



## Lesson Outline

### Big Picture

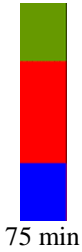
English language learners will:

- create their personal word study notebook;
- begin to work productively in flexible student groupings;
- begin to participate in small group discussions.

Day	Lesson Title	Language Goals	Expectations
1	Similar Triangles: Perimeter and Area Relationship	<ul style="list-style-type: none"> <li>• Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).</li> <li>• Participate in conversations by responding to specific questions, using short phrases (ESLAO).</li> <li>• Listen to others and stay on topic in group discussions (ESLBO).</li> <li>• Restate important information from presentations that include visual aids (ESLBO).</li> </ul>	MT1.01, MT2.01 CGE 2c, 3c
2	What Is Similarity?	<ul style="list-style-type: none"> <li>• Follow simple written instructions (ESLAO).</li> <li>• Use dictionaries to clarify word meanings (ESLAO).</li> <li>• Use context and familiar vocabulary in texts to infer the meaning of new words (ESLBO).</li> <li>• State the main idea of individual passages that contain familiar vocabulary (ESLBO).</li> </ul>	MT1.01 CGE 3b, 5a
3	Properties of Similar Triangles	<ul style="list-style-type: none"> <li>• Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).</li> <li>• Work cooperatively with a partner on shared classroom tasks (ESLAO).</li> <li>• Participate in group work, cooperative games and teamwork (ESLBO).</li> <li>• Use first language appropriately in classroom and social situations (ESLBO).</li> </ul>	MT1.01, MT1.02 CGE 3c, 4b

**ESLAO** – Beginning Communication in English builds on students’ previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment.

**ESLBO** – English in Daily Life expands students’ essential English communication skills and introduces the language of classroom studies.



**Math Learning Goals**

- Investigate the relationship between the perimeter and the area of similar triangles.
- Use the Pythagorean relationship to find information about triangles.

**Materials**

- BLM 1.1.1

**Assessment Opportunities**

**Minds On... Whole Class → Guided Discussion**

Draw several different types of triangles on the board, e.g., right, isosceles, scalene. Ask students to identify them by name, and note that some triangles may have more than one name, e.g., right isosceles. Discuss what information is required to find the perimeter and the area of each triangle. Lead students to recognize that finding the height may require the use of the Pythagorean theorem. Review the Pythagorean theorem.

**Groups of 3 → Making a Hypothesis**

Students discuss and make a hypothesis about the relationship between the area and the length of the perimeter of similar triangles, e.g., Given a triangle and a similar triangle whose perimeter is double, what is the effect on its area? Students include reasons for their hypothesis, e.g., their previous knowledge and understanding of area and perimeter, their conceptual understanding of the formulas, a guess resulting from a relevant sketch.

**Action!**

**Groups of 3 → Guided Investigation**

Groups work through BLM 1.1.1. Encourage students to show their work and present their solution in an organized manner. Different groups may come up with different solutions. Have these solutions placed on chart paper for sharing. If groups finish early, ask them if they can come up with an alternative way to solve the problem.

**Mathematical Processes/Problem Solving/Checklist:** Assess how students state a hypothesis, apply problem-solving strategies, and adjust their hypothesis based on new information.

**Consolidate Debrief Whole Class → Guided Discussion**

Consider the results of the investigation. Share different solutions. Facilitate a discussion by asking leading questions such as:

- Considering the formula for the area of a triangle, why do you think the area will be 4 times the original area when the perimeter is doubled?
- Does this logic hold true for halving the perimeter? Explain.
- What do you think will happen if the perimeter is tripled?
- How could you check this?
- What other tools could you use to solve this problem?

**Home Activity or Further Classroom Consolidation**

Investigate if your conclusion to today's problem will be true if the original shape is a rectangle.

Orient triangles in various ways so that not all have horizontal bases.

Some students may choose to use GSP<sup>®</sup>4.

There is an opportunity to discuss Pythagorean triples.

*Application*

**Terminology**

*right triangle*  
*isosceles triangle*  
*scalene triangle*  
*perimeter*  
*area*  
*Pythagorean theorem*  
*similar triangles*

**Language Goals**

- Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).
- Participate in conversations by responding to specific questions, using short phrases (ESLAO).
- Listen to others and stay on topic in group discussions (ESLBO).
- Restate important information from presentations that include visual aids (ESLBO).

**Materials**

- grid paper

**Assessment Opportunities**

**Minds On...**

**Whole Class → Guided Discussion**

Label pictures of various types of triangles with their names and ask English language learners to add them to their personal word list that includes the definition in their first language.

**Groups of 3 → Making a Hypothesis**

Group English language learners with at least one English-speaking student so that they can practise using the English terminology.

Provide grid paper to give visual support as they think about the hypothesis.

**Action!**

**Groups of 3 → Guided Investigation**

Ask students to provide visual support with their solutions and to underline key terminology so that English language learners will have additional cues when discussing the solutions.

**Consolidate Debrief**

**Whole Class → Discussion**

Record solutions as they are shared, and ask students to point to or highlight key parts when explaining their reasoning.

Post the additional questions asked on the board or overhead so that English language learners have time to look up important words for meaning before beginning the discussion.

**Home Activity or Further Classroom Consolidation**

*Application*

Define what a rectangle is and show some examples, to build their vocabulary.

Additional vocabulary may be necessary, e.g., hypothesis, double

**Make Sure They're Ready**

Explicitly review the terminology required for the task including perimeter, area and similar triangles.

**Incorporate Identity**

Create a poster for this unit with the terminology written in English and in the first languages of the English language learners in the class.

