CIIM

Curriculum Implementation Intermediate Math

Research Report

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Curriculum Implementation in Intermediate Math (CIIM) Research Project

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Curriculum Implementation in Intermediate Math (CIIM) Research Project

Introduction

In Fall 2005, the Ontario Ministry of Education introduced a revised curriculum in mathematics for Grades 1–10 that is a refinement of the previous mathematics curriculum introduced in elementary schools in 1997 and in secondary schools in 1999 for Grades 9 and 10. This curriculum encourages inquiry and the development of mathematical reasoning and engages students in mathematical activity to develop an understanding of mathematics concepts as well as mathematical skills. All teachers of intermediate mathematics in the province of Ontario are to implement this curriculum in their classrooms. As with all such important initiatives, those responsible for the initiative as well as those engaged in the initiative itself are interested in knowing how it is working. The Curriculum Implementation in Intermediate Mathematics (CIIM) research project was initiated in January 2006 to examine how this inquiry-oriented mathematics curriculum for Grades 7-10 in Ontario is implemented and understood by the multiple partners involved. In this report we describe the background, and evolution of this large-scale research initiative and share some of the research findings from year 1.

Background

For a very long time the content of mathematics education has been understood as a set of procedures, rules, and algorithms, and in that tradition the role of the teacher was to introduce these procedures and concepts in a logical sequence, and to provide enough examples and time for learners to practice applying the rules to a set of problems. The current reforms in mathematics education (NCEE, 1983; NCTM, 1989, NCTM 2000) have grown out of a concern over learners' limited understandings of mathematics when taught from this perspective. The mathematics education community increasingly acknowledges that doing and understanding mathematics goes well beyond knowing mathematical facts and procedures. It also means being able to reason mathematically and to have the ability to interpret and solve mathematical problems (Artelt, Baumert, Julius-McElvany, & Peschar, 2003; Ball 2003; Boaler, 2002; Hiebert, 1997; NCTM, 2000). Research shows that children learn more mathematics when instruction is based on students' ways of thinking, when students are engaged in problem solving, (Yackel & Cobb, 1996; Zack & Graves, 2001) and when teachers assist students in seeing the connections among various mathematical ideas (Lampert, 1990). Current mathematics curricula reflect the importance of engaging students in meaningful mathematics content through mathematical activity such as conjecturing, analyzing, investigating, and explaining.

The Ontario math curriculum for intermediate mathematics

The current Ontario math curriculum for intermediate mathematics is aligned with curricula in other jurisdictions and reflects current thinking and research in mathematics education. It has evolved through a collaborative process involving practitioners, policy makers and mathematics education researchers, and the most recent revisions of the curriculum began with the writing of background research reports to inform the curriculum writers of current research. These revisions also included discussions with various stakeholders so that the mathematics education community, as well as other communities, were participants in the curriculum revisions.

Both the current Grades 1 – 8 mathematics curriculum (OME, 2005a) and the Grades 9 and 10 mathematics curriculum (OME, 2005b) present progressive messages about the incorporation of technology, the value of mathematical communication, the importance of deepening mathematical understanding through inquiry and problem solving, and the accessibility of mathematics for all learners. The specific curriculum expectations in the curriculum documents use verbs such as *investigate*, *explain*, and *model*, to encourage a variety of mathematical processes and engage students with meaningful mathematics content. Following is an example of a specific curriculum expectation for Grade 7 that prompts students to recognize relationships between area and perimeter through investigation using a variety of tools such as concrete materials and technology.

> Students will determine, through investigation using a variety of tools (e.g., cans and string, dynamic geometry software) and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas. (OME, 2005a, p. 113)

A specific curriculum expectation for Grade 9 directs students to investigate linear relations with the use of technology to determine specific properties (the meaning of slope and y-intercept) of these relations.

> Students will identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$. (OME, 2005b, p. 34)

This active investigation of concepts with mathematical thinking tools, as required in the specific expectations described above, allows teachers and students alike the opportunities to engage mathematically as they jointly become aware of patterns, make conjectures, compare results, and provide explanations. This type of engagement creates a space of learning and of opportunity in which different representations often highlight different aspects of a complex concept or relationship which enrich the understanding of mathematical concepts.

Resources and professional development

Recognizing that the new ways of teaching mathematics are challenging for teachers, the implementation of the Ontario mathematics curriculum has been supported in a variety of ways. The Ontario Ministry of Education has provided funding for resources, technology, and professional development to support the curriculum. Resource documents related to intermediate mathematics, Grades 7-10, have been released by the Ontario Ministry of Education through provincial initiatives and include *Think Literacy,* 2003, 2004 and 2005; *Leading Math Success,* 2004; and *Targeted Implementation and Planning Supports (TIPS),* 2003, 2004 and 2005. The resource that appears most closely connected to teachers' classroom practice is the TIPS document which is a teaching resource that includes detailed course plans, unit plans, lesson plans, blackline masters and reference materials aimed at assisting teachers of Grades 7 and 8, as well as Grades 9 and 10 Applied courses. The TIPS documents also include a 32-page review of research on instructional and assessment practices. There is an online version of the resource that provides tutorials on Ministry-licensed software for the math classroom, and pre-made files for *Geometer's Sketchpad®*. In addition there are PowerPoint files for some lessons and videos of planning sessions. To assist teachers in becoming familiar with these materials, the TIPS resource was supported with numerous professional development opportunities.

Other professional development and resources have been developed in collaboration with leadership groups in mathematics education such as the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA). Also, the close collaboration of the Ontario Ministry of Education with OAME and OMCA has helped to develop a strong community in mathematics education, a community that has been able to support mathematics education initiatives.

What this means for teaching

This curriculum challenges teachers with new ways of thinking about mathematics teaching and learning. While the implementation of a reform mathematics approach requires the alignment of several components including curriculum, resources, activities, classroom structures, teaching approaches, and the role of the teacher, the research suggests that the actual practice of teaching, such as the way a teacher poses questions or responds to students' understandings, is the most critical element (Ball & Bass, 2002; Boaler, 2002). In our view the kinds of changes teachers are asked to undertake in order to successfully engage all learners in mathematical inquiry, are not simple and require a substantive re-orientation of their basic beliefs about mathematical ideas as well as mathematics teaching and learning. Even when curricula are available and are supported by professional development and resources, reform-oriented teaching practices are not necessarily evident (Ball, 2003; Frykholm, 1999). The posing of problems, the facilitation of discussion, and the consolidation of mathematical concepts are teacher practices that require a great deal of knowledge and attention. In many cases, teachers, themselves, have not learned mathematics in this way, nor have they had opportunities to learn and teach in inquiry-oriented settings. We know that even when teachers have had some inquiry-oriented learning experiences and acknowledge this approach as supporting their understanding of mathematics, there remains a visible tension between the reform-oriented and traditional approaches to teaching mathematics (Jacobs, Hiebert, Givven, Hollingsworth, Garnier, & Wearne, 2006).

The CIIM research project

The Curriculum Implementation in Intermediate Mathematics (CIIM) research project was initiated in January 2006 to examine how this inquiry-oriented mathematics curriculum for Grades 7-10 in Ontario is implemented and understood by the multiple partners involved. Our goal was to design and carry out a research project that strengthens our connections between researchers, teachers and policymakers in order to understand the complex process of implementation of inquiry-oriented mathematics curricula. It is important for us to conduct research that is useful to practitioners and policymakers so that they can make more informed decisions. From our perspective, however, it is not only a matter of bringing research into practice and policy, but of understanding policy and practice as we do research. Therefore our research design is multi-faceted and longitudinal as we work to understand how the new curricular and pedagogical experiences are understood by policy makers, mathematics teachers and researchers.

The research team

Our University of Ottawa research team is comprised of two university researchers, a project manager, and graduate student research assistants. The team has a variety of expertise and experience in curriculum, instruction and assessment, and teacher development. Several of the graduate students are or have been teachers in the Ontario system and therefore have first-hand experience of the curriculum and are aware of many of the math initiatives in Ontario. We are also working in collaboration with Dr. Geoffrey Roulet from Queen's University and several of his graduate students as research assistants.

Research design

This research project is designed to address the following aspects of the curriculum implementation:

- To determine how the current intermediate mathematics curriculum is understood and taught
- To understand how teachers have been supported in the implementation of this curriculum
- To describe learning environments that reflect the enactment of the curriculum

These research issues are addressed through a longitudinal, multi-faceted research design that includes focus group interviews, a web-based questionnaire, and case studies. Below is a summary of the design.

- Focus groups interviews with leaders in mathematics education (Winter 2006)
- Web-based questionnaire for math teachers in grades 7 - 10 (May to mid-July 2006)
- Case studies in classrooms that have been identified as implementing the curriculum (2007-2008)
    - Focus group interviews with teachers (Spring/Fall 2007)
    - Classroom site data collection (2007-08)

In this report we are presenting results from the focus group interviews with math education leaders and the web-based questionnaire for math teachers Grades 7-10.

Interviews with math education leaders in Ontario

Focus group interviews with mathematics coordinators, and leaders in mathematics education provide an important contribution to our understanding of the multiple perspectives of the curriculum and its implementation. These participants have been actively engaged in the curriculum implementation process and have worked extensively with teachers, administrators, and other leaders in the mathematics education community. It was important for us to listen to their expertise and experience to begin to understand the types of implementation supports that teachers have received, the level of curriculum implementation to date, and the professional development which they feel is still required. To recruit participants for this phase of the research we contacted the presidents of two professional associations, the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA) to inform them of the project and to request permission to address members of their respective organizations. Once they gave us permission we provided them with a recruitment email letter to distribute to the OMCA members and the OAME directors. The letter invited leaders in mathematics education to share their expertise and experience in focus group interviews or individual interviews to be scheduled at an upcoming meeting of their association. We informed them that all individual and focus group interviews would be audio taped. We suggested a number of possible meeting times and requested that those who were interested reply to us by email.

In February 2006 we conducted interviews with math education leaders who were attending the Board of Directors meeting of the Ontario Association for Mathematics Education (OAME). We held 3 focus group interviews with a total of 8 participants. We also attended the February meeting of the Ontario Mathematics Coordinators Association (OMCA) meeting in Waterloo, Ontario to present our research project and to conduct focus group interviews. At this meeting the participant groups were created through “open-space technology”, a strategy that allows participants to suggest relevant topics and to negotiate these topics to form discussion groups. Based on the interests of those present, three topic groups emerged. These were assessment, teachers’ use of tools and strategies to teach math, and the role of mathematical processes. Over 20 OMCA members participated in these 3 discussion groups. The discussions within these groups were not limited to the original topic. In fact, in all of our discussions with math education leaders our data suggest that topics were wide ranging and covered many aspects of curriculum implementation. All focus group interviews were audio taped and transcribed.

Our 6 focus group interviews revealed that the leaders in math education have a good understanding of the curriculum and are knowledgeable about current teacher practice including how the curriculum is enacted by teachers in their communities. They seemed to be in touch with the state of implementation, the successes to date and the work still to be done. A preliminary analysis of the data from these focus group interviews identify some recurring themes that helped inform the design of the teacher questionnaire.

Web-based Teacher Questionnaire

Another component of this research was a web-based questionnaire for teachers of Grades 7-10 mathematics that was made available online from May to mid-July 2006.

Questionnaire development. In designing the questionnaire, we examined a variety of other questionnaires that had been developed for teachers including those from TIMSS, PISA, and EQAO. We also consulted members of the Ontario Ministry of Education Advisory Panel for Connecting Practice and Research in Mathematics Education. Once the questionnaire was designed, members of the Advisory panel as well as researchers in other universities were consulted for feedback. A paper version of the revised questionnaire was then pre-tested by practicing teachers. The pre-testing led to further revisions and also helped to determine the amount of time needed for participants to complete the questionnaire. A web-design team then transformed the paper format into a web-based questionnaire. They then tested the web-based version extensively to ensure that it would perform in accordance with our specifications once it was online.

The content of the items included teaching experience, background in mathematics, understanding of the curriculum, instruction and assessment, and professional development. More specific topics were:

- What are the current practices of teachers of mathematics?
- What types of instructional activities are students engaged in?
- What forms of assessment are being used in mathematics?
- How do teachers use assessment data?
- What are teachers' understandings of the curriculum?
- How well do these current practices match the new curriculum?
- What are the supports that teachers have received for curriculum implementation?
- What are some of the challenges that teachers face in implementing the curriculum?
- What types of support are needed to enhance curriculum implementation?

The final questionnaire had a total of 45 items some of which were general questions while others were course specific. For course-specific items, we asked respondents to identify one particular math class that they were teaching during the school year and then use that class as the basis for their answers. The course-specific items allowed us to distinguish different practices, activities and attitudes associated with different courses. Following is an example of a general question and course-specific question.

Example of a general question

To what extent do you think your philosophy of teaching/learning mathematics is aligned with the philosophy of each of the following? Check one box in each row

	Not applicable	Not at all	A little	Somewhat	A lot
a) Your colleagues	□	□	□	□	□
b) Your mathematics consultant/coordinator	□	□	□	□	□
c) Your department head	□	□	□	□	□
d) Your principal	□	□	□	□	□
e) The Ontario mathematics curriculum	□	□	□	□	□

Example of a course-specific question

In **this class**, how important is each of the following? Check one box in each row.

Distribution of web-based questionnaire. One of the challenges of this research was how to identify and contact the teachers in the province who are teaching intermediate mathematics. When we began, we assumed that someone would have this information, but as we inquired we soon came to realize that it is not clear exactly who is teaching intermediate mathematics. Therefore we needed a strategy for contacting those teachers, and we developed the following approach. We sent letters to Directors of every English school board in the province inviting their board to participate in the web-based teacher questionnaire. If they accepted our invitation, the boards provided us with a contact person who could tell us the number of schools in their board where intermediate mathematics was taught. With that information, we designed a package for each school consisting of ten letters addressed to intermediate math teachers in the school and one letter for the school principal. In schools with large math departments we sent additional letters for teachers. The letters for the math teachers described the project and invited them to participate. Each letter included an individual access code and information about how to access the online questionnaire. The board contact persons distributed these packages to the schools in their boards. Out of 60 boards contacted, 42 accepted the invitation. In total, 1096 teachers responded to the questionnaire with a representative distribution of school boards and grades.

Who were the respondents? The sample was nearly evenly divided with 46% teaching Grades 7 and 8, and 51% teaching Grades 9 and 10. Only 3% of respondents reported "other" as their teaching assignment. The majority of respondents (93%) identified their primary role as a full time classroom teacher. Of the Grade 7/8 teachers, 42% reported that they teach math for 60 minutes or less each day while 31% report teaching math for more than 60 but less than 90 minutes daily. This suggests that these Grade7/8 teachers were responsible for the full range of subjects in the Grade 7 and 8 classes, rather than focusing on one subject on a rotary timetable. Of the Grade 9/10 teachers, 62% report that they teach math 3.5 hours or more daily. This suggests that more than half of the Grade 9/10 teachers in the survey teach a full timetable of math.

Table 1: The questionnaire respondents

	Frequency	Percent of respondents
Grade 7 and 8 teachers	506	46%
Grade 9 and 10 teachers	559	51%
Other	31	3%

Case Studies

In addition to the questionnaire and interview data, our research includes a number of case studies to obtain specific details about the curriculum implementation in classrooms and schools. These case studies focus on learning environments that reflect the enactment of the curriculum.

Focus group interviews with teachers (2006-2007). In preparation for the case studies we held with focus group interviews with selected teachers in various regions in the province, beginning with Eastern Ontario. These teacher focus group interviews provide us with information about specific classroom practices in intermediate mathematics and also provide an opportunity for developing a relationship with the teachers as collaborative partners.

Classroom sites. The case studies of classrooms are being conducted in various school boards across Ontario. The data from the case studies will include audiotaped interviews with teachers and principals, videotaped classroom observations of 5-7 mathematics lessons, collection of documents such as instructional and assessment activities, and board and school policies that affect classroom practice. The analysis of the case study data will help us to describe and understand the implementation of the curriculum in various contexts.

Organization of this report

This report began with an introduction that provided the context and background for this research project as well as the research design. The remainder of the report focuses on 5 specific themes that were discussed repeatedly in the data and that are important to share with the mathematics education community at this time. The 5 themes are:

- Transition from elementary to secondary mathematics

- The use of technology in the mathematics classroom
- The use of manipulatives in the mathematics classroom
- Assessment in mathematics
- Professional development in mathematics

Each theme is presented as an independent chapter so that readers can focus on the topic(s) in which they are most interested. For each theme, we include a review of the research literature related to the topic, a discussion of topic from the perspective of the Ontario mathematics curriculum (if applicable), the findings from the focus group interviews with the mathematics education leaders, and from the web-based questionnaire for teachers. Following the presentation of the data, we provide concluding comments that intergrate and reflect on these multiple perspectives. A reference list is provided at the end of each chapter.

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Minding the Gap: Transition from Grade 8 to Grade 9 Mathematics

What does the research literature say?

Research on students who are making the transition from elementary to secondary school shows that times of school transition can be problematic as students often face a different, more anonymous environment while being expected to negotiate greater social and academic demands (Braddock & McPartland, 1993; Hargreaves & Earl, 1991; Legters, 2000; Letgers & Kerr, 2001; Oakes, 2005; Roderick, 1993). While the differing cultures of elementary and secondary schools present their own challenges for students, the additional pressures of adolescent development serve to amplify their difficulties during transition (Fine, 1994; Rice, 1997; Seidman, Allen, Aber, Mitchell, & Feinman, 1994). Adolescent uncertainty and anxiety may undermine students' confidence and may contribute to feelings of alienation within the high school community. In some cases learners are faced with more demanding academic requirements, demands that can be overwhelming for those who may not be adequately prepared in elementary school courses. A number of research studies investigating academic failure during transition to high school, report that such failure may increase the probability of not graduating from high school (King, Warren, Boyer, & Chin, 2005; Lee & Burkam, 2003; Roderick, 1993). Furthermore, students coming from diverse backgrounds may face even greater challenges if adequate attention and support for their needs are not in place as they make the transition to high school (Legters, 2000; Roderick, 1993; Seidman et al., 1994).

In the context of Ontario education, Hargreaves and Earl (1991) presented similar findings based on an extensive review of the literature on the transition of upper elementary learners to secondary schools. In their report, the broad challenges that confronted students, including student anxiety in relation to attending a secondary school, and difficulties adjusting to a secondary school environment, may negatively impact learning, motivation, and engagement in school culture. Hargreaves and Earl (1991) also identified the impact of gaps or repetition in the curriculum on student learning. Recent research in Ontario discusses the connection between the early secondary years and success in completing high school. A study of students who are choosing to leave school before graduation (Ferguson, Tilleczek, Boydell, & Rummens, 2005), focused specifically on understanding the process of disengagement from school, and the issues associated with early school leaving. The results of this study revealed that early indicators of withdrawal or disengagement typically begin with experiences of school failure.

