 5b
2	Interpreting Proportional Relationships	<ul style="list-style-type: none"> <li>• Follow simple directions with support from visual cues (Stage 1).</li> <li>• Recognize frequently used classroom vocabulary (Stage 1).</li> <li>• Participate in conversations on familiar topics (Stage 2).</li> <li>• Participate in classroom and group discussions (Stage 2).</li> </ul>	8m26, 8m27  CGE 3b, 3g
3	Around the World in Eight Days	<ul style="list-style-type: none"> <li>• Answer specific questions using single words or short phrases (Stage 1).</li> <li>• Begin to work with a partner on a common academic task (Stage 1).</li> <li>• Respond with increasing confidence to a variety of teaching strategies (Stage 2).</li> <li>• Organize information around a central idea using graphic organizers (Stage 2).</li> </ul>	8m27  CGE 5a, 5b
4	Go Fish	<ul style="list-style-type: none"> <li>• Begin to adapt to a variety of teaching approaches and strategies used in a Canadian classroom (Stage 1).</li> <li>• Respond to oral instruction and information in standard Canadian English in school settings (Stage 1).</li> <li>• Respond with confidence to a variety of teaching approaches and strategies (Stage 2).</li> <li>• Participate in controlled, directed group work (Stage 2).</li> </ul>	8m26, 8m27, 8m68, 8m73  CGE 5a, 7i

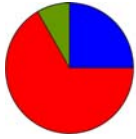
### **\*English Language Learner Language Goals**

*The Ontario Curriculum Grades 1–8, English As a Second Language and English Literacy Development (A Resource Guide) 2001*

Stage 1 – Using English for Survival Purposes

Stage 2 – Using English in Supported and Familiar Activities and Contexts



**Math Learning Goals**

- Investigate proportional situations using everyday examples.
- Identify proportional and non-proportional situations.

**Materials**

- relational rods
- measuring tapes
- BLM 8.1.1, 8.1.2, 8.1.3, 8.1.4
- assorted cylinders

**Assessment Opportunities****Minds On...****Pairs → Anticipation Guide**

Distribute BLM 8.1.1. Students highlight key words in each of the six statements, then complete the Before column of the Anticipation Guide for Proportional Reasoning. Upon completion students explain their reasoning to a partner. Volunteers explaining their reasoning.

See *Think Literacy Mathematics: Grades 7–9*, Anticipation Guide, p. 10.

**Action!****Small Groups → Investigation**

Explain the instructions at each station (BLM 8.1.2 and 8.1.4). Students rotate through three of them (or more if time allows). Students will record data on BLM 8.1.3.

**Whole Class → Discussion**

Compare the data collected at each station. Discuss data that doesn't fit due to incorrect measurements or calculations. Identify proportional and non-proportional situations (BLM 8.1.4).

**Communicating/Observation/Mental Note:** Observe as students rotate through the stations. Note any potential misunderstandings. These can be addressed in **Consolidate Debrief**.

See *Think Literacy Mathematics: Grades 7–9*, p. 38.

**Consolidate Debrief****Whole Class → Discussion**

Groups discuss their findings for each station.

Complete and post a class Frayer model for the word Proportion (BLM 8.1.4). Students revisit their original responses on the anticipation guide and complete the After column.

**Home Activity or Further Classroom Consolidation***Concept Practice*

Find some examples of proportional situations at home and add them to the Frayer model.

**Terminology**  
*proportional*

**Language Goals**

- Work with a partner on a shared academic task (Stage 1).
- Begin to apply knowledge of basic writing conventions (Stage 1).
- Participate in directed group work (Stage 2).
- Organize information around a central idea using graphic organizers (Stage 2).

**Materials**

- empty box of macaroni and cheese
- golf balls
- empty can of frozen lemonade

**Assessment Opportunities**

**Minds On... Pairs → Anticipation Guide**

As students highlight key words in each of the six statements demonstrate the meaning of some of the common words using concrete examples such as box of macaroni and cheese and golf balls.

Pair English language learners with an English-speaking partner to facilitate the reasoning discussion.

Additional vocabulary may be necessary, e.g., radius, circumference, diagonal, perimeter, rectangle

**Make Sure They're Ready**

Review the terminology used from previous units to reference their meanings.