**Assess with Sensitivity**

Ask questions of the English language learners as they work in their groups to determine if they are having difficulties with either the language or the mathematical concepts.



**Math Learning Goals**

- Investigate the properties of similar triangles using geoboards, e.g., corresponding angles are equal and corresponding sides are proportional.

**Materials**

- BLM 1.2.1, 1.2.2, 1.2.3
- 11-pin transparent geoboards
- geobands

**Assessment Opportunities**

**Minds On... Pairs → Guided Discussion**

One member of the pair is A and the other is B.

Ask: What does it mean to say that two shapes are similar?

A responds to B, then B responds.

If two triangles are similar, what might be true of their angles and their sides?

B responds to A, then A responds.

**Individual → Activating Prior Knowledge**

Option 1

Students complete the Before column of the Anticipation Guide (BLM 1.2.1).

Option 2

Students complete the What I Know and What I Want to Know columns (BLM 1.2.1).

Select one of the two options on BLM 1.2.1 to activate prior knowledge.

**Action! Pairs → Investigation**

**Learning Skills/Teamwork/Observation/Anecdotal Note:** Observe pairs of students for cooperative learning, sharing of responsibilities, on-task behaviour.

Students complete questions 1–4 on BLM 1.2.2.

Guide students through question 5 to establish properties of similar triangles before completing the remaining questions. Include how to write a similarity equation for the corresponding sides of similar triangles. For question 6, students represent each triangle on a separate geoboard to determine the corresponding angle measurements by translating, rotating, or reflecting.



Provide only the number of bands needed.

Establish that one unit is the horizontal or vertical length between two pegs on the geoboard.

**Consolidate Debrief Pairs → Reflecting**

Students complete the After column or the What I Learned column on BLM 1.2.1.

**Home Activity or Further Classroom Consolidation**

Complete worksheet 1.2.3.

*Application*

**Terminology**

*corresponding  
angles  
corresponding  
sides*

**Language Goals**

- Follow simple written instructions (ESLAO).
- Use dictionaries to clarify word meanings (ESLAO).
- Use context and familiar vocabulary in texts to infer the meaning of new words (ESLBO).
- State the main idea of individual passages that contain familiar vocabulary (ESLBO).

**Materials**

**Assessment Opportunities**

**Minds On... Pairs → Guided Discussion**

Pair English language learners with an English-speaking partner so that they can hear and use the terminology as they take turns during the discussion.

Post the discussion questions on the board or overhead for visual reference.

Have a pair of students model how the partner A and B discussion works, as you describe it.

**Individual → Activating Prior Knowledge**

English language learners can express their prior knowledge in their first language and then use their bilingual dictionaries to translate the key ideas into English.

**Action! Pairs → Investigation**

Pair English language learners with English-speaking partners to assist with reading of the instructions.

Demonstrate the use of geoboards and repeat instructions, if necessary.

When guiding students through question 5, record the properties of similar triangles and the conventions for writing similarity equations on the board. English language learners can copy them into their notes for future reference.

**Consolidate Debrief Pairs → Reflecting**

English language learners can complete their reflections in their first language and translate the key ideas into English afterwards.

**Home Activity or Further Classroom Consolidation**

*Application*

**Make Sure They're Ready**

Review corresponding angles and sides in similar triangles with a visual demonstration.

**Make It Language Rich**

Post the relevant content vocabulary poster created in the previous lesson so that English language learners can refer to as necessary during the lesson.



**Math Learning Goals**

- Investigate the properties of similar triangles, i.e., corresponding angles are equal and corresponding sides are proportional, using concrete materials.

**Materials**

- BLM 1.3.1, 1.3.2
- GSP<sup>®</sup> 4 (optional)
- protractors
- rulers
- square, legal- and letter-sized paper

**Assessment Opportunities**

**Minds On... Small Groups → Discussion**

Students complete a Frayer model for similar triangles based on their learning from the previous day's lesson (BLM 1.3.1). Students should keep this work for reference throughout the course.

Discuss briefly the differences and similarities between similar shapes and congruent shapes.

**Action! Whole Class → Instructions**

Discuss the purpose of the paper folding activity.

Students follow the fold instructions to create similar triangles (BLM 1.3.2).

**Groups of 3 → Exploration**

Each student in the group completes BLM 1.3.2 using a different-sized piece of paper ( $8\frac{1}{2} \times 11$ ,  $8\frac{1}{2} \times 8\frac{1}{2}$ ,  $8\frac{1}{2} \times 14$ ) and they compare their results.

**Reasoning and Proving/Oral Question/Anecdotal Note:** As students work, circulate, and ask questions so they can demonstrate they are using reasoning skills.

Using grid paper or GSP<sup>®</sup> 4 facilitates this activity.

Preview the activity prior to assigning it to class.

See Mathematical Processes in LMS Library.



**Consolidate Debrief Whole Class → Discussion**

Discuss answers from BLM 1.3.2, reinforcing that similar triangles have equal angles and sides that are proportional. Students should see this connection with the results of the exploration.

Groups share their summaries.

Consolidate how to determine a similarity equation for the corresponding sides of similar triangles, and how to solve the similarity equation using their knowledge of ratios and proportions.

**Home Activity or Further Classroom Consolidation**

Find the missing information for pairs of similar triangles.

*Concept Practice*

Provide students with several pairs of similar triangles, with some information given about each pair.

**Terminology**

*proportion  
similar shapes  
congruent shapes*

**Language Goals**

- Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).
- Work cooperatively with a partner on shared classroom tasks (ESLAO).
- Participate in group work, cooperative games, and teamwork (ESLBO).
- Use first language appropriately in classroom and social situations (ESLBO).