Similarly, in a recent study that investigated the graduation patterns of students in Ontario schools in relation to a restructuring of high schools from five to four years (King et al., 2005), the researchers discovered that certain courses had a high rate of failure particularly in the first year of high school. In their report, they identified the Applied courses in English and Mathematics as problematic. In particular, in 2002-03 the majority of students in the Grade 9 Applied Mathematics course were failing or close to it. A similar pattern was occurring in Grade 9 Essentials/Locally Developed Mathematics course. Further to this, they identified the correlation of an early loss of credits with failure to graduate. Even though the situation has begun to improve with the failure rates in these courses declining in 2003-04 and continuing to decline in 2004-05, the researchers continued to express concern as an increasing proportion of students were enrolling in the Applied Mathematics courses where failure rates were still higher than in the Grade 9 Academic Mathematics course.

Taken together, these findings highlight some of the complexity involved in making the transition from elementary to secondary school and caution that the inability to adjust can have an adverse effect on learning, grades, and success in completing high school. Among the recommendations to address these issues, these researchers stress the importance and value of ongoing communication and collaboration between teachers from both the secondary and elementary school communities as they discuss their programs of study, their students, their values, and their pedagogical approaches. Of particular importance are opportunities for teachers of both communities to come together to work on joint projects and activities. "It isn't just <u>talk</u> but <u>work</u> that binds" (Hargreaves & Earl, 1991, p.70). Other recommendations pertain to organizational reforms such as small learning communities, induction programs, de-streaming in favor of a common core curriculum, and interdisciplinary teaming which serve to create more responsive and caring learning environments at the high school level (Ferguson et al., 2005; Hargreaves & Earl, 1991; Lee & Burkam, 2003; Lee & Smith, 1995; Legters, 2000; Oakes, 2005).

The Ontario mathematics curriculum and transition

The curriculum documents, The Ontario Curriculum, Grades 1-8: Mathematics revised (OME, 2005a) and The Ontario Curriculum, Grades 9 and 10: Mathematics revised (OME, 2005b) speak directly to the issue of the transition from elementary school mathematics to secondary school mathematics.

> The transition from elementary school mathematics to secondary school mathematics is very important for students' development of confidence and competence. The Grade 9 courses in the Ontario mathematics curriculum build on the knowledge of concepts and skills that students are expected to have by the end of Grade 8. The strands used are similar to those used in the elementary, with adjustments made to reflect the more abstract nature of mathematics at the secondary level. Finally, the mathematics courses offered in secondary school are based on principles that are consistent with those that underpin the elementary program, a feature that is essential in facilitating transition (OME 2005a, p. 4).

Both sets of curriculum documents share a common vision and philosophy of mathematics teaching and learning. In both documents the justification for learning mathematics is related to the student's ability to participate in a future society that is oriented by knowledge and technology. This orientation is manifested in the opening paragraphs of both documents. In the Grades 9-10 curriculum documents, the study of mathematics will help students "meet the demands of the world in which . . . they will require the ability to use technology effectively and the skills for processing large amounts of quantitative information" (OME, 2005b, p. 3). In the Grades 1-8 curriculum documents, the "study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society" (OME, 2005a, p. 3).

Such actions and habits of mind are embedded in the mathematical processes that are foregrounded in both curriculum documents. The interconnected mathematical processes include problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. The mathematical processes are seen as central to developing a grounded and meaningful understanding of mathematics. They help students to "think critically, analyse and adapt to new situations, solve problems, communicate effectively" (OME, 2005a) and "learn independently" (OME 2005b).

The principles expressed in both sets of documents recognize diversity among students, and that all students can learn mathematics and deserve the opportunity to do so. Both curriculum documents support equity by promoting the active participation of all students. They also recognize that all students do not learn mathematics in the same way with the same resources, in the same time frame. There is a firm commitment to the idea that students develop mathematical understanding when they are given opportunities to investigate ideas and concepts through problem solving. The curriculum documents also support the view that mathematical knowledge develops over time.

What do Ontario mathematics education leaders say?

Our 6 focus group interviews with leaders in mathematics education revealed that they have a good understanding of the curriculum as put forth in the curriculum documents. Math leaders also are knowledgeable about current teacher practice including how the curriculum is enacted by teachers in their communities. They appear to be in touch with the state of implementation, the successes to date, and the work still needing to be done. With respect to their discussion regarding transition, their comments surfaced in the context of other topics. We did not directly ask them to discuss the transition from elementary to secondary school. Nevertheless, this was a topic that emerged repeatedly in the group interviews.

As the math education leaders spoke about transition, there was a sense that the transition of students between Grades 8 and 9 was not as smooth as it should or could be. The mathematics leaders noted that along with moving from Grade 8 to Grade 9, students are also moving from one school culture to another. In addition, students are being streamed academically based on past performance and decisions about future educational and career plans. Focus group participants also acknowledged that students are going through all the changes that young adolescents experience. They spoke of the different demands in elementary and secondary mathematics classes and gaps they perceived in the continuum of curriculum expectations. There was widespread agreement and emphasis on the importance of good communication between the elementary and secondary panels, while acknowledging the challenges to this communication. The mathematics leaders also suggested some ways to overcome these challenges.

In the discussions pertaining to transition, some of the mathematics leaders reported that they felt that the communication between teachers of Grades 7 and 8 and teachers of Grades 9 and 10 is not very strong. They offered a number of scenarios to capture what happens in their communities. In some cases, there is no conversation between panels. In others, when secondary and elementary teachers do meet, the communication is not necessarily effective. There often appears to be a lack of understanding of one another's needs, experiences, and context.

> And I think another aspect to it all is that there has not been a really good understanding on the part of the secondary teachers to understand what it is that elementary teachers do and even what the content is in Grades 7 and 8. (Frank, Math Leader Focus Group [MLFG] 2)

In other cases, one group may want to tell the other group how they should be teaching.

> I think the impression was that when high school teachers showed up to a meeting it was because they were here to tell us how to do our job better or how to do it right, that we are not doing it right, and they're just going to come and tell us how to do it. (Bob, MLFG 2)

The mathematics leaders reported that when groups of teachers from different panels have the opportunity to get together, there are often very different views of teaching and learning expressed. In addition, their discussions revealed that the positions of elementary and secondary teachers are not necessarily consistent within each panel. In one group, the elementary teachers were noted as being more traditional than the secondary, whereas another group of mathematics leaders described the secondary teachers as more traditional. For example, in discussions about the use of manipulatives, a number of mathematics leaders mentioned that teachers in Grades 7 and 8 may be reluctant to use manipulatives because they feel that they need to prepare students for high school where manipulatives won't be used. Other mathematics leaders mentioned that secondary teachers' desire to use manipulatives was being hindered by the fact that students are unfamiliar with such tools. They felt that the use of Algebra Tiles and other manipulatives in Grades 7 and 8 would be appreciated by secondary teachers.

> So, if I'm going to be teaching in Grade 10, whether it's Applied or Academic, I'm doing factoring and I want to use the Algebra-Tiles, I really don't want to invest the front-end time to teach them what they are, what the colours mean. It would be great if that was just part of their repertoire from Grade 7 and Grade 8. But we have to agree together as a community that we're all going to do it. (Kathy, MLFG 6)

At the same time, we hear of instances when secondary teachers tell the Grade 7 and 8 teachers to teach in traditional ways and not to bother teaching with manipulatives as it takes too much time.

The mathematics leaders described the need for enhanced communication across panels. For example, in a discussion of the pressures felt by Grade 9 teachers because of the EQAO testing, participants pointed out that while the Grade 9 teachers see the onus of students' results placed on them, what students learn in Grades 7 and 8 is important to their learning in Grade 9. Although the mathematics leaders identified a number of challenges, they also were aware of positive cross-panel collaborations. Most of the discussions centered on meetings that were specifically constructed to promote a positive dialogue. For instance, participants reported that professional development on the TIPS resource had helped to create a dialogue across panels because teachers from Grades 7 through 9 are invited to the same workshops.

> I would say TIPS in my particular experience moved people along because first of all, it opened a dialogue between 7, 8, and 9 teachers. And the PD was organized so that they were together and the dialogue started that hadn't been there before, in a way that they were all learning at the same time. (Joan, MLFG 2)

Mathematics leaders also emphasized the importance of having administrative support for cross-panel initiatives, and they suggested that new provincial and board initiatives that support communities of schools or new numeracy initiatives are helpful in bringing teachers together. Such initiatives are regarded as beneficial for cross-panel communication as well as for sharing ideas within panels since they provide opportunities to discuss the continuum of learning from Grades 7 to 10. In addition, these encounters often lead to further exchanges such as teachers visiting each other's classrooms to become more familiar with one another's contexts.

> He [intermediate math coordinator] has actually set up and put aside time for the last couple of years where the high schools would meet with their feeder schools and not only would they meet and discuss things but every Grade 9 teacher would spend a half-day in a Grade 8 class and a Grade 8 teacher would spend a half-day in a Grade 9 class, which was really interesting. (Steve, MLFG 1)

Communication was viewed as essential not only for building more meaningful relations but also to identify structural and pedagogical differences. The importance of ongoing communication among teachers also pertained to some concerns about the content of the various mathematics courses across grade levels as well as the rigor demanded of students by the curriculum in different grades.

> But there have been big gaps for, I think, our students between 7 and 8, there are missing things; 7 and 10 there are just whole concepts that disappear for two years, and I still think the rigor of Grade 8 is much less than the rigor of Grade 9, and I think students really struggle into that Grade 9. And I don't know if that's all curriculum or development or, but I think that probably secondary teachers, they seem to say to me that that is an issue (Joan, MLFG 2).

In summary, from the group interviews with the mathematics leaders, we heard not only some of the challenges facing students and teachers during transition, but they also proposed a number of practical solutions to address these concerns. Their proposed solutions to some of their concerns are consonant with the findings in the research literature.

The questionnaire data

Who were the respondents? There were 1,096 respondents to the web-based questionnaire that intermediate mathematics teachers could access online from May to mid-July, 2006. The sample was nearly evenly divided with 46% teaching Grades 7 and 8, and 51% teaching Grades 9 and 10. Only 3% of respondents reported “other” as their teaching assignment. The majority of respondents (93%) identified their primary role as a full time classroom teacher. Of the Grade 7 and 8 teachers, 42% reported that they teach mathematics for 60 minutes or less each day while 31% report teaching mathematics for more than 60 but less than 90 minutes daily. This suggests that these Grade 7 and 8 teachers were not on rotary but rather were responsible for the full range of subjects in the Grade 7 and 8 classes. Of the Grade 9 and 10 teachers, 62% report that they teach mathematics 3.5 hours or more daily. This suggests that more than half of the Grade 9 and 10 teachers in the survey teach a full timetable of mathematics.

Mathematics qualifications. We asked the teachers to indicate their qualifications with respect to mathematics. Table 1 illustrates the type of mathematics qualifications reported by grade level.

Table 1: Mathematics qualifications by panel

Mathematics Qualifications	Grade 7 and 8	Grade 9 and 10
Intermediate mathematics	24%	77%
Senior mathematics	5%	74%
Honours Specialist (mathematics)	1%	31%
P/J Mathematics - Part 1	4%	1%
P/J Mathematics - Part 2	1%	1%
P/J Mathematics - Specialist	1%	0%
Other mathematics qualifica-tions	7%	4%
No mathematics qualifications	69%	11%

When we look at the mathematics qualifications by panel, we see that 69% of the Grade 7 and 8 teachers report having no specific mathematics qualifications. It is important to note, however, that often when teachers find themselves teaching Grades 7 or 8 mathematics and want to take additional qualifications in Intermediate mathematics, they are unable to do so because entry into this course requires a particular number of university mathematics courses as a prerequisite which many teachers do not have.

For many of the items on the questionnaire, we asked respondents to identify one particular mathematics class that they were teaching during the school year and then use that class as the basis for their answers. This allowed us to distinguish different practices, activities, and attitudes associated with different courses. One question of interest was to find out how comfortable teachers felt with the content of the course or grade level they were teaching. Table 2 below shows the percentage of teachers by course/grade level who reported *very comfortable* with the content of the course they were teaching.

Table 2: Comfort with the content of the course for the category *very comfortable*

Course/grade level	**Very comfortable with course content**
Grade 7	57%
Grade 8	61%
Grade 9 Academic	80%
Grade 9 Applied	71%
Grade 9 Essential/Locally Developed	37%

From the table, we see that 57% of those teaching Grade 7, and 61% of those teaching Grade 8 reported that they were *very comfortable* with the content of the course compared with 80% of teachers in the Grade 9 Academic course and 71% in the Grade 9 Applied course who reported that they were *very comfortable*. This finding relates to the challenges confronting the Grade 7 and 8 teachers in the province as described earlier. At the same time, only 37% of those teaching Grade 9 Essential/Locally Developed reported being *very comfortable* with the content of their course. This may be due to their lack of mathematical expertise or their lack of experience with the content of the course.

Communication between panels

In this study it was important to learn about the communication between the two panels. Of particular interest was the frequency of meetings between teachers from both panels to discuss issues pertaining to transition. The question we posed is as follows:

> **Teachers of mathematics in Grades 7, 8, 9, and 10 may meet together to help ease the transition of their students from Grade 8 to Grade 9 mathematics. During the current school year, how often will you have met with your colleagues in the other panel?**

In response to this question, 46% of the teachers reported that they had not met with their colleagues at all. Table 3 shows the distribution of meetings.

Table 3: Meeting with colleagues to discuss transition

Frequency of meeting	Percentage
Have not met	46%
Once	25%
Twice	14%
Three times	7%
More than three times	7%

We then examined the frequency of these meetings by the type of school (see Table 4 below) in order to get a sense of what aspects of location or organizational structure might enable or constrain such meetings.

Table 4: Frequency of meeting by type of school

Frequency of meeting	**Type of School**				
	K-8	6-8	7-8	7-12	9-12
Have not met	35%	57%	40%	58%	50%
Have met once	25%	27%	32%	22%	24%
Have met 2 - 3 times	29%	14%	22%	14%	20%
Have met more than 3 times	12%	2%	7%	6%	6%

As Table 4 shows, the largest percentage of teachers who had no meetings with their colleagues from the other panel were those teachers in the Grade 7-12 schools. This finding was unexpected as teachers in both Grades 7 and 8, and Grades 9 and 10 are located in the same school. In this case, while location is not the main deterrent to meeting, the finding suggests, however, that the two panels exist separately in the same school. Regardless of school type, the separation of the elementary and secondary communities is very problematic. One particular concern pertains to the extent to which teachers have an understanding of the mathematics taught in the different courses.

Understanding the curriculum continuum

As stated in the curriculum documents:

> The Grade 9 courses in the Ontario mathematics curriculum build on the knowledge of concepts and skills that students are expected to have by the end of Grade 8. The strands used are similar to those used in the elementary, with adjustments made to reflect the more abstract nature of mathematics at the secondary level (OME, 2005a, p. 4)

In order to understand how familiar teachers in each of the different panels were with the curriculum expectations for different grades we asked the following:

> **How familiar are you with the current Ontario curriculum expectations for each of the following courses?**

While the question was developed for all courses and all grade levels, Table 5 shows how teachers in Grades 7 and 8 and those in Grades 9 and 10 characterize their familiarity of the curriculum expectations for Grade 8 and Grade 9 Academic Mathematics, respectively.

Table 5: Familiarity with mathematics curriculum expectations for Grade 8 and Grade 9 Academic Mathematics

Familiarity	Grade 8 Mathematics		Grade 9 Academic Mathematics	
	Grade 7 and 8 teachers	Grade 9 and 10 teachers	Grade 7 and 8 teachers	Grade 9 and 10 teachers
Not at all	3%	20%	24%	2%
A little	5%	36%	33%	5%
Somewhat	18%	36%	35%	18%
Very	75%	8%	8%	76%

While 75% of the Grade 7 and 8 teachers are *very* familiar with the curriculum expectations for Grade 8 mathematics, only 8% report being *very* familiar with the curriculum expectations for Grade 9 mathematics. The situation is reversed when we look at the Grade 9 and 10 teachers' familiarity with the curriculum expectations for those two courses. In that case, 76% of the Grade 9 and 10 teachers report being *very* familiar with the curriculum expectations for Grade 9 mathematics, while only 8% report being *very* familiar with the Grade 8 curriculum expectations. Another way to look at this table is to recognize that 56% of Grade 9 and 10 teachers are either *not at all* or only *a little* familiar with the Grade 8 mathematics curriculum. Similarly, 57% of the Grade 7 and 8 teachers are *not at all* or only *a little* familiar with the Grade 9 Academic Mathematics curriculum.

Classroom practices

Another area of interest was to document the different practices, routines, and activities that may be part of the mathematics classroom in the different panels. We asked a number of questions in this regard. With respect to the type of mathematical activity that may be prevalent in a given class, we asked:

In this class, how often do the following occur?

Table 6 shows the results for all questionnaire participants for a variety of classroom practices.

Table 6: Frequency of classroom practices

Classroom Practices	Overall		
	Never	Some lessons	Most or every Lesson
The teacher explains, demonstrates or provides examples	0%	8%	92%
Students work on problems with multiple solutions	4%	62%	34%
The teacher provides solutions to problems	1%	32%	67%
Students work on practice questions	0%	9%	91%
Students justify their answers and explain their reasoning	0%	34%	66%
Students work on investigations to determine relationships or mathematical ideas	2%	62%	36%
The teacher works with small groups of students	11%	56%	33%
Students work with concrete materials or manipulatives	7%	69%	23%
Students provide solutions to problems	1%	25%	74%
Students use computer software or graphing calculators	14%	73%	12%

When we look at these findings overall, we see that the most frequent classroom practices for *most or every lesson* are teacher explanation, demonstration or provision of examples (92%), followed by students working on practice questions (91%). The next most common classroom practices are students (74%) and teachers (67%) providing solutions to problems.

We also looked at each individual classroom practice listed to examine the differences and similarities of practices across the transition period of Grade 7, 8, and 9 courses. Some of the practices were fairly similar across the courses. “The teacher explains, demonstrates or provides examples” had fairly similar results across the panels and across the courses. “Students work on practice questions” also had fairly similar results except for the Grade 9 Essential/Locally developed course where this practice was more prevalent than in the other courses. “The teacher provides solutions to problems” and “students provide solutions to problems” also had similar results across the grades and courses. For these two practices, it is interesting to compare the results for the practices of students and teachers providing solutions to problems. Table 7 presents “students provide solutions,” and “teachers provide solutions” for each of the courses.

Table 7: Students and teachers provide solutions by course/grade level

Course/grade level	Most lessons/Every lesson	
	Students provide solutions to problems	**Teachers provide solutions to problems**
Grade 7	89%	67%
Grade 8	90%	61%
Grade 9 Academic	93%	70%
Grade 9 Applied	91%	59%
Grade 9 Essential/Locally Developed	90%	59%

From Table 7, it appears that in all of the mathematics courses, students provide solutions to mathematics problems more often than teachers as part of classroom practice.