**Action! Small Groups → Investigation**

Model the action at each station as you explain the instructions to connect the oral instruction to a physical action.

If possible, pair two English language learners with the same first language in a group with two English-speaking students to facilitate the development of the mathematical concepts and the appropriate use of the language used in the discussion.

**Make It Language Rich**

Advise groups to assist each other as they clarify instructions and collect data.

**Whole Class → Discussion**

The discussion can be accompanied by a written example of a recording sheet on the overhead as a visual cue to help them determine the station that is being discussed.

Describe which stations were proportional and non-proportional on the board so that the examples can be copied into students' notes.

**Assess with Sensitivity**

When observing English language learners, be aware that difficulties may be due to language acquisition and not the mathematical concept.

**Consolidate Debrief Whole Class → Discussion**

Small groups draft a Frayer model for the word *proportion* to prepare for the completion of a class model so English language learners can participate in a small group setting.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

Encourage English language learners to find examples of *proportional* situations that may be unique or representative of their culture to highlight the diversity in the class.



**Math Learning Goals**

- Use multiple representations to determine proportions.
- Through exploration and inductive reasoning, determine what makes a situation proportional.

**Materials**

- manipulatives
- BLM 8.2.1
- chart paper
- markers

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Add student examples to the Frayer model from the Home Activity in Day 1. Discuss why the student examples are proportional or non-proportional.

**Pairs → Problem Solving**

Students solve the problem and share how they came to their solution:  
 Jack and Jill were driving the same speed along a highway. It took Jack 25 minutes to drive 50 kilometres. How long did it take Jill to drive 125 kilometres? Explain different methods of arriving at the same solution.

Highlight methods for problem solving:

- unit rate strategy: unit rate (25 minutes for 50 km,  $\frac{1}{2}$  min for 1 km,  $\frac{1}{2} \times 125 = 62.5$  mins for 125 km)
- factor-of-change strategy: 125 is 2.5 times as far.  
 Therefore,  $25 \times 2.5 = 62.5$  mins
- fraction strategy:  $\frac{25}{50} = \frac{1}{2}$ ,  $\frac{1}{2} = \frac{62.5}{125}$
- cross-product algorithm:  $\frac{25}{50} = \frac{x}{125}$ ,  $x = \frac{125 \times 25}{50}$ .

Unit-rate strategy: how many for one?

Factor-of-change strategy: “times as many” method

Fraction strategy: use unit rates as fractions and create equivalent fractions

Cross product algorithm: set up a proportion, form a cross product, and solve the equation by dividing

“Connecting Research to Teaching Proportional Reasoning” by Kathleen Cramer and Thomas Post

([http://education.umn.edu/rationalnumberproject/93\\_2.html](http://education.umn.edu/rationalnumberproject/93_2.html))

**Action! Small Groups → Investigation**

Groups solve the given problems (BLM 8.2.1) using two methods. One person from the group explains the methods they used. Post the solutions.

Circulate to monitor progress, offer suggestions, and note the variety of strategies used. Distribute chart paper and markers to groups as they are ready.

**Reasoning and Proving/Demonstration/Anecdotal:** Observe reasoning skills during the investigation and select groups to present so that all methods are shared.

**Consolidate Debrief Whole Class → Discussion**

Revisit the posted solutions to reinforce the strategies used. All methods use multiplicative reasoning (unit rate strategy, factor of change strategy, fraction strategy, and cross-product algorithm) and it is this multiplicative property that makes a proportion.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

Solve the problem and validate your solution using a second strategy: If you can type 45 words per minute, how long will it take to type a 900-word essay? Show your work.

**Terminology**

*multiplicative reasoning*

**Language Goals**

- Follow simple directions with support from visual cues (Stage 1).
- Recognize frequently used classroom vocabulary (Stage 1).
- Participate in conversations on familiar topics (Stage 2).
- Participate in classroom and group discussions (Stage 2).

**Materials**

**Minds On...**

**Whole Class → Discussion**

Include examples from the homework that highlight the cultural diversity in the class.

**Pairs → Problem Solving**

Post the problem and provide time for English language learners to look up any unknown vocabulary and ask clarifying questions before beginning the solution.

Pair English language learners with students who speak the same first language, if possible, to facilitate sharing any unique perspective they might bring to the solution of the problem.