**Materials**

**Minds On...**

**Small Groups → Discussion**

English language learners add their first language definitions to the Frayer model.

When discussing the differences and similarities between similar shapes and congruent shapes, use visual examples to make the meaning clear.

**Action!**

**Whole Class → Instructions**

Model the fold instructions for English language learners.

**Groups of 3 → Exploration**

Group English language learners with a student whose first language is the same, if possible. Provide assistance with the instructions as necessary. Have a labelled, folded model available for reference.

**Consolidate  
Debrief**

**Whole Class → Discussion**

Groups of English language learners can share their summaries with groups of English-speaking students first before the whole class sharing to give them practise in a smaller setting.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

**Assessment Opportunities**

Additional vocabulary may be necessary, e.g., reasoning, ratio

**Incorporate Identity**

The math concepts learned in the English language learner's first language are transferable. Assist students with the English terminology as needed.

**Make It Language Rich**

Have English language learners highlight key ideas and new words during the exploration.

**Engage the Senses**

Mime the steps involved in the exploration for students having difficulty understanding what is expected.



## Lesson Outline

### Big Picture

English language learners will:

- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- begin to make short presentations.

Day	Lesson Title	Language Goals	Expectations
1	Going Around the Curve (Part 1)	<ul style="list-style-type: none"> <li>• Create individual and group language-experience stories (ESLAO).</li> <li>• Give and respond to straightforward directions and instructions (ESLAO).</li> <li>• Use short sentences and phrases to tell stories, recount events, provide directions or instructions, and stating opinions (ESLBO).</li> <li>• Use a variety of simple sentence patterns in their writing (ESLBO).</li> </ul>	QR2.01 CGE 5a, 7j
2	Going Around the Curve (Part 2)	<ul style="list-style-type: none"> <li>• Give and respond to straightforward directions and instructions (ESLAO).</li> <li>• Take turns in conversations and classroom discussions (ESLAO).</li> <li>• Maintain face-to-face conversation on familiar topics (ESLBO).</li> <li>• Restate important information from presentations that include visual aids (ESLBO).</li> </ul>	QR2.01 CGE 2c, 4f
3	Making a Difference	<ul style="list-style-type: none"> <li>• Take turns in conversations and classroom discussions (ESLAO).</li> <li>• Give and respond to straightforward directions and instructions (ESLAO).</li> <li>• Maintain face-to-face conversations on familiar topics (ESLBO).</li> <li>• Restate important information from presentations that include visual aids (ESLBO).</li> </ul>	QR2.02 CGE 5a, 5b
4	Features of Parabolic Graphs	<ul style="list-style-type: none"> <li>• Determine the meaning of unfamiliar words, using pictures and illustrations (ESLAO).</li> <li>• Include key conceptual vocabulary in personal word lists for classroom study (ESLAO).</li> <li>• Use vocabulary-acquisition strategies (ESLBO).</li> <li>• Maintain a vocabulary notebook or list for various subject areas (ESLBO).</li> </ul>	QR2.03 CGE 4f

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**Math Learning Goals**

- Collect data that can be modelled by a quadratic relation, using connecting cubes, and calculate first and second differences.
- Draw the curve of best fit on chart paper.
- Realize that the shape of the graphs are curves rather than lines.

**Materials**

- linking cubes
- chart paper
- grid chart paper
- BLM 4.1.1, 4.1.2, 4.1.3, 4.1.4, 4.1.5

**Assessment Opportunities**

**Minds On... Groups of 3 → Placemat**

Students complete a placemat with the phrase “linear relationship” in the centre. They reflect on everything they recall about the characteristics of linear relations in their own section and share their results within their groups. They write the characteristics they agree upon in the centre. Repeat the process using the phrase “non-linear relationship.”

Summarize characteristics on chart paper and post.

Recall that first difference implies a linear relationship.

Show students how to find second differences, using an example.

**Curriculum Expectations/Demonstration/Observation/Checklist:** Observe what characteristics students recall about linear and non-linear relations.

**Action! Groups of 3 → Experiments**

Each group completes an assigned experiment (BLMs 4.1.1–4.1.5). They record their data in a table on chart paper, and plot the data on grid chart paper.

Groups who do not complete their experiment can be given some time during the next lesson.

**Learning Skills/Teamwork/Observation/Checklist:** Observe how well students work as a productive team to complete the task.



These experiments provide “clean data” from which a constant second difference can be determined.

**Consolidate Debrief Groups of 3 → Sharing**

Groups who complete the activity post their graphs and tables of values and plan how they will present their work to the class.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

Complete the practice questions.

Provide students with appropriate practice questions.

**Terminology**

*first difference*  
*second difference*

**Language Goals**

- Create individual and group language-experience stories (ESLAO).
- Give and respond to straightforward directions and instructions (ESLAO).
- Use short sentences and phrases to tell stories, recount events, provide directions or instructions, and stating opinions (ESLBO).
- Use a variety of simple sentence patterns in their writing (ESLBO).

**Materials**

**Minds On...**

**Groups of 3 → Placemat**

Group English language learners with English-speaking students so that they can hear and use the terminology.

English language learners can write their portion of the placemat in their first language to capture their prior knowledge comprehensively and translate to English as the group agrees on what will be written in the centre portion.

**Action!**

**Groups of 3 → Experiments**

Keep the same groups as in the **Minds On**.

Encourage English language learners to consult with their group before they ask the teacher for assistance.