The other classroom practices in the list showed some differences across the Grade 7, 8, and 9 courses. There were some slight differences in the results of "students justify their answers and explain their reasoning." There were also differences in "students use computer software or graphing calculators," but we would suggest that the differences we saw are connected to the reference to the use of different technologies in the various course curricula. For instance, the use of graphing calculators is more prevalent in the Grade 9 Applied and Academic courses than in Grade 7, 8, or Grade 9 Essential/Locally Developed.

For the other practices, we provide tables to view how these practices are experienced in Grades 7, 8, and 9. Table 8 illustrates responses to the following:

In this class, how often do students work on problems with multiple solutions?

Table 8: Students work on problems with multiple solutions by course/grade level

Course/grade level	Students work on problems with multiple solutions	
	Never	Most lessons/ Every lesson
Grade 7	1%	49%
Grade 8	1%	43%
Grade 9 Academic	6%	26%
Grade 9 Applied	6%	24%
Grade 9 Essential/Locally Developed	23%	14%

As illustrated, teachers in Grades 7 and 8 reported that students worked on problems with multiple solutions *most or every lesson* (49% and 43%, respectively), while for teachers in Grade 9 Academic and Grade 9 Applied courses this occurred less often (26% and 24%, respectively). For the Grade 9 Essential/Locally Developed course, only 14% of the teachers reported that students worked on problems with multiple solutions *most lessons*, while 23% said that it *never* occurred.

With respect to the use of investigations in mathematics classes, responses to the following question are displayed in Table 9.

In this class, how often do students work on investigations to determine relationships or mathematical ideas?

Table 9: Students work on mathematical investigations by course/grade level

Course/grade level	Students work on investigations to determine relationships or mathematical ideas	
	Never	Most lessons/Every lesson
Grade 7	1%	56%
Grade 8	1%	47%
Grade 9 Academic	3%	22%
Grade 9 Applied	2%	28%
Grade 9 Essential/Locally Developed	5%	37%

The results reveal that for Grade 7, 56% of teachers report that students work on mathematical investigations *most or every lesson*, and for Grade 8, 47% of the teachers report doing so. The reported occurrence for this classroom practice is lower in the Grade 9 Academic Mathematics course (22%), Grade 9 Applied Mathematics course (28%) and Grade 9 Essential/Locally Developed course (37%).

Similarly, when we examine the use of concrete materials or manipulatives by course, in Table 10, we see differences. It should be recognized that these materials might not be expected to be used in every lesson, or in some cases most lessons. Hence, it is also important to look at the *never* column that would suggest the percentage of classroom teachers who would never provide opportunities for students to work with concrete manipulatives or materials.

Table 10: Students work with concrete materials or manipulatives by course/grade level

Course/grade level	Students work with concrete materials or manipulatives	
	Never	Most lessons/Every lesson
Grade 7	1%	43%
Grade 8	1%	28%
Grade 9 Academic	14%	3%
Grade 9 Applied	6%	20%
Grade 9 Essential/Locally Developed	9%	59%

For this classroom practice, 43% of the Grade 7 teachers, 28% of the Grade 8 teachers, and 59% of the teachers of Grade 9 Essential/Locally Developed course reported using concrete materials or manipulatives in most lessons. In contrast, 20% of those teaching the Grade 9 Applied course and only 3% the teachers of Grade 9 Academic course reported doing so. In the Grade 9 Academic Mathematics course, 14% say they *never* use them.

We also see some differences across the courses in the practice of teachers working with small groups of students. Table 11 presents the results of this practice.

Table 11: Teachers working with small groups of students by course/grade level

Course/grade level	Teachers work with small groups of students	
	Never	Most lessons/ Every lesson
Grade 7	10%	36%
Grade 8	5%	42%
Grade 9 Academic	22%	13%
Grade 9 Applied	10%	28%
Grade 9 Essential/Locally Developed	0%	82%

We can see that a large number of teachers work with small groups of students in the Grade 9 Essential/Locally developed course. This practice appears to occur least in the Grade 9 Academic course.

Assessment practices. Not only were there some differences with respect to the classroom practices for different mathematics courses, but there were also some differences in the ways teachers reported on how they gained access to their students' understanding of mathematics.

> **In this class to what extent do you use the following to get a sense of students' understanding in mathematics? Check one box in each row.**

Table 12 provides a breakdown of responses to this item showing what assessment methods are used *somewhat* or *a lot* by Grade 7 and 8 teachers and Grade 9 and 10 teachers.

Table 12: Types of assessments to get a sense of students' understanding for the combined category *somewhat/a lot*

Types of Assessment	Somewhat/A lot	
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**
Paper-and-pencil tests	96%	97%
Quizzes	88%	90%
Performance tasks	92%	76%
Response of students in class	79%	73%
Homework performance	74%	68%
Observation of students (notes/checklists)	74%	60%
Projects	48%	28%
Interviews/conferencing with students	41%	29%
Portfolios/dated work samples	32%	8%
Student presentations to other students	30%	11%

When examined by panel, all teachers rely most heavily on paper-and-pencil tests and quizzes to get a sense of what their students understand. At the same time the results also show differences by panel. For example, Grade 7 and 8 teachers more frequently report using students' journals, portfolios/work samples, and student presentations to access their students' understanding.

Given the range of assessment forms available, we also wanted to know which were most important for teachers in determining the marks for their students. To get this information, we asked the following question.

> **For this class, when determining a student's mark, how much weight do you give to each of the following types of assessments? Check one box in each row.**

Table 13 presents the results for the category *a lot* as we were interested in what types of assessment were relied upon most by teachers.

Table 13: Types of assessment used in determining students' mathematics marks for the category *a lot*

Types of Assessment	A lot	
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**
Paper-and-pencil tests	42%	64%
Quizzes	25%	22%
Performance tasks	28%	16%
Response of students in class	8%	1%
Homework performance	13%	4%
Observation of students (notes/checklists)	11%	1%
Projects	10%	6%
Interviews/conferencing with students	6%	1%
Portfolios/dated work samples	4%	1%
Student presentations to other students	2%	1%

All teachers report that they rely most on paper-and-pencil tests to determine a student's mark, with Grade 9 and 10 teachers placing a heavier emphasis on their use. The pattern of responses by panel, however, suggests that more teachers in Grades 7 and 8 report using other assessment practices such as interviews/conferencing, projects, and students' journals when determining a student's mark than teachers in Grades 9 and 10.

Additionally, as Table 14 illustrates, if we examine the use of paper-and-pencil tests by course/ grade level, paper-and-pencil tests appear to be more widely used in determining a student's mark in the Grade 9 Academic and Applied courses, than they are in the Grade 7, Grade 8, and Grade 9 Locally Developed Mathematics courses.

Table 14: Use of paper-and-pencil tests to determine students' mathematics marks for the category *a lot*

Course/grade level	Use of paper-and-pencil tests
	A lot
Grade 9 Academic	70%
Grade 9 Applied	53%
Grade 8	43%
Grade 7	41%
Grade 9 Essential/Locally Developed	40%

Perceptions and beliefs. We know that the perceptions and beliefs of teachers influence the way they understand and implement the curriculum. Therefore, to get a sense of the respondents' perspectives on the teaching and learning of mathematics, one question to ascertain this information was the following:

To what extent do you agree with the following statements?

- All students do not learn in the same way
- Students learn math most effectively when given opportunities to investigate ideas and concepts through problem solving
- Current technologies bring many benefits to the learning and doing of math
- Certain aspects of math need to be explicitly taught
- Problem solving helps students develop mathematics understanding and gives meaning to concepts and skills in all strands
- Through discussions students are able to reflect upon and clarify ideas and relationships between mathematics ideas
- Manipulatives are necessary tools to support effective learning of mathematics for all students
- Collaborative learning activities enhance student learning of mathematics

For each of the statements in the item we asked respondents to use the range of categories – *strongly disagree*, *disagree*, *agree* and *strongly agree*. In Table 15, we report the results in terms of the category *strongly agree*.

Table 15: Perspectives on mathematics teaching/learning for the category *strongly agree*

Perspectives on mathematics teaching and learning	Strongly agree	
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**
All students do not learn in the same way	86%	77%
Students learn math most effectively when given opportunities to investigate ideas and concepts through problem solving	41%	17%
Current technologies bring many benefits to the learning and doing of math	38%	24%
Certain aspects of math need to be explicitly taught	60%	59%
Problem solving helps students develop mathematics understanding and gives meaning to concepts and skills in all strands	55%	41%
Through discussions students are able to reflect upon and clarify ideas and relationships between mathematics ideas	46%	33%
Manipulatives are necessary tools to support effective learning of mathematics of mathematics for all students	48%	16%
Collaborative learning activities enhance student learning of mathematics	47%	20%

As the findings indicate, the majority of respondents *strongly agree* that "all students do not learn in the same way," and, to a lesser extent, that "certain aspects of math need to be taught explicitly." There is, however, less agreement on the remaining statements for teachers in both panels. In particular, there is a difference for the two panels with respect to the value of investigation and problem solving, the use of manipulatives, and the role of collaborative learning activities. For example, 41% of the Grade 7 and 8 teachers *strongly agree* that "students learn math most effectively when given opportunities to investigate ideas and concepts through problem solving" compared to 17% of the Grade 9 and 10 teachers. Similarly, 47% of the Grade 7 and 8 teachers *strongly agree* that "collaborative learning activities enhance student learning of mathematics" compared to 20% of the Grade 9 and 10 teachers. With respect to manipulatives, 48% of the Grade 7 and 8 teachers *strongly agree* that "manipulatives are necessary tools to support effective learning of mathematics of mathematics for all students" compared to 16% of the Grade 9 and 10 teachers.

At the same time, in order to understand which characteristics of learning were important if students were going to be successful at mathematics, we presented the following questionnaire item. For each of the statements in the item we asked respondents to select from the categories *not all*, *a little*, *somewhat*, *very*:

To be good at mathematics in this class, how important do you think it is for students to:

- Understand mathematical concepts, principles, and strategies
- Be able to provide reasons to support their solutions
- Understand how mathematics is used in the real world
- Be able to investigate mathematical ideas
- Think in a sequential and procedural manner
- Remember formulas and procedures

The findings are illustrated in Table 16 below.

Table 16: To be good at mathematics for the category *very important* across courses

Characteristics of learning	Very important
Understand mathematical concepts, principles, and strategies	69%
Be able to provide reasons to support their solutions	62%
Understand how mathematics is used in the real world	48%
Be able to investigate mathematical ideas	46%
Think in a sequential and procedural manner	45%
Remember formulas and procedures	24%

If we look at the responses of all the questionnaire participants, 69% of the teachers selected being able to “understand mathematical concepts, principles, and strategies” as very important for being good at mathematics. In contrast, only 24% chose being able to “remember formulas and procedures” as very important.

In Table 17 below, we have organized the responses to this question by the course/grade level for the category *very important*.

Table 17: To be good in mathematics by course/grade level for the category *very important*

Characteristics of learning	Very Important				
	Grade 7	**Grade 8**	**Grade 9 Academic**	**Grade 9 Applied**	**Grade 9 Essential/Local**
Understand mathematical concepts, principles, and strategies	71%	71%	84%	54%	19%
Be able to provide reasons to support their solutions	69%	69%	72%	49%	40%
Think in a sequential and procedural manner	39%	45%	53%	38%	19%
Be able to investigate mathematical ideas	58%	57%	53%	33%	24%
Understand how mathematics is used in the real world	56%	57%	37%	40%	67%
Remember formulas and procedures	22%	31%	27%	5%	0%

While across all the courses, it appears very important for students to be able to “understand mathematical concepts, principles, and strategies” in order to be good at mathematics, the degree to which this is the case varies by course. Table 17 reveals that 71% of the teachers in Grades 7 and 8 report this as very important, while the percentage of teachers in the Grade 9 Academic course is 84% and in the Grade 9 Applied course, the percent is 54%. In the Grade 9 Locally Developed course, only 19% of the teachers consider this very important to be good at mathematics in that course.

It was also valuable for us to know how mathematics teachers from the two panels perceive the characteristics of students who make up a typical Grade 9 Academic Mathematics class and a typical Grade 9 Applied Mathematics class. We asked the following two questions:

> **From your perspective, how many students would you expect there to be in typical Grade 9 ACADEMIC mathematics classes who have the following characteristics?**
>
> **From your perspective, how many students would you expect there to be in typical Grade 9 APPLIED mathematics classes who have the following characteristics?**

For each of these questions, respondents were provided with a list of characteristics and asked to select the number of students expected in each of the two courses with these characteristics from a scale of *none*, *a few*, *some*, or *many*. The following is the list of characteristics presented to them.

- Strong mathematics skills
- Behavioral challenges
- Good work habits
- IEP identification
- Good organization skills
- Gaps in their learning
- Ability to solve problems
- Support at home
- English as a second language
- Poor attendance
- Good social skills
- Motivation to learn
- Aspirations to attend university

The findings of teachers' perceptions of the characteristics of students in both the Grade 9 Academic and Applied courses are reported in Table 18 for the combined category *some/many*.

Table 18: Perceptions of student characteristics in Academic and Applied Mathematics courses for the category *some/many*

Characteristics of students	Grade 9 Academic		Grade 9 Applied	
	Some/Many		Some/Many	
	Grade 7 and 8 teachers	Grade 9 and 10 teachers	Grade 7 and 8 teachers	Grade 9 and 10 teachers
Strong math skills	96%	91%	58%	36%
Behavioural challenges	39%	24%	79%	86%
Good work habits	97%	94%	68%	44%
IEP identification	27%	23%	74%	82%
Good organization skills	94%	90%	61%	34%
Gaps in their learning	54%	61%	87%	93%
Ability to solve problems	94%	87%	62%	35%
Support at home	89%	89%	63%	55%
English as a second language	57%	33%	61%	39%
Poor attendance	17%	14%	65%	79%
Good social skills	94%	96%	87%	80%
Motivation to learn	96%	93%	66%	42%
Aspirations to attend university	97%	98%	38%	17%

The findings suggest that more positive characteristics are associated with students in a typical Grade 9 Academic Mathematics class, including strong mathematics skills, good work habits, good organizational skills, an ability to solve problems, motivation to learn and aspirations to attend university. In contrast, fewer students in a typical Grade 9 Applied Mathematics class are associated with positive characteristics and more are associated with challenges to learning. These include behavioural challenges, IEP identification, gaps in their learning, poor attendance, and lower motivation to learn.

Also, when considering the characteristics of students who make up a typical Grade 9 Applied class, the response patterns of the teachers in Grades 7 and 8 differed from those of teachers in Grades 9 and 10 with the teachers in Grades 7 and 8 appearing more positive. For example, 58% of Grade 7 and 8 teachers compared with 36% of Grade 9 and 10 mathematics teachers say that *some/many* students in this course have strong mathematics skills. Similarly, 61% of Grade 7 and 8 teachers and 34% of Grade 9 and 10 mathematics teachers say that *some/many* students in this course have good organizational skills, while 66% of Grade 7 and 8 teachers compared with 42 % of Grade 9 and 10 mathematics teachers say that *some/many* students in this course are motivated to learn. Since Grade 8 teachers make recommendations for placements in the Grade 9 mathematics courses, it is important for teachers of both panels to be aware of these different perceptions and expectations.

Professional development

In this study, we were also interested in identifying areas where teachers see a need for further professional development to support their work with the curriculum. We asked the following question:

> **To what extent do you feel that further professional development in these areas would help support you in implementing the curriculum in this class?**

Respondents were provided with the following list of areas for professional development and asked to consider how helpful they might be on a scale of *not at all*, *a little*, *somewhat*, *a lot*.

- Use of manipulatives
- Understanding how students learn mathematics
- Teaching through problem solving
- Use of Geometer's Sketchpad®
- Assessment in mathematics
- Using group work in mathematics
- Teaching strategies
- Use of graphing calculators
- Facilitating investigations
- Content knowledge for teaching
- Use of other computer software
- Other (please specify in comments section)

Table 19 reveals the results for the combined category *somewhat/a lot*.

Table 19: Areas of further professional development seen as helpful for the category *somewhat/ a lot*

Areas for professional development	Somewhat/A lot	
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**
Teaching through problem solving	84%	75%
Use of manipulatives	79%	57%
Assessment in mathematics	79%	55%
Understanding how students learn mathematics	78%	67%
Teaching strategies	76%	66%
Use of other computer software (other than GSP)	72%	42%
Using group work in mathematics	71%	52%
Use of Geometer's Sketchpad®	68%	40%
Facilitating investigations	67%	54%
Use of graphing calculators	52%	42%
Content knowledge for teaching	48%	27%
Other	30%	24%

For each of the suggested areas it appears that a higher percentage of teachers in Grade 7 and 8 are requesting professional development. "Teaching through problem solving" is the most requested area of professional development for both groups. Of particular notice is the percentage of teachers (78% of the Grade 7 and 8 teachers, and 67% of the Grade 9 and 10 teachers) who would like to learn more about "how students learn mathematics."

Concluding comments

The research literature on students who are making the transition from elementary to secondary school directs our attention to the complexity involved in transition and cautions that the inability to adjust can have an adverse effect on success in completing high school. The Ontario curriculum documents recognize that there should be a smooth transition from elementary to secondary school.

> The Grade 9 courses in the Ontario mathematics curriculum build on the knowledge of concepts and skills that students are expected to have by the end of Grade 8 . . . the mathematics courses offered in secondary school are based on principles that are consistent with those that underpin the elementary program, a feature that is essential in facilitating transition (OME 2005a, p. 4*)*.

Our data suggest that many teachers are not fully aware of the content and expectations of the mathematics curriculum of the other panel. In addition, our findings reveal that there are several differences between the teaching and learning cultures in Grades 7 and 8 and in Grades 9 and 10, a conclusion that is also found in the research literature. This then underscores the need for increased dialogue in order to address these issues, and to develop an awareness and understanding of both the curriculum and the cultures of schooling as experienced in the different panels. In fact, with respect to transition, both the research literature and many of the results from this study point to the importance of establishing the practice of collaborative communication among teachers from both panels.

There are many possible topics of conversation that would be of value for teachers including specific classroom practices, as well as engaging with each others' perceptions of what is required to be a good mathematics student and how that appears different depending on the grade level and specific course focus. Additionally, a conversation among teachers on the characteristics of "typical" students in applied or academic mathematics also appears warranted. These conversations can occur naturally within shared activities and in fact, there is evidence that such conversations might be more effective when they occur in the context of collaborative work. "It isn't just talk but work that binds" (Hargreaves & Earl, 1991, p.70). In our data, the effectiveness of such a shared activity was described in relation to a professional development initiative on how to use TIPS documents. As one mathematics leader stated, TIPS was effective because it "moved people along . . . it opened a dialogue between 7, 8 and 9 teachers . . . was organized so that they were together and . . . they were all learning at the same time (Joan, MLFG 2).

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The Use of Technology in Mathematics Class

What does the research literature say?