Observe English language learners as they work on the problem to identify any strategies that you have not seen before and include them in the highlight of the methods discussed.

**Action!**

**Small Groups → Investigation**

English language learners work in groups on a problem that is appropriate to the level of their mathematical understanding regardless of their facility in English.

**Consolidate Debrief**

**Whole Class → Discussion**

When revisiting the posted solutions have students demonstrate using manipulatives and underlining key ideas or words in their reasoning to provide visual cues.

Make notes on the board about the multiplicative property of proportions so that English language learners can copy the points as notes.

*Concept Practice*

**Home Activity or Further Classroom Consolidation**

**Assessment Opportunities**

Additional vocabulary may be necessary, e.g., unit rate, factor of change, fraction strategy, cross product

**Make It Explicit**

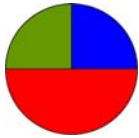
Use the terminology of the problem-solving methods and in writing during the highlighting of methods used.

**Engage the Senses**

Provide a variety of manipulatives that can be used as support to solve the problems.

**Assess with Sensitivity**

Provide opportunities for English language learners to describe their reasoning in a variety of ways.



**Math Learning Goals**

- Solve problems involving proportions using concrete materials.

**Materials**

- linking cubes
- pattern blocks
- grid paper
- BLM 8.3.1

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Pose the following problem:

Two players on the school basketball team scored all the points in the last game. The ratio of points scored was 2:5. The team scored 35 points in total. How many points did each player score?

Use manipulatives to model the problem (linking cubes, pattern blocks, grid paper). Students share a variety of strategies and their reasoning.

**Action! Whole Class → Instruction**

Demonstrate connections between ratio, proportion, and fractions using a graphic organizer.

**Pairs → Investigation**

Provide a number of packages with two items such as linking cubes, pattern blocks, coloured tiles in specific proportions that can be reduced to simplest form. Include a problem to be solved. Students use the contents to solve the problem and determine the ratio of the items in it. They reduce the ratio to simplest form.

They repeat the investigation with a different package.

Students present the problems they solved and their ratio. Classmates ask presenters questions so that they understand.

**Communicating/Presentation/Anecdotal:** Observe students' use of appropriate terminology and clarity of explanation.

Students requiring additional practice can complete BLM 8.3.1.

**Consolidate Debrief Pairs → Connecting**

Students create a mind map connecting the ideas and key information of proportion and share and compare with a partner.

**Home Activity or Further Classroom Consolidation**

Complete the problem:

Kerry said that the Japanese Bullet Train takes about 6 minutes to travel 22.2 km. Jerry said that at this rate, he could travel around the world at the equator in less than 8 days. Kerry disagrees – she thinks it will take longer.

Who is correct? Justify your response.

The diameter of Earth is approximately 38 250 km.

*Differentiated Concept Practice*

**Terminology**

*ratio  
proportion  
fractions*

**Language Goals**

- Answer specific questions using single words or short phrases (Stage 1).
- Begin to work with a partner on a common academic task (Stage 1).
- Respond with increasing confidence to a variety of teaching strategies (Stage 2).
- Organize information around a central idea using graphic organizers (Stage 2).

**Materials**

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Students discuss the problem with a partner for a few minutes before beginning the class discussion so they understand the problem and begin to think about it before the class discussion starts. Use manipulatives to demonstrate a solution to the problem to make the connection of the words, the strategy, and the concrete model.

**Make Sure They're Ready**

Review the terminology and include examples to determine if English language learners have the necessary prior knowledge.

**Action! Whole Class → Instruction**

Enhance the graphic organizer by including English language learners' first language on the graphic organizer. Post the completed organizer for reference.

**Make It Language Rich**

Provide a summary sheet of the key terms for reference during the lesson and for home study.

**Pairs → Investigation**

Pair English language learners with English-speaking partners so they can practise the presentation of their problems in English with support.

**Make It Comprehensible**

Check often with English language learners to make sure that they understand the activity.

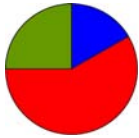
**Consolidate Debrief Pairs → Connecting**

English language learners include first-language words and phrases on their mind maps. If necessary, more English can be added when they compare with their partner.

**Home Activity or Further Classroom Consolidation**

Provide a written version of the problem for English language learners. Show a picture of the Japanese Bullet (high-speed commuter train) for reference. Students who have ridden the Bullet can share any experience that they may have.