**Consolidate  
Debrief**

**Groups of 3 → Sharing**

All three members of the group must share in the presentation of the work to the class. English language learners can point to the charts and graphs during the presentation if they are not comfortable speaking. Alternatively, groups could present their findings using choral speaking given time to write out the speaking notes and practise.

**Home Activity or Further Classroom Consolidation**

*Concept Practice*

**Assessment  
Opportunities**

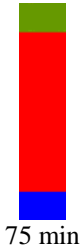
Additional vocabulary may be necessary, e.g., linear relationships, non-linear relationships

**Make It Language Rich**

English language learners write the new vocabulary into their personal word study notebook.

**Make It Explicit**

Define first differences and second differences explicitly with examples so that the expectations for the activity are clear for English language learners.



**Math Learning Goals**

- Complete the data collection from the previous lesson, if needed.
- Communicate students’ findings from the experiments to the entire class.
- Define the shape of the curves of best fit as a parabola.

**Materials**

- materials from Day 1
- large chart (graph) paper

**Assessment Opportunities**

**Minds On... Whole Class → Review**

Review the context of expectations for Day 1 experiments.

Tell students that the graphs obtained are only half of the full graph and direct them to extend them using symmetry. Demonstrate, as needed.

**Pairs → Peer Coaching**

Pair each student with someone who was not in their group on the previous day. Pairs discuss each experiment, checking the graph and the first and second differences.

- Word Wall
- parabola
  - quadratic equation/relation
  - vertex
  - axis of symmetry
  - x-intercepts
  - y-intercept
  - zeros
  - maximum/minimum value
  - symmetry

**Action! Groups of 3 → Presentation**

The original groups make any required changes, draw a large graph of their results, and present their findings to the class. Presentations should include the context provided in their problem; the models they constructed; the data they collected; an explanation of the pattern that they observed; the graphs they constructed; and the curve of best fit.

**Curriculum Expectation/Presentation/Checkbric:** Assess how students collect and represent data, and draw the curve of best fit. Observe results of experiments, ask oral questions, and clear up misconceptions.



**Consolidate Debrief Whole Class → Discussion**

Focus discussion on the shapes of the entire graph, including negative  $x$  values. Explain that this particular curve is called a parabola. Students take particular note of the constant second difference in the table of values, connecting to their understanding that linear relations have common first differences.

Explain that the common second difference identifies the resulting curve as a parabola. Students predict what their graph would look like if it was extended to negative values of the independent variable.

Students should see the full parabolic shape rather than just half.

**Home Activity or Further Classroom Consolidation**

- Complete a journal entry using the following writing prompt:  
Parabolic curves exist in the world in the following ways/places/actions...

OR

- Find pictures you think are parabolic and bring them to class.

*Reflection*

**Terminology**

*parabola  
symmetry*

**Language Goals**

- Give and respond to straightforward direction and instructions (ESLAO).
- Take turns in conversations and classroom discussions (ESLAO).
- Maintain face-to face conversation on familiar topics (ESLBO).
- Restate important information from presentations that include visual aids (ESLBO).

**Materials**

**Assessment Opportunities**

**Minds On...**

**Whole Class → Review**

Demonstrate the symmetry of the experiment graphs by folding on the vertical axis and tracing in the other half of a graph drawn from one of the experiments.

**Pairs → Peer Coaching**

Pair English language learners with same first language partners, if possible, so that they can discuss their understanding more fully.

**Action!**

**Groups of 3 → Presentation**

Circulate to ensure that English language learners have a role that they are comfortable fulfilling for the presentation

**Assess with Sensitivity**

Suggest various roles that English language learners could take during their group's presentation.

**Consolidate Debrief**

**Whole Class → Discussion**

Lead discussion providing visual cues. Have students repeat or paraphrase key ideas so that English language learners gain more experience in listening to the key ideas.

Using the same example as the demonstration in **Minds On**, work backwards using the first and second differences chart to fill in the points for the negative independent values to reinforce that the graph results in the same shape that was demonstrated during the folding exercise.

**Home Activity or Further Classroom Consolidation**

*Reflection*

**Incorporate Identity**

Encourage English language learners to write about or find examples of possible parabolic shapes that may be found in their home country or culture.



**Math Learning Goals**

- Determine that if the table of values yields a constant second difference the curve is parabolic and vice versa.
- Realize that there are other non-linear relationships that are not parabolic.
- Develop a word wall of new vocabulary related to the quadratic.

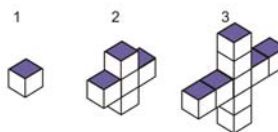
**Materials**

- connecting cubes
- graph paper

**Minds On...**

**Whole Class → Investigation**

Demonstrate a linear relationship, using the following construction.



Students individually complete a table of values with the headings **Model Number**, **Number of Cubes**, and **First Difference** and create a graph of the information. They connect the first difference with the slope of the line and reflect on the value of difference to determine if the relationship is linear or non-linear, and provide reasons for their conclusions.

**Assessment Opportunities**

**Action!**

**Pairs → Summarizing**

Students refer to data from any two experiments completed on Day 1. Engage student thinking by providing prompts: Prove that the data is non-linear. Further prompting: We know all the graphs are parabolas; we call these quadratic relations.

Ask:

- How could we know that they are parabolas just from the table of values?
- What pattern do you see in the work you've done on the tables?

Students complete the statement: The graph of a table of values will be a parabola if....

**Curriculum Expectation/Demonstration/Checklist:** Assess students' understanding that a constant second difference in a table of values determines that the relation is quadratic.

**Groups of 4 → Investigation**

Students build cubes of sides 1, 2, and 3, and record on a table of values the side length and volume. They calculate the volume for side lengths 4, 5, and 6, and put the data on a graph.