Current trends and research in mathematics education support the appropriate use of technology to enhance mathematics teaching and learning (Forgasz, 2006; Holyes & Noss, 2003; Kieran & Drijvers, 2006; Masalski & Elliott, 2005; NCTM, 2000, 2005: Noss & Hoyles, 1996; Peressini & Knuth, 2005; Shaffer & Kaput, 1999; Zhao & Frank, 2003). Technology contributes to teaching and learning in the mathematics classroom in numerous ways: by enabling easier communication, by providing opportunities to investigate and explore mathematical concepts, and by engaging learners with different representational systems which help them see mathematical ideas in different ways. In a recent position paper on the role of technology in mathematics, the National Council of Teachers of Mathematics proposed that integrating technology into the teaching and learning of mathematics is essential in order to develop and expand mathematical thinking (NCTM, 2005).

> In the context of a well-articulated mathematics program, technology increases both the scope of the mathematical content and the range of the problem situations that are within the students' reach . . . Using the tools of technology to work in interesting problem contexts can facilitate students' achievement of a variety of higher-order learning outcomes, such as reflection, reasoning, problem posing, problem solving, and decision making. (NCTM, 2005, p. 1)

Technology is not meant to replace the development of mathematical skills but, appropriately used, is a strong pedagogical tool that helps students explore and understand mathematics. As expressed in the NCTM discussion of the Technology Principle in *Principles and Standards for School Mathematics* (NCTM, 2000), "Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions" (p. 25).

Research on the ways in which specific technological tools mediate the development of mathematical thinking, emphasizes the important role that the multiple representations and the opportunities to investigate mathematical ideas, pose problems, and test conjectures play in the development of mathematical understanding. For instance, with the use of graphing technology, students have access to a graphic representation of a mathematical model that is not always easy to obtain by traditional paper-and-pencil methods. Jones (2005) suggests that graphing calculators "can enable students to approach situations graphically, numerically and symbolically, and can support students' visualization, allowing them to explore situations which they may not otherwise be able to tackle" (p.31). Technological thinking tools such as the Geometer's Sketchpad®, and computer algebra systems, invite learners to make mathematical conjectures, investigate those conjectures, and to engage in mathematical thinking and reasoning as they develop new understandings of mathematics concepts (Harvey, Waits, & Demana, 1995; Heid & Edwards, 2001; Hollar & Norwood, 1999; Kieran & Drijvers, 2006; Knuth & Hartmann, 2005; Laborde, Kynigos, Hollebrands & Strässer, 2006; Yerushalmy, 2006). Students develop a fluency not only to learn how to make appropriate use of technology, but also to create, modify, and test different mathematical ideas and representations.

> [Technological] tools used in mathematics not only make learners more capable. Tools also change learners in ways that affect the manner in which they construct and process knowledge. Tools mediate inasmuch as they are integrated in the internal and external activities and cognitive functioning of all learners. (Rivera, 2005, p. 136)

Being able to visualize and work with different technological tools, teachers and learners develop alternate ways of understanding mathematical concepts while having opportunities to examine their thinking and decision making during mathematical investigations. Shaffer and Kaput (1999) argue that "this ability to externalize the manipulation of formal systems changes the very nature of cognitive activity" (p. 97).

The role of teachers

In their work on computer algebra systems, Kieran & Drijvers (2006) underline the reciprocal relationship between the technological tool and the learner, since what the learner knows influences how the tool is appropriated, while the characteristics of the tool itself, its affordances and constraints, influence the students' mathematical thinking (see also Hoyles and Noss, 2003). Because of the dynamic mediating relationship between the various technologies, students, and the teacher, it is simplistic to expect that the mere presence of technology in the mathematics classroom will necessarily improve learning. Students need to learn how and when to apply the technology effectively in order to integrate it into their mathematics learning. In this endeavour, it is expected that students will be guided by teachers who are knowledgeable about how to integrate technology in mathematics learning in ways that enhance mathematical meaning (Mariotti, 2002; Pierce & Stacey, 2004).

While there has been a move by many teachers to incorporate the use of technology into their mathematics classroom, many middle and high school mathematics teachers report that they do not use computers for purposes other than for drill and practice, due to insufficient knowledge about how to use technologies effectively in teaching mathematics (Manoucherhri, 1999). In research on teachers' use of computers, a number of barriers have been repeatedly identified. These include the lack of release time to learn how to use computers, insufficient time during the school day to work with students and technology, and insufficient access to computers (Forgasz, 2006; Hadley & Sheingold, 1993; Manoucherhri, 1999; Smerdon, et al., 2000; Solomon 2002; Zhao & Frank, 2003). Additional barriers include lack of appropriate software or information about it, lack of maintenance and technical support, their own lack of knowledge about computers, as well as their own uncertainty in the use of technology. Practical constraints such as having to schedule lab time far in advance, or spending large amounts of time trouble-shooting were also identified.

In addition to the barriers listed, many teachers report that they "are not convinced of the usefulness of computers in their instruction, and of the potential of computers for enhancing the curriculum they teach" (Manoucherhri, 1999, p. 37). This suggests that in addition to becoming adept with a given technological tool, professional development initiatives should also help teachers develop an understanding of the ways in which mathematical content knowledge and teaching activities are mediated by the particular technology. For example, recent studies have shown that appropriate selection of activities when using dynamic geometry software, is essential to enable new knowledge and understanding (Hollebrands, 2007; Laborde, Kynigos, Hollebrands & Strässer, 2006). However, the selection of tasks and activities that make appropriate use of technology is not straightforward and requires substantial time for reflection and reorganization (Guin & Trouche, 1998).

Research on teachers who use computing technologies effectively as an integral component of their teaching reveals that such teachers tend to possess a high level of computer proficiency and more experience teaching in their subject areas (Becker, 1994; Higgins, Moseley, & Tse, 2001). At the same time, these teachers report that they work with others to engage technology in collaborative environments that have adequate resources in terms of hardware, software and technical support (Tiene & Luft, 2001). Recognizing the importance of effective professional development, other studies show that teachers who have spent more time in collegial professional activities regarding technology report feeling better prepared than other teachers and are more likely to use technology (Smerdon et al., 2000).

These findings imply that increased personal interest and knowledge about technology, more experience with technological tools, a supportive collegial environment, and a better understanding of the ways in which mathematical thinking may be mediated by different technologies, have the potential to support the successful integration of technology into the mathematics classroom. When considering the influence of institutional factors in relation to personal factors, “teachers need access to a healthy human infrastructure and a functional and convenient technical infrastructure” (Zhao, Pugh, Sheldon, & Byers, 2002, p. 512).

What do Ontario mathematics education leaders say?

The following summary of the OAME and OMCA focus group discussions highlights some of the themes associated with technology that were prevalent in the discussions with leaders in mathematics education. The discussions also reveal that the math education leaders have a good understanding of the curriculum, current teacher practice, and professional development needs.

The discussions about technology indicated that the Ontario leaders in mathematics education perceive the curriculum documents as being very supportive in the use of technology. They appreciate that technology is “explicit” within the curriculum and that teachers “can’t hide from it now.” In general, they saw technology as a useful resource in teaching mathematics. For some, technology is seen as a pedagogical tool as in this discussion on the use of Computer Algebra Systems (CAS) to support student learning of solving equations.

> Beth: They would show you the steps; they wouldn't actually write it down, but they would do it on their calculator and the screen would show it and you could tell that they knew how to solve equations based on what they were inputting in their calculator. Because they would say, "Oh, I have to subtract five from both sides", and then it would show them right away that "Yes, I made the right choice" or if they thought they needed to add five to both sides, it would show them "No, I made the wrong choice." So instead of doing, you know, ten questions incorrectly for homework, they had instant feedback, and they were excited about doing math because of the technology. And those kids in Grade 11 this year had no difficulty continuing on with their questions and stuff.
>
> Dave: And they got immediate feedback.
>
> Beth: Immediate feedback and that is so important to the Applied level. (Mathematics Leader Focus Group [MLFG] 1)

However, many challenges to the use of technology in mathematics classes were mentioned that included limited access to technology, lack of teacher understanding or confidence in using technology, and a lack of technology leadership in some school boards. Access to computers was seen as a problem as math leaders believed that computer allocation did not increase with the increased prominence of technology in the curriculum. For instance, mathematics teachers compete with teachers in other disciplines for computer time. Participants reported that teachers' access to a computer lab was often limited to a few days in a course. One participant also noted that it appeared that secondary schools had better access to computers due to more funding but even at the secondary school level, access is limited. Another barrier to technology use that was suggested was teachers' lack of understanding or confidence. For instance, teachers may not be able to connect the use of technology to meaningful mathematics due to their own understanding of mathematics.

> But what is it that this technology or/and manipulative, what is the mathematics that I'm going to draw out of the students and do I know enough of that mathematics to be able to identify when it's happening? (Peter, MLFG 6)

Participants suggested that, as with many new ideas and resources, teachers would need to move outside their comfort zone in using technology to teach mathematics.

> I think even before you get to the math that comes out of the technology or manipulative that you're using, there's a sense of risk that a teacher must be willing to take to sort of dive into the activity with their kids, even if they know that maybe their students know more about the calculator than they do. And so, there's that fear, that risk that needs to be sort of put to the wayside and just dive into it and let things fall where they may. (Erin, MLFG 6)

As well, some teachers may not recognize how to incorporate the use of technology in their teaching or view the use of technology as an event rather than as a tool for everyday teaching and learning.

> Beth: I think though there are still some teachers who feel that, "I need to use my graphing calculator for these three weeks," and they get it out and still treat it as it's, um...
>
> Dave: Like the scatter plots...
>
> Beth: A toy. Yes, and we'll do all these things and then we'll put it away after we finish that or there are a lot of other people who say, "If we need it today, we'll use it; if we don't need it today, we won't use it." (MLFG 1)

Further, the mathematics education leaders have observed that teachers feel that using technology may be more time-consuming and they often grapple with finding time to cover the curriculum. The following participant, who is a teacher himself, discussed the issue.

> I took time to use it [the technology]. … another teacher didn't use it because of time constraints, and I didn't get quite as much covered in the curriculum. There are still too many expectations sometimes or if you really want to do the kids with time to manipulate, play and discover, and that, you need the time and that's a concern. You have to cheat and cut a few things. (Dave, MLFG 1)

Participants also discussed a concern about there being an insufficient number of experts in the use of technology to teach mathematics within the province. They saw small boards as having difficulties given that coordinators are frequently expected to cover many subjects and may not have the knowledge of how to use the various technologies to teach mathematics.

The mathematics education leaders have also observed that interaction of teachers with their colleagues supports their learning. As individual teachers or teams of teachers receive training in new technology and develop their expertise, it is shared with others in the school.

> Beth: When we first went with the graphing calculator use, there were enough people that had gone to workshops and things that they knew what was going on, and there were others that didn't feel comfortable but the policy was if you're having difficulty, phone me, I'll come down. And one teacher in particular, as soon as you saw the calculators going down the hallway, you knew that partway through the period you were going to get a phone call.
>
> CS: Code Blue, Code Blue.
>
> Beth: Yes, but then it was that she wasn't confident with it but, as time progressed, she would then send students down to see me and then I wouldn't hear from her anymore because she felt more comfortable with what was going on. But she knew that there was somebody out there who could help, and if you were in a school that didn't have that somebody out there who could help, I think it took a lot longer to move forward or if you had someone who wasn't willing to ask for help, it also took longer. (MLFG 1)

Participants also suggested that another way of overcoming teachers' fear of using technology is to provide them with more resources such as pre-made sketches or applets.

What does the curriculum say?

The Ontario Curriculum Grades 1-8 Mathematics Revised (OME, 2005a) and the Ontario Curriculum Grades 9 and 10 Mathematics Revised (OME, 2005b) have identical introductory messages around the selection of tools and computational strategies as well as the discussion of calculators, computers, and communications technology. Both refer to the importance of students meeting the demands of today's world by using technology effectively and "to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems," (OME, 2005a, 2005b, p. 14). Students "learn in the presence of technology" and technology "should influence the mathematics content taught and how it is taught" (OME, 2005b, p. 3).

Across both documents, teachers are responsible for familiarizing students with these technologies such that use of these tools "should not be laborious or restricted to inputting and learning algorithmic steps" and computing technologies "should be used seamlessly in teaching, learning and assessment" (OME, 2005b, p.14). Furthermore, emphasis is placed on students' learning of appropriate computational strategies, and how and when to use technology for computation.

In both documents, calculators and computer-based software, including spreadsheets, statistical software (e.g., TinkerPlots®, Fathom®), dynamic geometry software (e.g., The Geometer's Sketchpad® [GSP]) are mentioned as specific technologies seen as important problem-solving tools that extend students' capacity to investigate and analyse mathematical concepts. Students are encouraged "to select and use communications technology that would best support and communicate their learning" (OME, 2005a, p. 15). In both elementary and secondary school, students are encouraged to develop and represent mathematical ideas through the use of multiple representations that include numeric, geometric, graphical, algebraic, pictorial and concrete.

Both of the curriculum documents use the same language in discussing the role of technology in mathematics and information and communication technology (ICT) tools that include simulations, multimedia resources, data bases, sites that give access to large amounts of statistical data, and computer-assisted learning modules. Within curriculum expectations, specific mention is made of applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, and graphing software. The Grades 9 and 10 curriculum document also incorporates the use of computer-algebra systems (CAS), and graphing calculators. Of particular interest is that the 2005 revision marks the first time that CAS technologies are identified within the Ontario mathematics curriculum.

A close examination of the curriculum expectations demonstrates the incorporation of technology in the curriculum. Table 1 shows, by grade level and course, the percentages (and number) of specific expectations within the curriculum documents, and the percentage (and number) of lesson plans within the TIPS resources that refer to the use of technology.

Table 1: Percentage of references to technology in the curriculum and TIPS by grade level

Course/grade level	**Curriculum Specific Expectations referring to technology**	**TIPS Lesson Plans that use technology**
Grade 7	38% (25)	21% (18)
Grade 8	29% (18)	38% (20)
Grade 9 Applied	41% (18)	41% (24)
Grade 9 Academic	43% (23)	-
Grade 10 Applied	45% (15)	50% (12)
Grade 10 Academic	61% (23)	-

In many cases, the use of technology is referred to in the examples that are provided in the curriculum expectation. For instance, across the Grades 7, 8, and 9 curricula and strands, the use of technology is predominantly (~75%) found within the examples provided within the specific expectations. In the Grade 10 Applied and Academic courses, while the use of technology is still mentioned frequently in the examples of the expectation (~50%), specific technologies are also mentioned explicitly within the body of the expectation (~30%).

We have also examined the curriculum with respect to the reference to specific technologies. Dynamic geometry software (specifically The Geometer's Sketchpad® software) is the most commonly mentioned computing technology across Grades 7 to 10. Graphing (or graphical) calculators is the most commonly mentioned non-computer-based technology, initially introduced to students in Grade 9. Computer Algebra Systems (CAS) is defined as "a software program that manipulates and displays mathematical expressions (and equations) symbolically" and is introduced in the curriculum in Grade 9.

Table 2 provides a summary of the number of curriculum expectations that refer to specific technologies. It should be noted that some expectations may mention more than one technology.

Table 2: Number of specific expectations in the curriculum that refer to the use of graphing calculators, CAS, and dynamic geometry software by grade level and course

	Graphing Calculators	Computer Algebra Systems (CAS)	Dynamic Geometry Software (GSP)
Grade 7	-	-	10
Grade 8	-	-	5
Grade 9 Applied	3	3	6
Grade 9 Academic	5	4	9
Grade 10 Applied	4	4	2
Grade 10 Academic	2	2	10

What do the questionnaire data say?

One of the principal data gathering instruments of the CIIM project was the 45 item online survey that gathered data about participants' teaching assignment, instruction and assessment practices in mathematics, and professional development experiences and needs. Within this questionnaire, several questions specifically dealt with teachers' views about, and use of, technology in the classroom. These questions included comfort level with using technology for teaching mathematics, importance placed on promoting the use of multiple representations of ideas (concrete materials, technology, etc.), access to computers, amount of time students spend using technology, and the professional development needs with respect to the use of technology.

Beliefs about the use of technology

There are several items that can give us a sense of teachers' views about the use of technology. In one item, teachers were asked to identify their level of agreement with a number of belief statements about teaching and learning mathematics. Each of the statements was asked independently such that participants were not required to rank one against the others. One of the statements was the following:

To what extent do you agree with the following statements?

Current technologies bring many benefits to the learning and doing of mathematics.

Participants were asked to rate their agreement with the statement using a four-point scale ranging from *strongly disagree* to *strongly agree*. Table 3 reports the percentage of participants who *strongly agreed* with the statement.

Table 3: Percentage of teachers who *strongly agree* that technologies bring benefits to the learning and doing of mathematics by panel

	Strongly Agree	
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**
Current technologies bring many benefits to the learning and doing of mathematics	38%	24%

This data would suggest that a slightly higher percentage of teachers of Grades 7 and 8 *strongly agree* that current technologies bring many benefits to the learning and doing of mathematics.

Teachers were also asked to reflect on a particular class that they teach and to identify the importance of particular classroom practices. Respondents were provided with a list of classroom practices and were asked to rate the importance of each statement using a four-point scale ranging from *not at all* to *very*. Each of the six statements required a separate response. The data are presented in Table 4 by what teachers indicated is *very important*. The table is organized in descending order.

Table 4: Percentage of teachers who rated as *very important* the following items

Classroom practices	**Very Important**
Providing students with opportunities to practice skills	87%
Providing students with many examples	69%
Providing opportunities for students to explain their reasoning	58%
Providing engaging problems	58%
Encouraging student discussion of mathematical ideas	50%
Promoting the use of multiple representations of ideas (concrete materials, technology, etc.)	48%

This particular question is not specific to the use of technology but rather examines the use of multiple representations that include the use of concrete materials and technologies. We notice that fewer teachers report the use of multiple representations as very important compared to other practices. It is interesting to examine the response to the use of promoting the use of multiple representations by grade and course. The results for this particular item are presented in Table 5.

In this class how important is … promoting the use of multiple representations of ideas (concrete materials, technology, etc.)?

Table 5: Percentage of teachers who rated promoting the use of multiple representations as *very important* by course/grade level

Course/grade level	Very Important
Grade 7	60%
Grade 8	53%
Grade 9 LDCC	59%
Grade 9 Applied	50%
Grade 9 Academic	37%
Grade 10 LDCC	57%
Grade 10 Applied	47%
Grade 10 Academic	30%

These data suggest that there are some differences among the teachers of different grades with respect to the importance of promoting multiple representations. For instance, teachers of the Grades 9 and 10 Academic courses did not rate the importance of multiple representations as highly as teachers of other courses.

Comfort with the use of technology

Participants were asked to rate their level of comfort with a variety of aspects of the curriculum. One of these aspects was the use of technology to teach mathematics. The question offered a four-point scale response from *not at all*, *a little*, *somewhat*, and *very*. Table 6 provides the full list of aspects and a display of the response for *somewhat* and *very* comfortable.