*Differentiated  
Concept  
Practice*



**Math Learning Goals**

- Solve problems involving proportions.
- Connect to a everyday sampling problem.

**Materials**

- paper bags
- linking cubes
- masking tape

**Assessment Opportunities**

**Minds On... Whole Class → Investigation**

Students create ratios by moving to different areas within the classroom, based on an attribute chosen by the teacher. Record the appropriate ratios on the board that reflect the class population. Possible ratios: 1) boys: girls: adults; 2) shirt colour – light: dark: medium.

Discuss the ratios and demonstrate when they can be reduced to simplest form.

Use the term *simplest form*, as it will be required in Lesson 6.

All parts of the ratio together represent the whole class.

**Action! Small Groups → Exploration**

Each group receives a paper bag filled with 30 linking cubes of one colour. One student removes six cubes, puts a piece of masking tape on each cube, and returns them to the bag. Another group member shakes the bag, takes out five cubes, records how many of these cubes are taped and how many are not, and returns the cubes to the bag. Each group member repeats this process of taking out five cubes, recording, and returning cubes to the bag. Compare results and estimate how many cubes are in the bag.

Lead a discussion on how this experiment can be used to determine the total number of cubes in the bag (equivalent ratio – 6 out of 30 equivalent to 1 out of 5).

Repeat with 20 cubes, 5 of which are taped. Students take out 4 cubes at a time, determine the ratio of taped cubes to those that are not taped, and make predictions using the ratios of taped cubes to total cubes to estimate the number of cubes in the bag.

Students do not know how many cubes are in the bag.

**Reasoning and Proving/Observation/Anecdotal:** Observe groups as they work through their exploration and listen to their reasoning.

**Consolidate Debrief Whole Class → Connecting**

Groups share their estimates and explain their thinking. Work through the estimation for the problem: Scientists often use the catch, band, and release method to estimate the size of wildlife populations. For example, 250 trout were caught, banded, and released into a small lake in Northern Ontario. One month later, another 250 trout were caught in the lake, 30 of them had bands. From this information scientists could estimate the size of the trout population of the lake. (Approximately 1708 trout were in the lake.)

Students explain why they wait for a month to catch fish.

Provide the population for your school and community.

Remind students that all parts of whole ratios represent the total population.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

Assuming that the ratio of eye colour of the class is the same within the wider community, estimate how many people have eye colour that is blue, brown, or other in the whole school, the community, the province, and the country.

Students record any assumptions that they make.

Population: Ontario – approximately 11.5 million; Canada – approximately 33 million (July 2005)

**Terminology**  
*sampling*

**Language Goals**

- Begin to adapt to a variety of teaching approaches and strategies used in a Canadian classroom (Stage 1).
- Respond to oral instruction and information in standard Canadian English in school settings (Stage 1).
- Respond with confidence to a variety of teaching approaches and strategies (Stage 2).
- Participate in controlled, directed group work (Stage 2).

**Materials**

**Minds On...**

**Whole Class → Investigation**

Student volunteers can demonstrate the attribute and the motion so that English language learners have non-verbal cues to which they may respond.

**Assessment Opportunities**

Additional vocabulary may be necessary, e.g., simplest form, total population

**Make It Language Rich**

The focus on the story of counting populations helps to connect mathematics to the environment and links math language to a context.

**Action!**

**Small Groups → Exploration**

Share a story about counting fish in a lake, or animals on a game preserve from the newspaper or Internet to provide a context and to clarify that the goal of this activity is to determine the number of cubes in the bag without actually counting them. Use pictures to support your story, if possible.

Have all the groups mark six cubes at the same time and then have one group demonstrate one trial of the activity for clarity of instruction and mime the repetition of the process for the whole group.

**Make It Explicit**

Define the parts of the ratio and show how the parts form the whole population to clarify how ratios are different from fractions. (Symbolically they look the same.)

**Consolidate Debrief**

**Whole Class → Connecting**

Write the estimates for each group on the board before the discussion. Students point to and speak to their estimate as they explain their thinking. Summarize the process for estimating a population at the end of the discussion so that students can record the key ideas in their notes.

*Concept Practice*

**Home Activity or Further Classroom Consolidation**