Ask:

- Is this linear or non-linear?
- Is the curve a parabola?
- How can you verify this?

**Consolidate Debrief**

**Whole Class → Discussion**

Discuss and summarize facts:

- The table of values associated with parabolas always has a common second difference.
- There are other curves that are not parabolas. These curves do not have common second differences.

**Home Activity or Further Classroom Consolidation**

Complete one task about parabolas:

- Determine if the situations are linear, quadratic, or neither, and provide reasons for your answers.

OR

- Place the parabolic picture that you brought to class on a grid and determine some of its points. Verify that it is or is not a parabola.

Before continuing, students should have the understanding that the first differences are not the same and therefore the data is non-linear.

Students can confirm their predictions by selecting other sets of data from the experiments.

*Concept Practice*

Provide a variety of situations and/or data that are linear, quadratic, or neither.

**Terminology***quadratic  
relationship***Language Goals**

- Take turns in conversations and classroom discussions (ESLAO).
- Give and respond to straightforward directions and instructions (ESLAO).
- Maintain face-to-face conversations on familiar topics (ESLBO).
- Restate important information from presentations that include visual aids (ESLBO).

**Materials****Assessment Opportunities****Minds On... Whole Class → Investigation**

Draw the appropriate table of values on the board showing how to complete the first difference column. Refer to the Word Wall for definitions to review the terminology for English language learners.

After discussing what has been learned, ask students to explain it to each other using their own words in an inside/outside circle. This provides several opportunities for English language learners to hear and explain the reasoning for determining if a relationship is linear or non-linear based on a first difference table.

**Make It Language Rich**

Using the inside/outside circle provides a safe opportunity for English language learners to state the important ideas aloud.

**Action!****Pairs → Summarizing**

Pair English language learners with same first language students, if possible, so that they can summarize first in their own language and then translate into English.

Alternatively, pair English language learners with English-speaking students who can assist with language for the summary.

Post all questions so that English language learners can determine what they are being asked.

**Groups of Four → Investigation**

Post instructions for the investigation as well as questions to be answered so English language learners can think in their first language before composing a response in English.

**Engage the Senses**

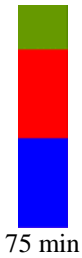
Provide linking cubes and graph paper for the investigation to help English language learners to make connections between words and meanings and to demonstrate their understanding in non-verbal ways.

**Consolidate Debrief Whole Class → Discussion**

Summarize discussion using concrete models, posted charts and written definitions so that English language learners have a variety of ways to consolidate their understanding.

Have students summarize the second difference property by explaining it to each other in an inside/outside circle.

**Home Activity or Further Classroom Consolidation***Concept Practice*



**Math Learning Goals**

- Identify the key features of parabolic graphs created on Days 1 and 2 (the equation of axis of symmetry, the coordinates of the vertex, the y-intercept, x-intercepts (zeros), and the max/min value).
- Develop and use appropriate vocabulary related to parabolic curves.

**Materials**

- BLM 4.4.1, 4.4.2, 4.4.3
- scissors
- glue sticks

**Assessment Opportunities**

**Minds On... Pairs → Timed Retell**

Provide partners A and B with a different graph of a quadratic relation (BLM 4.4.1). They should not see each other's graph.

Student A describes the key features of the graph to student B. Student B sketches the graph from student A's oral prompts. Students compare how close the sketch was to the given graph. The partners switch roles.

**Whole Class → Discussion**

Discuss the terms they used to describe their graphs to their partners. Emphasize the key features that were focused on in the descriptions.

Ask: Did you have difficulties identifying or describing any of the key features? If so, what was the difficulty?

- Word Wall
- axis of symmetry
  - vertex
  - the y-intercept
  - zeros
  - max/min value

The "Zero" is the value of x when y = 0

**Action! Whole Group → Guided Instruction**

Guide students in defining the terms that describe the features of a parabola and label the graphs, using appropriate terminology (BLM 4.4.2).

Connect the terminology to parabolic graphs created from real data.

**Consolidate Debrief Groups of 3 → Discussion**

Students rejoin groups of 3 and revisit the graphs they created on Days 1 and 2. They cut out the key terminology (BLM 4.4.3) and glue them in the appropriate location on their graphs providing a rationale for their choice.

**Curriculum Expectation/Demonstration/Mental Note:** Observe students' interpretation of the key features of a parabola as they identify these features of their graph.

**Home Activity or Further Classroom Consolidation**

Complete the practice questions.

*Application  
Concept Practice*

Provide students with questions that include parabolas that represent real data.



**Terminology**

*axis of symmetry  
vertex  
y-intercept  
zeros  
maximum or  
minimum values*

**Math Learning Goals**

- Determine the meaning of unfamiliar words, using pictures and illustrations (ESLAO).
- Include key conceptual vocabulary in person word lists for classroom study (ESLAO).
- Use vocabulary-acquisition strategies (ESLBO).
- Maintain a vocabulary notebook or list for various subject areas (ESLBO).

**Materials**

**Assessment Opportunities**

**Minds On...**

**Pairs → Timed Retell**

Pair English language learners with an English-speaking partner. Designate the English-speaking partner as Partner A to model the activity for their English language learner partner by going first. (Note: Partner A should speak slowly, and may have to repeat the key features several times)

**Whole Class → Discussion**

Point to the terms on the Word Wall and on the graph to help English language learners connect the words to the graph.