How comfortable are you with the following aspects of the curriculum for this class?

Table 6: Percentage of teachers who rated as *somewhat* and *very* comfortable with the following aspects of the curriculum

Aspects of the curriculum	Somewhat	Very
a. Using a variety of assessment methods	50%	41%
b. Using technology to teach mathematics	42%	32%
c. The philosophy of the curriculum	51%	26%
d. Teaching through problem solving and investigating	51%	36%
e. Students working in groups	43%	37%
f. The Achievement Chart	36%	24%
g. The content of the course	27%	68%
h. Using concrete materials to teach mathematics	43%	40%
i. Integrating concepts from across the strands of mathematics	44%	39%

Combining the categories of *somewhat* and *very*, we see that overall, 74% of teachers perceive themselves as being *somewhat* or *very* comfortable using technology to teach mathematics.

It is also interesting to examine this response to the comfort in using technology to teach mathematics by grade and course. Table 7 presents the responses by course for the *very* category.

How comfortable are you with the following aspects of the curriculum for this class? Using technology to teach mathematics?

Table 7: Percentage of teachers who responded that they are *very* comfortable in using technology to teach mathematics.

Course/grade level	Very comfortable
Grade 7	22%
Grade 8	24%
Grade 9 LDCC	24%
Grade 9 Applied	43%
Grade 9 Academic	42%
Grade 10 LDCC	22%
Grade 10 Applied	38%
Grade 10 Academic	40%

These data suggest that, in general, fewer teachers in the Grades 7 and 8 courses and the Locally Developed courses report being *very* comfortable in using technology to teach mathematics.

Technology use in the classroom

Several items helped to provide a sense of teachers' classroom practice. For instance, we provided the following question with a list of classroom practices.

In this class, how often do the following occur?

In this question, teachers were asked to respond to a list of classroom practices and to choose their response from a four-point scale ranging from *never*, *some lessons*, *most lessons*, or *every lesson*. The results of the *most lesson* or *every lesson* categories are presented in Table 8.

Table 8: Percentage of teachers who reported that the following activities occurred in *most* or *every lesson*

	Most lessons and Every lesson
The teacher explains, demonstrates or provides examples	93%
Students work on problems with multiple solutions	34%
The teacher provides solutions to problems	67%
Students work on practice questions	91%
Students justify their answers and explain their reasoning	66%
Students work on investigations to determine relationships or mathematical ideas	36%
The teacher works with small groups of students	33%
Students work with concrete materials or manipulatives	24%
Students use computer software or graphing calculators	13%

Overall, 13% of teachers reported that students work with computing software or graphing calculators during *most* or *every* lesson. However, it might not be expected that computing software or graphing calculators would be used in most lessons or every lesson. Furthermore, each course taught might have a different technology focus. For instance, graphing calculators are not referred to in the Grades 7 and 8 curriculum.

Thus, we examined the data about the use of computer software or graphing calculators by grade and course. We are including the responses to *never, some*, *most lessons*, and *every lesson* categories in Table 9.

Table 9: Percentage of teachers reporting how often students use computer software or graphing calculators by course.

The data show that students in Grades 7 and 8 are less likely to use computer software or graphing calculators than students in Grade 9 and Grade 10 Applied and Academic classes. The data also suggest that there is a high percentage of teachers of LDCC classes who use computer software and graphing calculators in *most* or *every* lesson. The variety in responses could reflect the different emphases on specific technologies in the curriculum as well as reflecting teachers' views of technology and their comfort level.

	Never	Some lessons	Most lessons and Every lesson
Grade 7	27%	57%	17%
Grade 8	20%	66%	15%
Grade 9 LDCC	32%	46%	23%
Grade 9 Applied	4%	89%	8%
Grade 9 Academic	5%	86%	9%
Grade 10 LDCC	26%	70%	4%
Grade 10 Applied	0%	85%	15%
Grade 10 Academic	10%	83%	9%

We, therefore, want to look at the use of specific technologies in the courses where they are specified in the curriculum. The use of graphing calculators does not appear in the curriculum until Grade 9 and is referenced in the Grades 9 and 10 Academic and Applied curricula. Thus, Table 10 looks at the responses of Grades 9 and 10 Academic and Applied teachers to:

In this class, how often do students use each of the following resources? Graphing calculators

In this question teachers were asked to choose their response from a four-point scale ranging from *never, some lessons*, *most lessons*, or *every lesson*.

Table 10: Percentage of teachers reporting how often students use graphing calculators by course.

Course	Never	Some lessons	Most lessons and Every lesson
Grade 9 Applied	8%	88%	5%
Grade 9 Academic	4%	87%	9%
Grade 10 Applied	2%	84%	14%
Grade 10 Academic	9%	83%	9%

The data show similarities in how often students use graphing calculators in Grade 9 and Grade 10 Applied and Academic classes and imply that nearly all students in these courses use this technology in at least *some lessons*.

Dynamic geometry software is referenced in courses from Grades 7 – 10. Table 11 provides a display of the responses to the following question for each of those grades.

In this class, how often do students use each of the following resources? The Geometer's Sketchpad® (GSP)

In this question, teachers were asked to choose their response from a four-point scale ranging from *never*, *some lessons*, *most lessons*, or *every lesson*.

Table 11: Percentage of teachers reporting how often students use GSP by grade and course.

Course/grade level	Never	Some lessons	Most lessons and Every lesson
Grade 7	46%	52%	2%
Grade 8	40%	60%	0%
Grade 9 Applied	41%	59%	0%
Grade 9 Academic	38%	62%	0%
Grade 10 Applied	54%	46%	0%
Grade 10 Academic	53%	46%	1%

The data show some similarities in how often students use GSP in Grades 7 through Grade 10 and across Applied and Academic classes and implies that for Grades 7, 8, and 9, between 54% - 62% of the students in these courses will use this technology in at least some lessons. For the Grade 10 courses, 46% and 47% of the students will have used this software. It is also interesting to examine this data with respect to the mention of this software in the curriculum. GSP would not be expected to be used in *most* or *every lesson* as it does not appear in most or every curriculum expectation. However, it might be important to examine the classroom use of GSP in particular courses with respect to the reference to GSP in the curriculum of that course. For instance, while 10 of the curriculum expectations in Grade 10 Academic suggest the use of GSP, 53% of the teachers of Grade 10 Academic report that the students have never used the software. This may indicate areas for further support and professional development.

Access to technology

Two questions were asked with respect to access to computers for teaching mathematics. One concerned the availability of computers in the teachers' classrooms and the other concerned the access to a computer lab in the school. Table 12 presents the results for the following question, organized by panel:

> **Approximately how many computers are available in your classroom for mathematics instruction for your students in this class?**

Participants were asked to select one of the responses shown in the table.

Table 12: Percentage of teachers reporting number of computers in their classroom by grade category

	Grade 7 and 8 teachers	Grade 9 and 10 teachers
One computer for every student	5%	5%
One computer for every 2 students	3%	4%
One computer for every 3 students	2%	1%
One computer for more than 3 students	26%	3%
One computer only	44%	25%
No computers	20%	62%

The data suggest that 80% of the teachers of Grades 7 and 8 and 38% of the teachers of Grades 9 and 10 report having at least one computer in their classroom. However, by examining the other percentages, it appears that few classrooms have sufficient computers for students to be working individually, in pairs, or in small groups.

In order to determine student and teacher access to the use of a computer lab, the following question was asked.

When you need to use the computer lab with this class, to what extent is it available?

The responses are presented in Table 13.

Table 13: Percentage of teachers reporting extent of availability of computer lab

Availability of computer lab	
Never available	3%
Rarely available	21%
Sometimes available	48%
Often available	23%
Always available	5%

We also examined this finding by school type to see if there were differences in K - 8 schools, 9 -12 schools, or 7 - 12 schools and when viewed this way, the pattern of responses in each type of school was similar to the pattern displayed in Table 13. The data suggest that 28% of the teachers can *often* or *always* use the computer lab when needed. Conversely, 24% of the teachers report that the computer lab is *never* or *rarely* available and 48% of the teachers report that it is *sometimes* available.

The final questionnaire item provided a space for teachers to make additional comments with respect to any aspect of implementing the curriculum. Of the 363 participants who responded to this last open-ended question, 12% (45) commented on technology. Of these, 40% (18) cited challenges to getting access to computers, having too few for the number of students, and not having appropriately capable and reliable computers. Although most of the comments regarding access to technology were directed towards the availability of computers, participants occasionally referred specifically to computer software and graphing calculators. As one respondent suggests:

> If the internet was available for laptops in the classroom, and a projector was readily available, I would be able to do many more exciting things with my students. The technology is simply not up to date and ready to use. (16060)

Professional development and technology

In order to determine teachers' professional development needs, we asked the following question:

> **To what extent do you feel that further professional development in these areas would help support you in implementing the curriculum in this class?**

We also provided a list of professional development areas and for each area, participants were asked to rate their response on a four-point scale response from *not at all*, *a little*, *somewhat*, and *very.* The responses are presented in Table 14.

Table 14: Percentage of teachers that would find further professional development in each of the areas to be helpful in implementing the curriculum

	Somewhat and Very Helpful
Use of manipulatives	67%
Understanding how students learn mathematics	72%
Teaching through problem solving	79%
Use of Geometer's Sketchpad®	53%
Assessment in mathematics	66%
Using group work in mathematics	61%
Teaching strategies	70%
Use of graphing calculators	46%
Facilitating investigations	60%
Content knowledge for teaching	37%
Use of other computer software	56%

Overall, teachers responded that further professional development in the use of The Geometer's Sketchpad® (53%), use of graphing calculators (46%), and use of other computer software (56%) would be *somewhat* or *very* helpful in implementing the curriculum.

Table 15 specifically focuses on these three areas of professional development connected to the use of technology and presents the data for teachers of Grades 7 and 8 and teachers of Grades 9 and 10.

Table 15: Percentage of teachers that would find further professional development in computing software, GSP and use of graphing calculators helpful in implementing the curriculum .

	Somewhat and Very Helpful	
	Grade 7 and 8 teachers	Grade 9 and 10 teachers
Use of other computer software (other than GSP)	72%	42%
Use of The Geometer's Sketchpad®	68%	40%
Use of graphing calculators	62%	42%

In all three cases, a higher percentage of teachers of Grades 7 and 9 reported that further professional development in these areas would be helpful.

The following open-ended response question was asked of teachers with regard to their professional development experiences.

> **Please describe a professional development experience that has positively influenced the way you teach mathematics.**

Of the 757 participants who responded to this open-ended question, 11% (86) commented on positive professional development experiences focusing specifically or partially on technology. These participants referred to attending or experiencing professional development that focused on specific technologies: 24 mentioned graphing calculators and related technologies and 30 referred to workshops on GSP. Although almost all comments were positive, a few participants identified challenges to technology-based professional development including issues of access to technology, and the lack of time to implement technology within the classroom.

The issue of time also came up in another questionnaire item when teachers were asked the extent to which they felt they had adequate time for a variety of activities. With respect to having sufficient time to learn new technologies, 71% of the respondents reported that they had *little* or *insufficient* time to learn new technologies.

Also, with respect to time, some of the 45 participants who discussed technology issues when responding to the last questionnaire item asking for additional comments, discussed the issue of lack of time for teachers to both use technology in the classroom, and to learn about, and explore, technology so that they would be able to incorporate it into their classrooms. While recognizing the value of professional development, the following comment summarizes the issue:

> Along with PD, teachers need release time to work and play with these tools to develop a comfort level that they can take into the classroom. Technology is great, but only if we have time to learn how and when to use it in our classroom to best help our students. (19221)

Further, in responding to this open-ended item, participants listed a number of specific areas relating to technology for which they saw a need for further professional development: GSP, interactive whiteboards, CAS systems, as well as other common computer programs and new technology.

Concluding comments

A number of observations and connections can be made from the various sources of data. One is the observation that the positioning of technology within the curriculum is very much aligned with the current thinking expressed in the research literature. Other observations include the recognition from the questionnaire data that a fair number of teachers are using technology in the teaching of mathematics, that access to computers is an issue in the use of technology, and that teachers are suggesting that they would like more professional development in the use of technology as well as time to explore the use of technology to teach mathematics.

Use of technology

The data suggest that approximately 90% of teachers of Grades 9 and 10 Applied and Academic courses report that their students use graphing calculators in at least some of the lessons. These data suggest that these teachers perceive themselves as comfortable with the technology and are making it readily available for student use. This is also reflected in the mathematics education leaders in the focus group data. The focus group data also highlights the importance of continued support for the use of technology and the collaboration of peers.

Also, nearly half of the teachers across Grades 7-10 report that students in their classes use dynamic geometry software (specifically, GSP) during at least some lessons. While this provides evidence of many teachers using dynamic geometry software, it also prompts us to seek ways to help more teachers provide opportunities to their students to investigate mathematical ideas with dynamic geometry software. Professional development and collegial support may provide opportunities for teachers to work with GSP, as over one third of teachers who offered comments about technology in the open-ended question regarding professional development reported positive influences on the value of GSP workshops. Also, we see support for GSP within the TIPS resources as they include many pre-made sketches included as part of complete lesson plans for immediate use by teachers. The availability of pre-made sketches and applets was identified by focus group participants as assisting teachers in overcoming fear of using dynamic graphing software.

Access to computers

While there is an increased prominence of technology in the curriculum, only 1 out of 4 teachers report *often* or *always* being able to use the computer lab when they need it. The issue of access to computers is echoed by both the focus group and questionnaire participants. The research literature also points to the challenge that teachers face with easy and timely access to technology, whether in computer labs or within the classroom and suggests that teachers with access to a healthy, encouraging, and technology-competent human infrastructure, and a functional, convenient, and easy to access technical infrastructure, are more likely to develop positive attitudes towards, and effectively integrate technology into their practice.

The role of professional development

Teachers' lack of comfort with technology as well as their lack of knowledge about the technology and its appropriate use in teaching mathematics are cited in the research literature as barriers to implementing technology effectively. The focus group data from the leaders in mathematics education and the teacher questionnaire data suggest similar challenges. As the focus group participants noted, many teachers are anxious as they are asked to move out of their comfort zone and integrate technology in the teaching of mathematics.

However, both questionnaire and focus group data suggest that teachers recognize the importance of ongoing professional development in developing expertise in using technology. This is supported by the research literature that shows that professional development helps to prepare teachers to use technology in their classroom and integrate it more effectively into their learning plan. Teachers responding to the questionnaire also mention the importance of having time to 'play' with the technology to increase their own competence and comfort level. In addition to formal professional development, participants in the focus group and questionnaire support the research literature in acknowledging the importance of teacher support and opportunities for collaborative learning that comes from interactions with other mathematics teachers, and especially those peers who have been able to integrate technology effectively.

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The Use of Manipulatives in Mathematics Class

Introduction

Piaget's constructivist theory (1952) of how children develop an understanding of number revealed the importance of concrete materials in children's learning and provided a powerful theoretical impetus for incorporating concrete materials into mathematics instruction. More recently, Vygotsky's sociocultural theory (1978, 1986) of human development has focused our attention on the social and distributed nature of cognition, while demonstrating how all cognition/learning is mediated by symbolic resources such as language and numeracy systems, and by material resources such as computers. Research in mathematics teaching and learning that draws on this perspective and views mathematical inquiry and its facilitation as a network of social practices that are mediated through mathematical communication, artifacts and resources (Forman, 2003; Greeno, 1998; Lave, 1996; Lerman, 1996; Sfard, 2006) provides the current theoretical encouragement for educators to incorporate concrete materials/manipulatives, physical or virtual, into the learning of mathematics. By manipulatives we are referring to a wide range of mathematical thinking tools such as fraction circles and strips, geoboards, algebra tiles, and area models that are explicitly designed to provide a representation of abstract mathematical ideas as well as other materials, such as linking cubes, pattern blocks, and paper folding that can be used to construct representations of a variety of mathematical ideas. These materials are designed for all learners to use, not only young children or struggling learners. The National Council of Teachers of Mathematics views their potential influence for students and teachers as follows:

> [For example] Students can develop a deep understanding of rational numbers through experiences with a variety of models, such as fraction strips, number lines, 10times10 grids, area models, and objects. These models offer students concrete representations of abstract ideas and support students' meaningful use of representations and their flexible movement among them to solve problems. (NCTM, 2000, p. 215)

With respect to teachers, they suggest that:

> Teachers can gain valuable insights into students' ways of interpreting and thinking about mathematics by looking at their representations. They can build bridges from students' personal representations to more-conventional ones, when appropriate. (NCTM, 2000, p. 67)

Engaging with manipulatives allows teachers and students alike the opportunities to behave and think mathematically as they jointly become aware of patterns, make conjectures, compare results, and provide explanations. Shaffer and Kaput (1999) suggest that to be able to "externalize the manipulation of formal systems changes the very nature of cognitive activity" (p. 97). This type of engagement creates a space of learning and of opportunity in which

> Different representations often illuminate different aspects of a complex concept or relationship. For example, students usually learn to represent fractions as sectors of a circle or as pieces of a rectangle or some other figure. Sometimes they use physical displays of pattern blocks or fraction bars that convey the part-whole interpretation of fractions . . . Other common representations for fractions, such as points on a number line or ratios of discrete elements in a set, convey some but not all aspects of the complex fraction concept. Thus, in order to become deeply knowledgeable about fractions—and many other concepts in school mathematics—students will need a variety of representations that support their understanding. (NCTM, 2000, p. 68)

Using these mathematical thinking tools also provides opportunities to 'externalize' the many modes of mathematical reasoning including conjecture, explanation, evaluation, and argument. These opportunities, in conjunction with their potential to enhance learners' mathematical understanding through multiple representations, make a compelling case for incorporating manipulatives into the classroom.

What does the research literature say?

Prior to the 1980s, studies regarding the effectiveness of manipulatives on student learning of mathematics provided inconclusive results (Hall, 1998). More recent research, however, has found that the appropriate use of different categories of manipulatives has improved student attitude and understanding of mathematics (Clements, 1999; Erickson & Niess, 1996; Hall, 1998; Leinenbach & Raymond, 1996; Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1986; Tooke, 1992). At the same time, the research cautions that the appropriate use of manipulatives is a complex issue and suggests that there are many elements to be considered. The critical element, however, is the recognized importance of the mediating role of the teacher in the development of learners' mathematical understanding (Ball, 2003; Ball & Bass, 2002; Ball & Even, 2004; Boaler, 2002).

In order to use manipulatives appropriately, teachers need to know how to interpret or represent the mathematical ideas they wish their students to learn in order to facilitate learning with understanding (Beattie 1986; Moyer, 2001; Post, 1981). Since the physical attributes of the manipulatives are not the source of the meaning of the mathematical concept (Ball 1992; Baroody, 1989; Clements, 1999), this creates a complex challenge for teachers as many have not had much experience with manipulatives and are unaware how to work with them (Lesh, Post, & Behr, 1987).

> Physical apparatus does not offer unmediated mathematical experience in itself, it can neither contain nor generate mathematics. Only people can do this, with their minds, and it is a central part of a teacher's role as teacher to help pupils to become able to do this for themselves. (Pimm, 1995, p. 13)

At the same time, using manipulatives effectively to promote mathematical understanding is also a function of the tasks a teacher chooses, and how she views manipulatives in relation to that task (Marshall & Swan, 2005; Moyer, 2001). While a mathematical task can serve as a location for joint/

shared understanding and collective exploration, Pimm (1995) suggests caution in setting up tasks so that that the students are not merely manipulating materials but are manipulating materials with the purpose of mathematical meaning. It would seem that before teachers are ready to set up such tasks for students, they, themselves, need time to experience and explore manipulatives and the mathematical meanings that they might entail.

Sowell (1989) in a meta-analysis found that the effective use of manipulatives in mathematics learning was enhanced when they were used for extended periods of time. Additional studies suggest that long term use of the same manipulatives to teach multiple ideas can help students identify connections between mathematical concepts (Beattie, 1986; Gilbert & Bush, 1988).

> If there is any risk related to the use of manipulatives in these [intermediate] grades, it derives from their being ignored or abandoned too quickly (Driscoll, 1981, p. 24).

In a recent study of teachers and students working with manipulatives in classrooms, Moyer (2001) found that the teachers often saw manipulatives as toys or playthings that could supplement the learners' mathematical inquiry, or be used as a reward for good behaviour. Their use was not central to the real math of the classroom.

> Teachers often view their use of manipulatives for math instruction as playing, exploring, or a change of pace . . . By aligning manipulatives with 'fun math' teachers were, in essence, distinguishing manipulative use from their regular mathematics teaching (Moyer, 2001, p. 188).

These attitudes constitute some of the biggest impediments to the effective use of manipulatives in the mathematics classroom as they point to an established practice in mathematics education that privileges the symbolic. For example, algebraic manipulation still plays a large role in school mathematics. As a consequence, mathematical concepts are introduced without considering (accounting for) the human experiences from which they originally emerged (Davis, 1996). To privilege the symbolic exclusively is to be blind to the fact that mathematical understanding emerges as part of our experience with the world.

> There are traces of the mathematical in the earliest of human records . . . [involving] a noticing of sameness, pattern, and regularity amid one's explorations . . . it involves comparing, ordering, creating and naming. (Davis, 1996, p. 93)

With respect to the use of manipulatives in high school, several studies reported a decline in manipulative use in later years of schooling (Gilbert & Bush, 1988; Perry & Howard, 1997). A suggested reason for this was that teachers appeared to lack the knowledge of the mathematics behind the manipulatives and were also uncertain about how to use them.

> As students move through the curriculum, the focus tends to be increasingly on presenting the mathematics itself, perhaps under the assumption that students who are old enough to think in formal terms do not, like their younger counterparts, need to negotiate between their naive conceptions and the mathematical formalisms. Research indicates, however, that students at all levels need to work at developing their understandings of the complex ideas captured in conventional representations. A representation as seemingly clear as the variable x can be difficult for students to comprehend. (NCTM, 2000, p. 67)

This view is also supported by Dewey who saw the same processes of learning at work in the child and the adult.

> The first stage of contact with any new material, at whatever age of maturity, must inevitably be of the trial and error sort . . . This is what happens when a child at first begins to build with blocks and it is equally what happens when a scientific man in his laboratory begins to experiment with unfamiliar objects. (Dewey, 1966 [1916], p.180-181)

While the research reviewed supports the value of incorporating manipulatives into the teaching and learning of mathematics, it also includes evidence to suggest that the issue is complex and interwoven with many factors and that outright use of manipulatives in classrooms will not necessarily lead to learners' developing greater conceptual understanding (Ball, 1992; Meira, 1998; Moyer, 2001). Teachers' views about using manipulatives will be situated in their own assumptions and experiences with mathematics education, and this will affect the way that they will use new materials (Sherin, Mendez & Louis, 2004). The findings from these studies suggest that teachers may need help to modify their pedagogical approach in order to allow the time and space necessary for meaningful inquiry and exploration with manipulatives. However, even if teachers have learned appropriate strategies for using manipulatives, their beliefs about how students learn mathematics may influence how and why they use manipulatives as they do (Moyer, 2001). Emphasizing the important role that teachers must play in making strong and multiple connections between the manipulatives and the underlying mathematical concepts, Stein and Bovalino (2001) cautioned that "if not used with careful thought, manipulatives can be come little more than window dressing, they are nice to look at and play with but superfluous to overall learning" (p. 357).

The Ontario mathematics curriculum and manipulatives

The Ontario Curriculum Grades 1-8: Mathematics revised (OME, 2005a) and the Ontario Curriculum Grades 9 and 10: Mathematics revised (OME, 2005b) describe manipulatives as "necessary tools for supporting the effective learning of mathematics by all students" (OME, 2005a & 2005b, p. 23). In particular the curriculum documents acknowledge the important affordances of manipulatives that allow learners to move within and between multiple representations of mathematical concepts and constructs in order to develop their mathematical understanding. Manipulatives are given equal weight to other representations (numeric, geometric, graphical, algebraic and pictorial), described as helping students to see "patterns and relationships; make connections between the concrete and the abstract; test, revise, and confirm their reasoning; remember how they solved a problem; and communicate their reasoning to others . . . A variety of manipulatives helps deepen and extend students' understanding of mathematical concepts" (OME, 2005a & 2005b, p. 25). It is noteworthy that manipulatives are identified in both documents as "necessary tools for supporting the effective learning of mathematics by all students" (OME, 2005a & 2005b, p. 23) having value across all grades and courses. In addition, manipulatives are seen as providing "a valuable aid to teachers" allowing them to see the 'concrete' mathematical reasoning of their students (OME, 2005a & 2005b, p. 25).

Our analysis of the curriculum documents shows that many of the specific curriculum expectations make reference to the use of manipulatives and provide explicit support for their use. In several specific expectations, students are directed to investigate mathematical concepts using a variety of tools and strategies and particular manipulatives are given as examples of the kinds of tools that students can use. Further, supporting resources developed by the Ontario Ministry of Education such as the Targeted Implementation and Planning Strategy (TIPS) also support the use of manipulatives. Table 1 provides a summary of the percentage of both the curriculum expectations and TIPS lesson plans that make reference to the use of manipulatives.

Table 1: Percentage of references to manipulatives in the Curriculum and TIPS by course/ grade level

Course/grade level	Curriculum expectations	TIPS lesson plans
Grade 7	45%	57%
Grade 8	34%	58%
Grade 9 Applied	23%	36%
Grade 9 Academic	23%	n/a
Grade 10 Applied	21%	46%
Grade 10 Academic	26%	n/a

What do Ontario math education leaders say?

The following summary of the OAME and OMCA focus group discussions highlights some of the themes associated with manipulatives that were part of the discussions. The math education leaders discussed the benefits of the use of manipulatives and stressed the fact that their use is featured prominently in the curriculum and that funding has been available for the purchase of manipulatives. Many math education leaders suggested that manipulatives have the potential to address misconceptions that students develop when they rely solely on rules and procedures. In addition, the math leaders were sensitive to the ways in which manipulatives encourage multiple representations.

> But it does say in the expectations that students have to use a variety of tools. So my thinking is that every student has to be able to show in more than one way, that's the expectation. It might be pencil and paper and then it might be a manipulative drawing, verbal explanation and that would depend on the level or the style of learning. (Kathy, Math Leader Focus Group [MLFG] 6)

While many of the participants suggested that students at all levels develop a deeper understanding of math through the use of manipulatives, some believed that manipulatives were for certain types of students such as for kinesthetic and hands-on learners. Others suggested that manipulatives help students move from the concrete to the abstract. The math education leaders stated that many teachers, primarily those at the secondary level, believe manipulatives are only necessary for assisting struggling learners or those with IEPs to gain understanding. They reported that teachers do not feel they are necessary or beneficial for students who are working at an average level or above in their classes. In addition, participants reported that teachers who work with students in higher grades view manipulatives as tools more suited for younger students. They seem to regard them as toys that real mathematicians wouldn't use.

> Sure. The other thing is I don't think they [teachers] realize that mathematicians use manipulatives, use all kinds . . . even in post-graduate work, there are things used to help people to understand mathematics and if we could sort of impress them that it's a useful tool, and it might help them even to understand the math, then I think we'll see more of it with their students, but I think we have a way to go. (Joan, MLFG 2)

Teachers were seen as being resistant or fearful of using manipulatives due to the lack of understanding of the mathematical concepts behind these tools. Some suggested that teachers may be intimidated by the mathematical possibilities raised by using manipulatives, as manipulatives move teachers outside their comfort zone with mathematics more than when using traditional algorithms or methods.

> Yes. As just a sample at one of our workshops on fractions, we were working with Cuisenaire rods, and to start off I gave them a scenario about a student who was mixed up because the teacher had said that a quarter divided by a half was equal to a half and he couldn't understand why it wasn't an eighth. I said, what would you do to help the child? So they worked in their groups just to think about it. Right away, they drew a picture, right, and they drew a quarter, then they drew like this. And they said, well, if I have a quarter and if I divide by a half, oh, I do get an eighth. They were like, we know the answer isn't an eighth, but you do get an eighth when you divide by half. And the total misconception was they were dividing by two and they don't have that understanding, because they just learned rules. So, then we got out materials and we showed them what divided by a half means. We want to find how many of the half rods, or pattern blocks, or whatever, fit into the quarter one. Only half of it fits into. The light bulbs just went crazy. And yet, they still are resistant to use it with their students because I think they're afraid of what they don't know. (Kathy, MLFG 6)

While it was acknowledged that teachers were also afraid "of looking stupid in front of the kids" (Erin, MLFG 6), it was pointed out that even when teachers saw the value in using the manipulatives to increase their understanding, they were often still resistant to using them with their students. The math education leaders talked about how teachers didn't necessarily know the ways in which manipulatives were to be used and perhaps lacked training in teaching through the use of manipulatives. Some suggested that if teachers had a better understanding of their value for learners at all levels, then they might be implemented more often with all students. While participants recognized the value of manipulatives, they indicate their use in the intermediate grades is still emerging. Participants suggested that many teachers feel that there is not enough time to cover the material in the curriculum. As such, teachers may avoid using manipulatives as they believe that manipulatives are more time-consuming and less efficient than traditional methods. Participants however, suggested that a quicker traditional manner does not guarantee understanding and teachers may have to revisit the same topic at a later date.

The math education leaders also discussed a variety of ways that they see teachers' use of manipulatives being supported such as through their presence in textbooks and resource documents (e.g., TIPS). As well, particular aspects of professional development supported teachers' implementation of manipulatives. For instance, teachers who had opportunities to learn about and practice using manipulatives in professional development activities became more open to using them in their classrooms as they personally had gained a better understanding of the math concepts and saw how the manipulatives could support students' understanding. The math leaders reported that professional development that was sustained over time helped teachers continue to feel supported. Developing a community in which teachers could come together and work with manipulatives to teach math was seen as a very positive approach that supported teacher and student understanding. This was the case when teachers worked with other teachers within the same grade, and when they worked with colleagues from across various grades.

The questionnaire data

One of the principal data gathering instruments of the CIIM project was a 45-item online survey that elicited information about math teachers' teaching assignment, instruction, and assessment practices in mathematics, and professional development experiences and needs. There were 1,096 respondents to the web-based questionnaire that intermediate math teachers could access online during May and June 2006. The sample was nearly evenly divided with 46% teaching Grades 7 and 8, and 51% teaching Grades 9 and 10. Only 3% of respondents reported "other" as their teaching assignment. The majority of respondents (93%) identified their primary role as a full time classroom teacher. Of the Grade 7 and 8 teachers, 42% reported that they teach math for 60 minutes or less each day while 31% report teaching math for more than 60 but less than 90 minutes daily. This suggests that many of these Grade 7 and 8 teachers were not on rotary but rather were responsible for the full range of subjects in the Grade 7 and 8 classes. Of the Grade 9 and 10 teachers, 62% report that they teach math 3.5 hours or more daily. This suggests that more than half of the Grade 9 and 10 teachers in the survey teach a full timetable of math.

Within this questionnaire, several items dealt specifically with teachers' views about the use of concrete materials and mathematical manipulatives in teaching mathematics as well as their actual use in classrooms. Participants were asked to rate their agreement with the following statement using a four-point scale ranging from *strongly disagree* to *strongly agree*.

> **To what extent do you agree with the following? Manipulatives are necessary tools to support effective learning of mathematics for all students.**

The findings showed that 48% of the Grade7 and 8 teachers *strongly agreed* that "Manipulatives are necessary tools to support effective learning of mathematics for all students" compared with 16% of the Grade 9 and 10 teachers.

In another item, respondents were asked also to rate the importance of each of six classroom practices using a four-point scale ranging from *not at all* to *very important*. Each of the activities required a separate response. One of these classroom practices was related to the use of multiple representations. The specific question asked was:

> **In this class, how important is each of the following? Promoting the use of multiple representations of ideas (concrete materials, technology, etc.)**

In general, 48% of the respondents rated this as *very important*. However, for many of the items on the questionnaire, we asked respondents to identify one particular math class that they were teaching during the school year and then use that class as the basis for their answers. This allowed us to distinguish different practices, activities, and attitudes associated with different courses. Table 2 below presents the responses for the above item when we analysed it in relation to a specific course or grade level.

Table 2: "Promoting multiple representations of ideas (concrete materials, technology, etc.)" by course/grade level for the category *very important*

Course/grade level	Very Important
Grade 7	60%
Grade 8	53%
Grade 9 LDCC	59%
Grade 9 Applied	50%
Grade 9 Academic	37%
Grade 10 LDCC	57%
Grade 10 Applied	47%
Grade 10 Academic	30%

It appears that more teachers in Grades 7 and 8 and in the Grade 9 and 10 Locally Developed courses view the use of multiple representations as *very important* than teachers of Grade 9 and 10 Academic and Applied courses. The data also suggest that teachers in Grade 9 and 10 see the use of multiple representations as being more important for teaching students in Applied courses than for those in Academic courses.

In another item participants were asked to rate their level of comfort with a variety of aspects of the curriculum. The question offered a four-point scale response: *not at all, a little*, *somewhat*, and *very comfortable*.

How comfortable are you with the following aspects of the curriculum for this class?

Table 3: Percentage of respondents who rated their comfort level as *somewhat* and *very comfortable*

Aspect of the curriculum	Somewhat	Very
Using a variety of assessment methods	50%	41%
Using technology to teach mathematics	42%	32%
The philosophy of the curriculum	51%	26%
Teaching through problem solving and investigating	51%	36%
Students working in groups	43%	37%
The Achievement Chart	36%	24%
The content of the course	27%	68%
Using concrete materials to teach mathematics	43%	40%
Integrating concepts from across the strands of mathematics	44%	39%

From this table we see that with respect to the use of concrete materials to teach math, 83% of teachers responded that they were *somewhat* or *very comfortable* teaching mathematics with concrete materials. Table 4 below presents the findings of the responses of those who were *very comfortable* using concrete materials when analyzed by course and grade level.

Table 4: Percentage of teachers who are *very comfortable* using concrete materials to teach mathematics by course/grade level

Course/grade level	Very comfortable
Grade 7	46%
Grade 8	43%
Grade 9 LDCC	52%
Grade 9 Applied	48%
Grade 9 Academic	30%
Grade 10 LDCC	57%
Grade 10 Applied	35%
Grade 10 Academic	27%

These data suggest similar trends to the previous table that examined the importance that teachers place on the use of concrete materials. In terms of comfort, fewer teachers of Grade 9 and 10 Academic report that they feel *very comfortable* with their use.

As well as examining the importance and comfort with the use of multiple representations and concrete materials, we also asked teachers to respond to the extent of their use. This was part of a question where we asked teachers:

In this class, how often do the following occur?

This question was followed by a list of activities that might occur in a classroom. In this question teachers were asked to choose their response from a four-point scale ranging from *never*, *some lessons*, *most lessons*, or *every lesson*. Table 5 shows the results for the combined categories *most lessons/every lesson*.

Table 5: Percentage of respondents who report that the activities occur in *most lessons/every lesson*

Classroom practices	Most lessons/ Every lesson
The teacher explains, demonstrates or provides examples	93%
Students work on problems with multiple solutions	34%
The teacher provides solutions to problems	67%
Students work on practice questions	91%
Students justify their answers and explain their reasoning	66%
Students work on investigations to determine relationships or mathematical ideas	36%
The teacher works with small groups of students	33%
Students work with concrete materials or manipulatives	24%
Students use computer software or graphing calculators	13%

Overall, 24% of teachers reported that students work with concrete materials in *most* or during *every lesson*. However, it may not be expected that concrete materials or manipulatives are used in most or every lesson and so it is useful to examine the full range of responses for this particular item. Table 6 presents the data for "students work with concrete materials or manipulatives" by course/ grade level:

Table 6: Percentage of respondents rating how often students work with concrete materials or manipulatives by course/grade level.

Course/grade level	Never	Some lessons	Most lessons/ Every lesson
Grade 7	1%	55%	43%
Grade 8	1%	71%	28%
Grade 9 LDCC	9%	32%	59%
Grade 9 Applied	6%	75%	20%
Grade 9 Academic	14%	84%	3%
Grade 10 LDCC	9%	48%	43%
Grade 10 Applied	10%	82%	8%
Grade 10 Academic	20%	74%	7%

These data show several differences in the use of concrete materials or manipulatives in the various grades or courses. An argument could be made that the curriculum may not be suggesting that concrete materials or manipulatives be used in most or every lesson but their use is certainly suggested for some lessons. Hence, examining the *never* column gives us an indication that there are some classes in which students do not have the opportunity to work with manipulatives or concrete materials. This seems to be more prevalent in the Grade 9 and 10 courses, particularly in the Academic stream.

Another item that examines classroom practice asked teachers to report often students used manipulatives such as algebra tiles, geoboards etc. in their course. In this question teachers were asked to choose their response from a four-point scale ranging from *never*, *some lessons*, *most lessons*, or *every lesson*.

Table 7: Percentage of respondents rating how often students work with manipulatives by course/grade level.