**Action!**

**Whole Group → Guided Instruction**

Make an acetate version of BLM 4.4.2 and project it on the board so students can stick the key terms in the appropriate locations during the discussion

**Consolidate Debrief**

**Groups of 3 → Discussion**

Make heterogeneous ability groupings, so stronger students can play a leadership role. Place English language learners in appropriate groups based on their understanding of mathematics rather than their language skills.

**Home Activity or Further Classroom Consolidation**

*Application  
Concept Practice*

Additional vocabulary may be necessary, e.g., opens up, opens down

**Make It Comprehensible**

Review the terminology explicitly to assist English language learners in participating in this lesson. Encourage English language learners to add the key terms to their personal word list.

**Engage the Senses**

Use as many visual aids as possible during this lesson so that English language learners see the connection between the key words and their location on the parabola.

# Unit 7: Quadratic Relations of the Form

$$y = ax^2 + bx + c$$

Grade 10 Applied

## Lesson Outline

### Big Picture

English language learners will:

- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- use strategies for decoding and comprehension;
- communicate orally using simple phrases and appropriate terminology.

Day	Lesson Title	Math Learning Goals	Expectations
5	Factoring Quadratic Relations of the Form $y = ax^2 + bx + c$ , Where $a$ Is a Common Factor	<ul style="list-style-type: none"> <li>• Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).</li> <li>• Work cooperatively with a partner on shared classroom tasks (ESLAO).</li> <li>• Participate in group work, cooperative games and teamwork (ESLBO).</li> <li>• Use first language appropriately in classroom and social situations (ESLBO).</li> </ul>	QR1.02, QR2.04  CGE 5b
6	Using Intercepts to Graph Quadratic Equations	<ul style="list-style-type: none"> <li>• Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).</li> <li>• Adapt learning strategies to a task and to the conditions of learning (ESLAO).</li> <li>• Listen to others and stay on topic in group discussions (ESLBO).</li> <li>• Ask questions of teachers and peers for clarification and to obtain information (ESLBO).</li> </ul>	QR1.02, QR1.03, QR2.04  CGE 3c
7	We Have a Lot in Common	<ul style="list-style-type: none"> <li>• Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).</li> <li>• Communicate information about various cultures (ESLAO).</li> <li>• Use short sentences and phrases to tell stories, recount events, provide directions or instructions, and give opinions (ESLBO).</li> <li>• Compare and contrast the traditions and behavioural norms of a number of cultures (ESLBO).</li> </ul>	QR1.03, QR2.04  CGE 3c, 5b

**ESLAO** – Beginning Communication in English builds on students’ previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment.

**ESLBO** – English in Daily Life expands students’ essential English communication skills and introduces the language of classroom studies.

# Unit 7: Day 5: Factoring Quadratic Relations of the Form $y = ax^2 + bx + c$ , Where $a$ Is a Common Factor (TIPS4RM)

Grade 10 Applied



## Math Learning Goals

- Determine the connections between the factored form and the  $x$ -intercepts of a quadratic relation.
- Factor binomials and trinomials involving one variable up to degree two, by determining a common factor using algebra tiles.

## Materials

- algebra tiles
- graphing calculators
- BLM 7.5.1, 7.5.2, 7.5.3
- glue stick

## Assessment Opportunities

### Minds On... Whole Class → Four Corners

Review the forms of equations and representations of quadratics and label four stations: factored form, standard form, graphical form, and chart or algebra tiles representation. Randomly hand out the cards (BLM 7.5.1).

Students move to the corner for the form represented on their card and check that everyone has the same form. They can help one another express their representation in the other forms and jot them on the back of their cards. They then “find” the other three people holding the related cards.

Groups justify they all have different forms of representations of the same quadratic. Paste these on a single page and post them for a gallery walk.

**Curriculum Expectations/Demonstration/Marking Scheme:** Collect worksheets and assess students’ understanding of the concepts needed to complete the Home Activity from Day 4.

### Action!

#### Pairs → Investigation

Students complete BLM 7.5.2.

Discuss the ideas investigated and extend to relations that include negative coefficients for  $b$  and  $c$ , e.g.,  $y = x^2 - x - 6$ . Students should understand that  $(r) \times (s) = c$  and  $r + s = b$ , yielding factors  $(x + r)(x + s)$  and  $x$ -intercepts of  $-r$  and  $-s$ .

Students complete BLM 7.5.3 and share their solutions on the overhead.

#### Whole Class → Guided Instruction

Introduce common factoring by displaying overhead tiles for  $3x + 3$ .

Demonstrate that this is 3 groups of  $(x + 1)$ , thus this can be expressed as  $3(x + 1)$ . The “common factor” is 3. Repeat for  $2x + 4$  and  $5x - 10$ .

Display the overhead tiles needed to factor  $2x^2 + 2x + 2$ .

Ask:

- Can these be placed into groups where each type of tile is equally represented in all groups? Why or why not?
- How many groups did you get? What are the contents of each group?

Show how the factored answer would be expressed.

Complete several more examples, such as:  $3x^2 + 6x + 12$ ,  $2x^2 + 6x - 8$ ,  $2x^2 - 8x + 10$ ,  $4x^2 + 8x + 8$ .

For  $4x^2 + 8x + 8$ , some students may think that 2 groups of  $(2x^2 + 4x + 4)$  would be appropriate when 4 groups of  $(x^2 + 2x + 2)$  is the most appropriate answer. Use this example to discuss the greatest common factor.

### Consolidate Debrief

#### Individual → Journal

Students summarize how to factor a trinomial, including conditions that make it possible for a trinomial to be factored and how to determine a common factor.

#### Home Activity or Further Classroom Consolidation

Complete the practice questions.