Course/grade level	Never	Some lessons	Most lessons/ Every lesson
Grade 7	1%	65%	34%
Grade 8	4%	79%	17%
Grade 9 LDCC	14%	55%	32%
Grade 9 Applied	12%	80%	8%
Grade 9 Academic	40%	60%	0%
Grade 10 LDCC	9%	83%	9%
Grade 10 Applied	21%	78%	1%
Grade 10 Academic	48%	52%	0%

A comparison of Tables 6 and 7 shows different response patterns with respect to the percentage of the respondents reporting that “students never work with concrete materials or manipulatives.” This may be due to the variation in the way the questionnaire items were worded. The two items on the questionnaire were:

> **In this class, how often do the following occur? Students work with concrete materials or manipulatives.**

Compared with

> **In this class, how often do students use each of the following resources? Manipulatives (e.g. algebra tiles, geoboards)**

The differences in responses may be due to different interpretations of the two phrases ‘concrete materials or manipulatives’ and ‘manipulatives’. The phrase ‘concrete materials or manipulatives’ may have been understood to include additional instructional materials (e.g., graph paper) not generally considered to be mathematical manipulatives while the phrase ‘manipulatives’ (e.g., algebra tiles; geoboards) is more restrictive. In both tables, however, two trends can be seen. The first is that as students move up the grades, they use manipulatives in fewer classes. The second is that students in Grade 9 and 10 Applied classes use manipulatives more frequently than their peers enrolled in Academic classes.

Teachers were also asked about areas where they felt that further professional development would be of value. Overall, 67% of the teachers responded that further professional development in the use of manipulatives would be *somewhat* or *very* helpful. When analzed by grade level, 79% of the Grade 7 and 8 teachers felt that further professional development in the use of manipulatives would be *somewhat* or *very* helpful, compared with 57% of the Grade 9 and 10 teachers.

The findings from the web-based questionnaire suggest that teachers of Grades 7 and 8 have their students use manipulatives more frequently, generally feel more comfortable with using manipulatives for mathematics instruction, more often agree that manipulatives are necessary tools to support effective learning of mathematics for all students, and view the promotion of the use of multiple representations of ideas as more important than teachers of Grade 9 and 10. At the same time, however, they are also the ones who request additional professional development in this area. One interpretation of this finding may be that the Grade 9 and 10 teachers do not attribute as much value to the use of manipulatives in mathematical instruction, consequently use them less frequently, and thus see less need for additional professional development in their use.

Concluding comments

The research literature on the use of manipulatives in mathematics teaching and learning directs our attention both to the educational value of manipulatives as well as to the complexity involved in using them effectively. The Ontario curriculum documents acknowledge the important affordances of manipulatives that allow learners to move within and between multiple representations of mathematical concepts and constructs in order to develop their mathematical understanding. In the curriculum documents, manipulatives are given equal weight to other representations (numeric, geometric, graphical, algebraic and pictorial) and are described as “necessary tools for supporting the effective learning of mathematics by all students” (OME, 2005a & 2005b, p. 23).

The data from the teacher questionnaire and focus group discussion with math education leaders suggest that not all teachers see value in the use of manipulatives for all of their students. Rather the data suggest that teachers view the use of manipulatives as more appropriate for students in Grades 7 and 8 and in the Applied and Locally Developed courses in Grade 9 and 10. The math education leaders stated that many teachers, primarily those at the secondary level, believe that manipulatives are only necessary for assisting struggling learners as they help students move from the concrete to the abstract. They also reported that teachers who work with students in higher grades view manipulatives more as toys that real mathematicians wouldn’t use.

These results are echoed in the research literature that reports that manipulative use is often valued more for instructing younger students and that their use declines in later grades. Several studies (Gilbert & Bush, 1988; Perry & Howard, 1997) suggest that one reason for this appears to be lack of teacher knowledge in the use of, and mathematics behind, the manipulatives. This theme was echoed by the math education leaders who suggested that teachers may be intimidated by the mathematical possibilities raised by using manipulatives, as these materials move teachers outside the comfort zone established when using more familiar algorithms and methods.

The data from the questionnaire, however, show that 67% of respondents considered that additional professional development related to the use of concrete materials and manipulatives would be of value. The math education leaders reported that when teachers were presented with professional development opportunities to learn about and experience using manipulatives, they became more open to using them in their classrooms. These findings are supported by the research as well.

> Designing professional development that provides teachers with deeper understandings of concepts such as representation and its role in mathematics teaching and learning may shape not only how the manipulatives are used but also what beliefs motivate teachers to use them (Moyer, 2001, p. 194)

The math education leaders reported that professional development that was sustained over time seemed to work better than short-term initiatives and was greatly appreciated by teachers. Participation in a community of teachers working together with manipulatives to teach math was seen as a very effective approach to support teachers. Such initiatives allowed teachers to personally deepen their mathematical understanding as well as see how working with manipulatives could do the same for their students. This was the case when teachers worked with other teachers within the same grade, and when they worked with colleagues from across the grades.

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Assessment

What does the research literature say?

Developing new assessment practices are an essential part of implementing new approaches to teaching mathematics (Earl, 2003; Graue & Smith, 1996; National Council of Teachers of Mathematics (NCTM), 1995; Wiliam, 2007). As teachers encourage students to engage in mathematical investigation and problem solving, to develop their own methods of solution, and to argue and discuss mathematical ideas, teachers recognize that assessment strategies must go beyond paper-and-pencil tests (Barnes, Clarke, & Stephens, 2000; Gearhart & Saxe, 2004; Romberg, 2004; Shepard, 2000, 2001; Suurtamm, 2004). These new assessment strategies no longer merely focus on how well a student uses a memorized algorithm or procedure but are designed to elicit students' mathematical understandings and engagement in mathematical processes (NCTM, 1995; Romagnano, 2001). Hence, along with traditional pencil and paper assessments, assessment strategies may also include conferencing, performance tasks, interviews, projects, portfolios, presentations and observations so that teachers can gain access to students' mathematical understanding.

New developments in assessment of mathematics are also set within the context of assessment reform which affirms that while assessment is conducted for many reasons such as reporting on students' achievement or monitoring the effectiveness of an instructional program, that the central purpose of assessment is to support and enhance student learning (Joint Committee on Standards for Educational Evaluation, 2003; NCTM, 1995; Wiliam, 2007). As such, assessment is viewed as integrated with learning, focused on the learning process, and consistent with instructional activities (Gearhart & Saxe, 2004; Shepard, 2000). Further, formative assessment plays a central role as assessment opportunities are imbedded in instruction and provide opportunities for informative feedback as does having regular opportunities for students to demonstrate what they know and can do (Black & Wiliam, 1998; Gearhart & Saxe, 2004; Shepard, 2000; Wiliam, 2007). Black and Wiliam (1998) highlight the impact of formative assessment on improved student learning and after reviewing over 250 studies, they contend that improving formative assessment practices results in gains in student achievement that are larger than most other educational interventions and are particularly evident for low achieving students. Taken together, new assessment ideas challenge traditional assessment practices of merely end-of-unit tests and prompt educators to provide regular and varied assessment opportunities that align with instruction, provide meaningful feedback to students and teachers and are part of regular classroom practice.

Drawing on current assessment literature and particularly on literature regarding assessment in mathematics, we are able to provide a list of several characteristics of effective assessment practice in mathematics. These include:

- Assessments should focus on important mathematical concepts, present a comprehensive view of mathematics, include the full range of mathematical activity, and reflect the important role of problem solving (Goldin, 1992; NCTM, 2000).

- A variety of assessment methods should be used to address the full range of mathematical processes, to ensure that all students have an opportunity to demonstrate their learning, and to compensate for limitations inherent in any one kind of assessment (NCTM, 1995; Shepard, 2001; Wiggins & McTighe, 1998).

- When technology and manipulatives are used in instruction (e.g., graphing calculators, dynamic geometry software, algebra tiles, etc.), they should also be available for students to use in assessments (NCTM, 1995)

- Opportunities for students to become more active participants in assessment should be provided including chances for students to develop criteria, and to evaluate, reflect on, and improve their work. Along with this, the use of peer and self-assessment helps to develop students' meta-cognitive awareness and can substantially enhance learning in mathematics (NCTM, 1995; Shepard, 2001, 2005; Wiliam, 2007).

- Informal assessments gathered on an ongoing basis through observing students and listening to them describe their work are critical as they enable teachers to gain valuable insights into students' mathematical understanding (Gearhart & Saxe, 2004; Shepard, 2001; Stiggins, 2004).

- Students should be informed about what they need to know, how they will be expected to demonstrate that knowledge, the criteria used in judging their work and the consequences of the assessment (NCTM, 1995; Romagnano, 2001).

- Providing students with detailed and meaningful feedback about the approaches they are using is an essential feature of effective mathematics assessment (Black & Wiliam, 1998; NCTM, 1995; Nicol & Macfarlane-Dick, 2006; Shepard, 2001; Wiliam, 2007).

Research on developing new assessment practices

While new forms of assessment are clearly aligned with new ways of teaching and provide important information about learning to students, parents and teachers, they may challenge traditional views of assessment and require a change in the way assessment is viewed. For teachers, this may involve reconsidering deeply held notions about mathematical knowledge, teaching and learning, assumptions about students, and perceptions about oneself as a teacher (Ball, 1994; Black & Wiliam, 1998; Shepard, 2000, 2001). In a recent review of research examining changes in classroom assessment practices, Tierney (2006) observes “although the rhetoric for assessment reform is strong, the way in which student learning is assessed in classrooms on a regular basis seems resistant to change” (p. 239). Many researchers have confirmed that meaningful changes in assessment practice occur gradually and will require the sustained and concerted effort of individuals across the entire educational system (Black & Wiliam, 1998; Fullan, 2003).

We draw on several studies that have focused on improving instructional and assessment practices in mathematics classrooms to better understand the challenge of shifting assessment practice. In one such study, Manouchehri (1998) examined the classroom practices of 51 Grades 6, 7, and 8 mathematics teachers involved in implementing inquiry-based mathematics programs. In another study, Barnes, Clarke and Stephens (2000) reviewed classroom documents, administered questionnaires and conducted interviews with teachers in 23 schools in two states in Australia to examine the impact of two different state-mandated assessment policies for Grades 11 and 12 courses on the classroom assessment practices of teachers in Grades 7 through 10. More recently, Wiliam, Lee, Harrison and Black (2004) worked with 24 mathematics and science teachers at the secondary level in two regions in the United Kingdom. In this study, researchers worked collaboratively with small groups of teachers over a period of six months to provide support in developing and planning new assessment strategies. As teachers tried out their new approaches the following September, their experiences were recorded along with measures of the impact of the new assessment approaches on students’ learning.

Such research studies have identified a number of factors which may interfere with shifting assessment practice and have also suggested some approaches which may encourage the implementation of assessment changes. Some of these issues are situated at the systemic or organizational level while others are more closely related to teachers' classroom practices (Tierney, 2006). We are highlighting some of the most salient factors that were identified.

At the organizational level, a number of researchers have noted that changing assessment practice requires strong leadership and extensive professional support. For example, Manouchehri (1998) found that the presence or absence of progressive leadership at the school and district level from individuals such as math coordinators, principals or expert teachers was instrumental in the success of all teachers who participated in the study. Wiliam, Lee, Harrison and Black (2004) found that the board-level person who acted as an intermediary and provided additional support for teachers was a valuable part of successful implementation.

The messages from curriculum and assessment policy documents also create challenges at the systemic level. One is the issue of alignment. Barnes, Clarke and Stephens (2000) note that in their study of secondary teachers in New South Wales, the teachers were faced with somewhat contradictory messages that caused confusion and hindered assessment changes. In this study, the curriculum encouraged the use of investigations, problem solving and modeling but as these were not required in externally mandated assessments, these practices were not well implemented in classroom instruction and assessment practices. However, in their study in Victoria, the mandated assessment practices included an equal emphasis on (i) problem solving/investigations, (ii) skills and (iii) extended projects and this had a positive impact on classroom instructional and assessment practices. They found that where the mandated assessment policies required the use of a centrally-set extended mathematics investigation to be completed by students over a two-week time period, there was greater use of alternative forms of assessment and increased use of problem solving and investigations. With respect to curriculum and assessment documents, teachers who participated in the Manouchehri (1998) study indicated that they needed more specificity and support with several aspects of the curriculum in order to implement appropriate assessment practices. Even when a prescription is offered in policy documents, the intent of the prescribed practice may not be understood and thus not implemented appropriately. As Wiliam et al. (2004) found, teachers benefit from having a variety of examples of new assessment practices by teachers with whom they can identify.

There are other non-systemic issues that create challenges to changes in assessment practices. These include the experiences, knowledge, and beliefs that teachers bring to the classroom that may not align with new instructional and assessment practices, as well as the constraints of time and opportunities to learn about and create new assessment practices. In some cases, teachers' understanding of mathematics creates a barrier to more open-ended assessment tasks. Assessment practices that include designing meaningful tasks and analyzing and interpreting students' responses to these tasks are complex and require substantial mathematical content knowledge that some teachers may not possess (Ball, 1994; Gearhart & Saxe, 2004; Manouchehri, 1998; Romagnano, 2001). Manouchehri (1998) observed that teachers' lack of mathematical knowledge caused them to avoid units with which they did not feel comfortable and that teachers often felt insecure opening up a dialogue about mathematics concepts because students asked 'why' questions that teachers thought they could not answer. As well, Manouchehri found that a lack of knowledge about multiple approaches to problems and investigations were limitations to implementing the inquiry-based instruction and assessment for most of middle school mathematics teachers who participated in the study.

Barnes, Clarke & Stephens (2000) cite time as the reason most frequently offered by classroom teachers in Grades 7 through 12 for limiting their use of problem solving and investigations in both instruction and assessment as they view this as taking away from time needed to cover the curriculum. Manouchehri (1998) found that all teachers, regardless of their years of teaching experience, felt as though they lacked sufficient time for designing lessons, conducting activities in class, and assessing student progress using inquiry-based approaches. In addition, some teachers noted that they needed additional time to teach students how to use manipulatives and to work in collaborative learning activities before new assessment approaches could be used.

Many studies indicate that changing assessment practice occurs with greater success when a collaborative environment is established and where school schedules include time for teachers to meet with colleagues (Ball, 1994; Rowley, Brew, & Ryan, 1996; Tierney, 2006). Overall, research suggests that effective professional development in assessment requires long-term support, the use of various forms of communication and the opportunity for teachers to establish a sense of community with their colleagues. To make meaningful changes in their mathematics assessment practices, teachers need to be actively involved in planning for and implementing changes and must have an opportunity to try out and reflect on new approaches in the context of their own classrooms (Ball, 1994; Black & Wiliam, 1998; Shepard, 2000; Tierney, 2006; Wiliam, Lee, Harrison, & Black, 2004). Teachers also need to be supported at the administrative level and with policies that align curriculum, instruction, and assessment, including large-scale assessment.

What does the curriculum say about assessment?

Messages about assessment and evaluation are threaded throughout the Ontario Curriculum Grades 1-8 Mathematics Revised (OME, 2005a) and the Ontario Curriculum Grades 9 and 10 Mathematics Revised (OME, 2005b). These two documents have nearly identical messages with regard to assessment and both explicitly state that "the primary purpose of assessment and evaluation is to improve student learning" (OME, 2005a, p. 18, 2005b, p. 17). The documents state that assessment information “helps teachers to determine students’ strengths and weaknesses,” guides them in “adapting curriculum and instructional approaches to students’ needs” and helps them to assess “the overall effectiveness of programs and classroom practices” (p. 18, p. 17). The importance of giving students feedback and clear directions for improvement and of providing opportunities for students to set specific goals and assess their own mathematics learning are indicated as ways of ensuring that assessment supports learning and is aligned with current thinking on formative assessment. The necessity of using a variety of assessment methods over a period of time in order to provide more opportunities for students to demonstrate the full range of their learning is also strongly emphasized in the curriculum. Suggested methods include assignments, day-to-day observations, demonstrations, tests, conversations or conferences, performances and projects.

Problem solving and the use of an investigative approach that provide opportunities for students to search for patterns and relationships and to engage in inquiry are strongly endorsed in the assessment of intermediate mathematics. Problem solving is described as offering “excellent opportunities for assessing students’ understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas” (OME, 2005b, p. 13). The curriculum documents also recommend the use of technology and manipulatives in the assessment and evaluation of mathematics. By “analyzing students’ concrete representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students’ thinking and provide supports to help enhance their thinking” (OME, 2005b, p. 23).

Teachers are advised that evaluation should focus on overall expectations.

> All curriculum expectations must be accounted for in instruction, but evaluation focuses on students' achievement of the overall expectations. A student's achievement of the overall expectations is evaluated on the basis of his or her achievement of related specific expectations (including the mathematical process expectations). (OME, 2005a, p.19)

In addition, "teachers will use their professional judgment to determine which specific expectations should be used to evaluate achievement of the overall expectations, and which ones will be covered in instruction and assessment (e.g., through direct observation) but not necessarily evaluated" (OME, 2005a, p.19).

Both curriculum documents include an Achievement Chart for teachers to use in the evaluation of students' work. It is suggested that "teachers will ensure that student work is evaluated in a balanced manner with respect to the four categories of knowledge and skills" (OME, 2005b, p. 19) which are included in the chart. The Achievement Charts in both the elementary and secondary curriculum documents use the same categories of knowledge and skills, achievement level descriptions and evaluation criteria. The Ministry suggests that the Achievement Chart will "guide the development of quality assessment tasks and tools" (p. 18), and help teachers both in planning instruction for learning and in providing feedback to students.

The curriculum documents suggest that teachers conduct assessments over a period of time rather than only at the end of the learning process. For Grades 9 and 10 teachers the final grade for students in each course must include both evaluations conducted throughout the course (70% of final grade) and evaluations administered towards the end of the course (30% of final grade). The Grades 9 and 10 curriculum document indicates that the 70% portion of the grade "should reflect the student's most consistent level of achievement throughout the course, although special consideration should be given to more recent evidence of achievement" (OME, 2005b, p.22). This most consistent/more recent requirement adds an additional dimension to the task of combining various pieces of assessment information for Grades 9 and 10 teachers. Grades 7 and 8 teachers are not required to ensure the 70%/30% balance and the most consistent/more recent requirement is also not included in the elementary curriculum document.

The curriculum also provides information about the assessment and evaluation of exceptional students and of those who are learning the language of instruction. While some of these students may require modified expectations, for many, changes in assessment procedures such as allowing additional time to complete tests or using alternative forms of assessment such as oral interviews, learning logs, or portfolios may be more appropriate (OME, 2005b, p.26).

What do Ontario mathematics education leaders say?

The following summary of the OAME and OMCA focus group discussions highlights the themes related to assessment that emerged in the discussions of math education leaders. These include the evidence of some teachers' shifting assessment practices and the recognition that challenges in developing new assessment practices still remain.

Evidence of shifting assessment practices

The focus group participants suggested that some teachers are now talking about assessment in new ways. This includes using terms such as "assessment for learning" and "assessment as learning". There is also evidence that some teachers are practicing new ways of assessing students such as in the following incident discussed by two mathematics education leaders:

> Nancy: This teacher sat down and interviewed every single child. There's 30 kids and he sat down with every child for 15 minutes. […] What ends up happening is when he interviews them, if they had not quite met some of the expectations, he says take this and show me now. Like, he actually gave them an opportunity to rework some of the work.
>
> Sarah: It sounds as though the teacher's looking to say, you know, the job of the student is to learn and then to show me that they know it and gave them multiple opportunities to show that they knew it. (Math Leader Focus Group [MLFG] 4)

In another example, one participant described a large culminating activity that integrated a variety of subject areas and involved the entire school. The assessment project incorporated tasks such as group work, class presentations, and verbal reports with teachers from many disciplines collaborating in the process.