Concept Practice

$r \times s = -6$   
 $r + s = -1$   
 $-3$  and  $2$   
 factors are  
 $(x - 3)(x + 2)$   
 $x$  intercepts  $3$   
 and  $-2$

In Grade 10 Applied, students only factor  $ax^2 + bx + c$  when  $a$  is a common factor.

Answer:  
 $y = 2x^2 + 2x + 2$   
 $y = 2(x^2 + x + 1)$   
 i.e., 2 sets of  
 $x^2 + x + 1$

Another similar example:  
 $y = 4x^2 + 12x + 12$

Provide questions that require checking answers by expanding or graphing with technology.

## Unit 7: Day 5: Factoring Quadratic Relations of the Form $y = ax^2 + bx + c$ , Where $a$ Is a Common Factor

Grade 10 Applied

### Terminology

### Language Goals

- Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).
- Work cooperatively with a partner on shared classroom tasks (ESLAO).
- Participate in group work, cooperative games and teamwork (ESLBO).
- Use first language appropriately in classroom and social situations (ESLBO).

### Materials

### Assessment Opportunities

#### **Minds On...**

#### **Whole Class → Four Corners**

When reviewing the forms and representations of quadratics, use the names and examples for each station. Post examples as a visual reference.

**Incorporate Identity**  
English language learners can write the names of representations in their own language on the posted examples.

**Make It Comprehensible**  
Demonstrating one example to the whole class using student math talk helps English language learners to understand what is expected.

**Engage the Senses**  
Providing algebra tiles and graphing calculators for students will help English language learners to make connections between area, factoring, and  $x$ -intercepts.

#### **Action!**

#### **Pairs → Investigation**

Demonstrate the area of  $x^2 + 4x + 3$  using overhead algebra tiles by having students arrange the algebra tiles so that all the pieces fit into a rectangle. Ask students to explain their thinking and encourage the class to ask questions. Have students use the overhead graphing calculator to follow up on the second part of the investigation, explaining and asking each other questions as they proceed. This models the solution in a concrete way and gives English language learners a chance to hear about and see how they are to conduct the investigation.

#### **Whole Class → Guided Instruction**

When demonstrating common factoring using algebra tiles, summarize the ideas on the board for visual reference. When completing extra examples, have small groups of students try it first on their own with algebra tiles so English language learners can work through and discuss the ideas in a small group before discussing as a whole class.

#### **Consolidate Debrief**

#### **Individual → Journal**

English language learners summarize their understanding in their first language; share with a partner with the same first language, if possible; then translate their understanding into English using short phrases and diagrams.

#### **Home Activity or Further Classroom Consolidation**

*Concept Practice*



75 min

**Math Learning Goals**

- Consolidate factoring.
- Connect factors to the  $x$ -intercepts of the graph of the quadratic relation.
- Graph quadratic relations, using intercepts.

**Materials**

- BLM 7.6.1
- large chart (grid) paper

**Assessment Opportunities**

**Minds On...**

**Pairs → Review**

Students discuss their journal entry from Day 5 and refer to their answers on BLM 7.5.1 and 7.5.2.

Pairs factor the following quadratic expressions, discuss what is the same and different about each, and how the results would correspond to the  $x$ -intercepts of the corresponding relation:

$$x^2 + 11x + 30, x^2 - 11x + 30, x^2 - x - 30, x^2 + x - 30.$$

**Curriculum Expectation/Observation/Checklist:** Listen to students' conversations to identify level of understanding of factoring trinomials and the connection between the factors and the  $x$ -intercepts.

**Action!**

**Whole Class → Demonstration**

Using large chart (grid) paper, place a dot on the  $x$ -axis at  $-4$  and  $2$ , and a dot on the  $y$ -axis at  $-8$ . Point out the  $-4$  and  $2$  are the  $x$ -intercepts and  $-8$  is the  $y$ -intercept of a parabola.

Ask volunteers to sketch the parabola that goes through these points. Point out that, at this time, they are focusing on the symmetry of the parabola, and that the parabola intersects correctly on the  $y$ -axis, and the  $x$ -axis. Repeat this several times using other values for the intercepts.

Discuss question A on BLM 7.6.1 and do question B together. Refer to question F to be sure students understand what is different in this question.

**Individual → Practice**

Students complete BLM 7.6.1.

Students who finish early can assist students who are having difficulty, and/or put an answer on the overhead and make a sketch of the relation on large chart (grid) paper.

**Consolidate Debrief**

**Whole Class → Discussion**

Students check their answers using the response on the overhead and on the chart paper grid.

Lead a discussion that compares graphing linear relations using intercepts and quadratic relations using intercepts. Discuss how factoring provides an efficient method for making a sketch of the graph, recognizing that it is a sketch only.

**Home Activity or Further Classroom Consolidation**

Complete the practice questions.



Include some examples where the parabola is opening downward, and where there is a double root.

Note the use of the word *sketch*. The graph is not intended to be exact, but to show the symmetry, the intercepts, and the general movement of the curve.

Students can visualize and draw their solution in the air also.

Some students may still need algebra tiles to factor.

Provide students with appropriate practice questions.

*Differentiated  
Exploration  
Reflection*

**Terminology**

**Language Goals**

- Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).
- Adapt learning strategies to a task and to the conditions of learning (ESLAO).
- Listen to others and stay on topic in group discussions (ESLBO).
- Ask questions of teachers and peers for clarification and to obtain information (ESLBO).

**Materials**

**Assessment Opportunities**

**Minds On...**

**Pairs → Review**

Pair English language learners with same first language students, if possible, so that they can discuss their understanding in their own language. Follow up by having English language learner pairs share with English-speaking pairs so that they have an opportunity to explain their understanding in English.

**Make It Language Rich**

Encourage English language learners to highlight key words on BLM 7.6.1 and determine they are clear on the meaning.

**Engage the Senses**

Provide algebra tiles, as necessary.

**Assess with Sensitivity**

When listening to students' conversations, take into account English language learners facility with English language.

**Action!**

**Whole Class → Demonstration**

Refer to the Word Wall when reviewing the key words/concepts, symmetry,  $x$ -axis,  $y$ -axis,  $x$ -intercepts,  $y$ -intercept and vertex.

Highlight or point to the key concepts as volunteers sketch the parabolas demonstrated. It is important that English language learners understand that this is a sketch only and may not be exact; students could draw more than one curve that fits the criteria to get this idea across.

**Individual → Practice**

Encourage English language learners to ask other students for help if they are having difficulty. Include English language learners in the preparation of solutions that will be used for **Consolidate Debrief**.

**Consolidate Debrief**

**Whole Class → Discussion**

Provide visual cues to make it easier for English language learners to participate or follow.

**Home Activity or Further Classroom Consolidation**

Assign work based on English language learners understanding of concepts rather than their facility with the English language.

*Differentiated  
Reflection  
Exploration*



**Math Learning Goals**

- Determine the connections between the factored form and the  $x$ -intercepts of the quadratic relations  $y = x^2 + bx$ .

**Materials**

- graphing calculators
- algebra tiles
- overhead algebra tiles
- BLM 7.7.1, 7.7.2, 7.7.3

**Assessment Opportunities**

**Minds On... Whole Class → Discussion**

Use the context of kicking a soccer ball. Depending on the kick, the ball may travel different heights and distances. Lead a discussion as to how to draw a height vs. horizontal distance graph for three scenarios in which there are different height and distance measures.

Ask:

- If you were to represent this scenario graphically:
  - what are appropriate labels for the axes? [height above ground in metres; horizontal distance from the kicker’s toe in metres]
  - where would the point be that represents the kick?
  - what would one of the  $x$ -intercepts of the graph be?
  - what might the general equation of the graph look like?

**Action!**

**Pairs → Group Investigation**

Students work through the graphing calculator investigation (BLM 7.7.1).

Provide each pair with one of the following equations:  $y = x^2 + 4x$ ,  $y = x^2 + 6x$ ,  $y = x^2 - 2x$ ,  $y = x^2 - 5x$ . Pairs of students complete questions 1–3 on BLM 7.7.1.

Form groups of four where each student in the group has a different equation. Students complete questions 4–9 (BLM 7.7.1). They compare what is the same about their graphs and what is different and comment on the  $x$ -intercept,  $y$ -intercept, and the general shape. Lead a discussion about the algebraic representations, e.g.,  $y = x(x + 4)$ ,  $y = (x - 0)(x + 4)$  and  $y = (x + 4)(x - 0)$  being equivalent ways of expressing the equation  $y = x^2 + 4x$  in factored form.

Use algebra tiles to demonstrate common factoring of  $y = x^2 + 6x$ , then do Example 1 and 2 of BLM 7.7.2. Students complete BLM 7.7.2. (See Teacher BLM 7.7.3.)

**Mathematical Process/Using Tools/Observation/Anecdotal Note:** Assess students’ use of graphing calculators and algebra tiles to identify the factors.

**Whole Class → Demonstration**

Relate the algebra tile method of factoring to the algebraic method. Demonstrate the algebraic method using several examples, including some with negatives.

**Consolidate Debrief**

**Individual → Journal**

Students describe the three methods of common factoring that were investigated in the lesson, and describe the connections of the factors to the  $x$ -intercepts of the corresponding relation.

**Home Activity or Further Classroom Consolidation**

*Concept Practice* Complete the practice questions.

**Answers**  
Same: zero as a  $y$ -intercept, zero as one  $x$ -intercept, shape, and size, opening up

Different: the other  $x$ -intercept

Provide students with appropriate practice questions.

**Terminology**

**Language Goals**

- Demonstrate an understanding of classroom directions and activities, and of key vocabulary (ESLAO).
- Communicate information about various cultures (ESLAO).
- Use short sentences and phrases to tell stories, recount events, provide directions or instructions, and give opinions (ESLBO).
- Compare and contrast the traditions and behavioural norms of a number of cultures (ESLBO).

**Materials**

**Assessment Opportunities**

**Minds On...**

**Whole Class → Discussion**

Provide a video clip of a soccer game, use a nerf ball, or show some pictures and diagrams to set the context for this problem and to connect the context to the mathematics. Post the questions being considered to provide visual cues.

Additional vocabulary may be necessary, e.g., soccer kick, height, horizontal distance

**Action!**

**Pairs → Group Investigation**

Provide visual cues during the discussion about algebraic representations.

**Whole Class → Demonstration**

Share both algebra tile and algebraic solutions for the demonstration. Ask students to explain their reasoning and paraphrase each other’s explanations to provide several opportunities for English language learners to make sense of the discussion. Ask English language learners to paraphrase an explanation if they feel comfortable doing so.

**Make It Comprehensible**  
Stress that the “common” factoring introduced previously (the common factor was numerical) is extended in this context (the common factor is a variable).

**Engage the Senses**  
Provide algebra tiles, as necessary.

**Consolidate Debrief**

**Individual → Journal**

Encourage English language learners to describe their findings, using their own language, pictures, and diagrams as well as English for the journal.

*Concept Practice*

**Home Activity or Further Classroom Consolidation**