The participants also mentioned that the Achievement Chart has prompted teachers to move beyond teachers' focus on assessing merely knowledge.

> Even if they're not required to have 25 per cent on this, 10 per cent on this, they actually do make a conscious effort to try and get all of the categories assessed during that chapter somehow. And I think that's been a step forward. (Beth, MLFG 1)

While there was evidence that some teachers are assessing in new ways, these incidents were not reported frequently. Rather, the focus group discussions centered on the challenges that remain in adopting new assessment practices.

Recognizing assessment challenges

While some teachers are cognizant of the multiple purposes of assessment, participants suggested that many other teachers still see reporting as the sole purpose of assessment.

> Kerry: Even just the idea that for a lot of them, assessment equals reporting.
>
> Nancy: Yes.
>
> Kerry: That's their purpose, not for learning but for reporting purposes. (MLFG 4)

In addition, participants suggested that some teachers may even view assessment as a form of punishment.

> I remember back in my department wonderful teachers and all the rest. But every once in a while, kids weren't good today, I'm going to give them a quiz tomorrow. It would be punishment by quiz. (Sarah, MLFG 4)

Participants stated that the assessment messages in the 1997/1999 curriculum documents were quite new for teachers at that time and many teachers are still struggling with understanding those messages. When first introduced, many assessment ideas were perceived as very challenging and difficult to address. For instance, one participant related the complexity of assessing problem solving and another discussed the problem of assessing mathematical processes.

> It's teachers perhaps not understanding "What am I assessing when I'm assessing problem solving? Is it complicated problems or complex problems? How do I assess it? How do I ensure that kids' marks don't get affected by it?" (Liz, MLFG 5)
>
> Getting back to the talk about the processes, teachers in Grades 9 and 10, we tell them that they have to assess processes, but they don't know what that means. They don't know how to go about it, so that kind of ties in with what Michelle was saying. (Scott, MLFG 4)

Along with not fully understanding the meaning of some of the new assessment ideas and policies, teachers struggle with putting some policies in place. As identified by participants, these policies include such things as aligning assessment with the Achievement Chart, reporting multiple content strands in elementary mathematics, moving back and forth between levels and percentages, reporting students' most recent and most consistent achievement, and principles of assessment for learning such as credit recovery opportunities. In trying to address the policies, it was suggested that the intent of policies is often missed.

One example that was often discussed is the mandate that elementary teachers must report mathematics marks by content strands. Participants reported that many elementary teachers may not understand the intent of this policy and that when teachers attempt to integrate the content strands, they may have difficulty separating the assessment data into those strands. The participants suggested that some teachers view this way of reporting as a barrier to integrating the strands. Further, when teachers are reporting by strand, they may be hard pressed to cover multiple strands in one term possibly impacting their ability to effectively implement the curriculum.

Participants discussed the idea that teachers conform to the policies that are mandated, particularly in report card mandates, but that they may not necessarily embrace the policies or agree with the philosophy behind some of the assessment messages. Focus group participants also talked about some of the reasons why teachers may be confused by new assessment approaches. In some cases, this related to mixed messages in policy documents. For instance, it is not clear to teachers or to the participants how content strands, curriculum expectations, process expectations, and the Achievement Chart can all be represented in assessment and reporting practices. In addition, assessment policies were perceived as being understood and implemented in different ways by boards, schools, administrators and departments.

A few participants noted that some school boards promote new assessment ideas through school board policies. For instance, some school boards have forced teachers to pay attention to the Achievement Chart or other assessment items in Ministry documents by designing policies. Many school boards expect secondary teachers to organize their marks by Achievement Chart categories.

> And the other thing too is that there seems to be a … lack of understanding of the assessment categories, and how you should implement those on assessment. So some Boards will say you have to have all four categories on every test. And then you record, or they record four distinct marks. And then do you have to do that or is it just that you have to make sure that you use all categories in terms of getting an assessment. (Dave, MLFG 1)

> I think our Board has a document for weighting the categories in every single course. (Steve, MLFG 1)

Boards were also noted as mandating other assessment practices such as particular approaches to summative assessments.

Another topic that came up in the focus group sessions is accountability. Participants noted that issues of accountability can often drive teachers to revert to traditional methods if they feel that those are easier to defend.

> Assessment, you know […] that is always something that I have problems with. It's not that I don't believe in what I'm doing, but if I need to be accountable to parents and my principal and the teachers who get the kids after I do, I need to be able to show them what I'm doing is valid. And so I've been a teacher for a while, but I'm still struggling with that kind of thing. (Sue, MLFG 3)

With regard to EQAO testing, participants offered a number of comments. One participant mentioned that EQAO scores may be used to determine directions for professional development. EQAO scores were also seen as being used to determine who would teach particular classes. In the discussion, a participant spoke of schools deciding to put their best teachers into the Grade 9 Applied class to raise their student scores. In some cases, participants mentioned that EQAO data is not used effectively and that provincial testing took away time from classroom teaching when there was already much to be covered in the curriculum.

Questionnaire data

The CIIM questionnaire included several questions concerning teachers' assessment practices in mathematics. Teachers were asked to indicate the assessment methods they use to get a sense of students' understanding as well as the methods they use in determining students' marks. They were also asked to indicate their level of comfort with using a variety of assessment methods and with the Achievement Chart, the extent to which they feel they have adequate time to assess students and provide them with feedback, the extent to which they use EQAO assessment resources and results, and their need for professional development with regard to assessment. In addition to these specific questions, participants had the opportunity to respond to two open-ended questions; one focused on professional development experiences which they found helpful in implementing the mathematics curriculum and the final question which allowed space for participants to provide additional comments.

In this report, we present findings that relate to the variety of assessment tools which teachers are using, teachers' use of EQAO assessment resources and results and those responses which suggest some of the challenges teachers identified in mathematics assessment.

Using a variety of assessment methods

In order to get a sense of teachers' use of a variety of assessment methods in intermediate mathematics classes, the following two specific items were asked:

In this class to what extent do you use the following to get a sense of students' understanding in mathematics? Check one box in each row.

For this class, when determining a student's mark, how much weight do you give to each of the following types of assessments? Check one box in each row.

Responses to these items have been summarized for the whole group of intermediate mathematics teachers who participated (n=1096) as well as by panel, identifying the responses of teachers of Grades 7 and 8 (n=506) or teachers of Grades 9 and 10 (n=559).

Table 1 provides a summary of the responses to the item that asks about the forms of assessment used to get a students' understanding of mathematics. The table presents the results for all intermediate mathematics teachers who participated in the questionnaire.

Table 1: Assessment forms used to get a sense of students' understanding

	Overall			
	None	**A little**	**Some**	**A lot**
Paper-and-pencil tests	0%	4%	41%	55%
Quizzes	2%	9%	47%	42%
Performance tasks	2%	15%	62%	21%
Homework performance	5%	24%	43%	28%
Observations of students (notes/checklists)	8%	26%	44%	22%
Interviews/conferencing with students	20%	45%	27%	8%
Responses of students in class	4%	20%	45%	31%
Students' journals	58%	25%	13%	5%
Portfolios / dated work samples	58%	23%	14%	5%
Projects	21%	42%	33%	5%
Self assessment	41%	43%	14%	2%
Peer assessment	55%	35%	9%	1%
Student presentations to other students	40%	41%	17%	3%
Computer/graphing calculator output or demonstrations	49%	38%	12%	1%
EQAO mathematics assessment results	64%	24%	11%	1%

While responses to this item indicate that pencil-and-paper tests and quizzes are the most frequent forms of assessment that teachers use to get a sense of students' understanding in mathematics, the questionnaire results also suggest other assessment methods are used. For instance, the data would indicate that performance tasks, homework, observations, and student responses in class are quite widely used.

Table 2 provides a breakdown of responses to this item showing results for the top ten responses for Grades 7 and 8 teachers and the Grades 9 and 10 teachers.

Table 2: Assessment forms used to get a sense of students' understanding focusing on *somewhat* or *a lot*

Top 10 responses	**Somewhat or A lot**	
	Grade 7 and 8 teachers	**Grade 9 and10 teachers**
Paper-and-pencil tests	96%	97%
Quizzes	88%	90%
Performance tasks	92%	76%
Response of students in class	79%	73%
Homework performance	74%	68%
Observation of students (notes/checklists)	74%	60%
Projects	48%	28%
Interviews/conferencing with students	41%	29%
Portfolios/dated work samples	32%	8%
Student presentations to other students	30%	11%

When examined by panel, the dominance of paper-and-pencil tests and quizzes remains evident but the results also suggest that mathematics assessment practices may be different in Grade 7 and 8 and Grade 9 and 10 settings. For example, teachers of Grades 7 and 8 more frequently report the use of students' journals, portfolios/dated work samples and student presentations in getting a sense of students' understanding.

We also wanted to get a sense of the kinds of assessments that teachers use in determining a student's mathematics mark. Table 3 provides a summary of the responses for all questionnaire participants.

Table 3: Assessment forms used in determining students' mathematics marks

	Overall			
	None	**A little**	**Some**	**A lot**
Paper-and-pencil tests	0%	4%	42%	53%
Quizzes	6%	15%	55%	24%
Performance tasks	6%	16%	57%	22%
Homework performance	32%	31%	28%	8%
Observations of students (notes/checklists)	38%	30%	26%	6%
Interviews/conferencing with students	60%	25%	12%	3%
Responses of students in class	44%	29%	23%	5%
Students' journals	69%	18%	12%	2%
Portfolios / dated work samples	68%	18%	12%	2%
Projects	25%	34%	34%	7%
Self assessment	65%	26%	8%	1%
Peer assessment	73%	22%	5%	0%
Student presentations to other students	52%	32%	14%	1%
Computer / graphing calculator output or demonstrations	65%	27%	8%	0%
EQAO mathematics assessment results	73%	16%	10%	1%

Responses to this item suggest that paper-and-pencil tests, quizzes and performance tasks are the three most frequently used forms of assessment when determining a student's mark. However, the table also indicates that some teachers may use a variety of assessment forms to help to determine a student's mark.

Table 4 provides a breakdown of the top ten responses to this item showing results for the teachers of Grades 7 and 8 and the teachers of Grades 9 and10.

Table 4: Assessment forms used in determining students' mathematics marks

Top 10 responses	**Grade 7 and 8 teachers**		**Grade 9 and 10 teachers**	
	Some	A lot	Some	A lot
Paper-and-pencil tests	54%	42%	32%	64%
Quizzes	61%	25%	51%	22%
Performance tasks	60%	28%	54%	16%
Response of students in class	37%	8%	10%	1%
Homework performance	39%	13%	19%	4%
Observation of students (notes/checklists)	43%	11%	11%	1%
Projects	41%	10%	27%	6%
Interviews/conferencing with students	20%	6%	5%	1%
Portfolios/dated work samples	19%	4%	5%	1%
Student presentations to other students	22%	2%	8%	1%

When examined by panel, some differences in mathematics assessment practices in these settings are suggested. In general, the pattern of responses to this item suggests that Grades 7 and 8 teachers may use a wider variety of assessments in determining a students' mark than do Grades 9 and 10 teachers. As well, particular forms of assessment such as interviews/conferencing with students, projects, and students' journals were indicated more frequently by teachers of Grades 7 and 8.

Another way to look at this item is to look at the use of a specific assessment strategy in the individual courses. For instance, Table 5 indicates the use of paper-and-pencil tests to determine a students' mathematics mark in the different courses. In this particular case, we are reporting on the *a lot* column to see the variety in the strong use of tests among the teachers of the different courses. We have also ordered the chart from the greatest use of tests to the least use of tests.

Table 5: Use of paper-and-pencil tests to determine students' mathematics marks by course

	A lot
Grade 10 Academic	82%
Grade 9 Academic	70%
Grade 10 Applied	53%
Grade 9 Applied	53%
Grade 8	43%
Grade 7	41%
Grade 9 Essential/Locally Developed	40%
Grade 10 Essential/Locally Developed	35%

An examination of this table suggests that tests are more widely used in the Grade 9 and 10 Academic courses, followed by the Applied courses, Grades 7 and 8, and then the Locally Developed courses.

The final question on the questionnaire asked teachers to provide additional comments that could be related to any aspect of teaching intermediate mathematics. Some of these responses provide insight into teachers' use of a variety of assessment forms. For instance, some participants stated that finding time to develop and use other forms of assessment is challenging.

> For some of the questions concerning the time I have to complete certain tasks... I use my own time a great deal. I complete all research at home in the evenings and on weekends. I see students daily throughout my lunch hour for Math resource. I would love to be able to use a variety of assessments, but seem to have no time to prepare other assessments besides tests. (Participant 06127)

> Discussion can be useful, but realistically I don't usually have time to get around to interview and observe every student and make comments for each task so a worksheet keeps them on task and fills in the gaps I miss. (Participant 06430)

Others suggested more professional development opportunities are needed to learn to develop and use other forms of assessment.

> My board has done so much in the way of PD for manipulatives. The PD I am looking for is more student-based. I'd like to learn more about how to use group work and rich assessment tasks in my classroom. (Participant 22013)

> I think it would be helpful to have good examples of performance assessment tasks for the course, a better idea of how many to give and a timeline for them. (Participant 06952)

Teachers' use of EQAO assessment resources and results

A number of items on the CIIM questionnaire concerned teachers' use of EQAO assessment resources and results. The results from these items as well as some open-ended item responses related to EQAO are summarized below.

In the questionnaire, teachers were asked:

To what extent have the following resources or learning opportunities helped your implementation of the mathematics curriculum in this class?

They were presented with a list of 20 different response options including resources such as mathematics textbooks, Ministry curriculum documents, mathematics workshops, and TIPS. One option in this list was EQAO assessment resources. Table 6 shows participants responses to their use of EQAO assessment resources. Both the overall responses and the responses organized by panel are indicated.

Table 6: Teachers' use of EQAO assessment resources

	Responses			
	Not at all	**A little**	**Somewhat**	**A lot**
Overall	62%	26%	10%	2%
Grades 7and 8 teachers	75%	19%	5%	1%
Grades 9 and 10 teachers	51%	33%	14%	2%

With regard to teachers' use of EQAO assessment results, our data show that EQAO assessment results are used primarily by Grade 9 teachers. Teachers were asked to indicate the different ways that they use EQAO assessment results, with the questionnaire item asking specifically:

In this class, to what extent do you use EQAO assessment results in mathematics to:

- **Identify how well students are meeting the curriculum**
- **Communicate with parents**
- **Identify areas of program strength**
- **Identify areas for improvement**
- **Form part of the student's final mark**

Table 8 presents the ways that Grade 9 teachers responded to this item, specifically identifying the *somewhat* and *a lot* columns. It should be noted that official EQAO assessment results are not received by schools until the fall following the Grade 9 assessment. However, in some schools, teachers mark a portion of the Grade 9 assessment and use it as part of the students' final mark. The table shows that many Grade 9 mathematics teachers use results as part of the mark and some teachers use the results to determine how well students are meeting the curriculum and to identify areas of program strength and improvement.

Table 8: Grade 9 teachers' use of EQAO assessment results to:

	Grade 9 Applied	**Grade 9 Academic**
	Somewhat /A lot	**Somewhat /A lot**
Identify how well students are meeting the curriculum	43%	49%
Communicate with parents	15%	22%
Identify areas of program strength	38%	36%
Identify areas for improvement	42%	39%
Form part of the student's final mark	55%	55%

A few teachers also included comments concerning the EQAO assessment in response to the two open-ended items. In responding to the item asking for teachers to identify a positive professional development experience, a number of teachers indicated that they had found their involvement with EQAO test marking to be beneficial in improving their assessment practice.

> Marking the Grade 9 EQAO assessment during the summer gave me a better understanding of the four categories of Knowledge and Understanding, Application, Problem Solving and Communication. This allowed me to better design my assignments and tests in order to assess students in the categories. (Participant 03400)

> Marking EQAO in the summer. It made me really think about how different students can give different answers and it still be in the same level. It also made me try to analyze whether or not the student can demonstrate their learning by explaining their thinking in their answers. (Participant 03970)

In responding to the last item on the questionnaire that asked for additional comments, a few teachers commented on some of the impacts of the Grade 9 EQAO assessments on their classes. For example, one teacher stated:

> In Grade 9 we find we don't have enough time to do all the "extras" with computers and manipulatives because we are trying to cover the material before the EQAO testing. We are a rural school that typically misses 5 days a semester due to snow and fog days. This time could have been used to do the extras. As it is we barely cover the course before EQAO and we teach to the "big ideas" of the course so that the students have to chance to do well on the EQAO, not that EQAO really matters to anyone but the government. (Participant 14388)

Challenges in mathematics assessment

Analysis of the questionnaire data suggests that teachers find a number of challenges with regard to mathematics assessment. One challenge that teachers identify is related to the Achievement Chart. The questionnaire included an item asking teachers to indicate how comfortable they are with various aspects of the mathematics curriculum.

How comfortable are you with the following aspects of the curriculum for this class?

Responses to this item are summarized in Table 9 and suggest that while most teachers indicate that they are quite comfortable in using a variety of assessment methods, they are considerably less comfortable with the Achievement Chart. In fact, the Achievement Chart is the area of the curriculum which teachers identified as being the least comfortable. This was also true when the data was examined by panel and by course.

Table 9: Teachers level of comfort with aspects of the curriculum

	Somewhat comfortable	Very comfortable
Using a variety of assessment methods	50%	41%
Using technology to teach mathematics	42%	32%
The philosophy of the curriculum	51%	26%
Teaching through problem solving and investigating	51%	36%
Students working in groups	43%	37%
The Achievement Chart	36%	24%
The content of the course	28%	68%
Using concrete materials to teach mathematics	43%	40%
Integrating concepts from across the strands of mathematics	44%	39%

In response to the open-ended item that invited participants to provide additional comments, several made reference to the Achievement Chart. The following participant offered an extensive comment concerning assessment. While the comment may seem rather lengthy to include, it is an interesting description of the challenges that the Achievement Chart presents within the broader context of mathematics assessment in Ontario.

> The biggest concern that I have with implementing the curriculum is with assessment. This seems to be a major concern (or bother, rather) for most of the teachers I work with. In our board, we are required to report on the 4 Achievement Chart categories and this makes no statistical sense. I am a strong believer that there must be a balance of the categories represented in all assessment but to report on them and split marks into categories is like fitting a square peg into a round hole. The Achievement Chart categories overlap so fitting assessment into categories is up to the teachers' interpretation leading to inconsistencies. Rather than report on the categories, I propose that teachers are encouraged to represent all of the categories in a balanced way but then when evaluating student work, providing the student with ONE mark to indicate how well they did. A strongly worded comment or the use of a rubric can address why they achieved the mark they did. Parents, students and teachers have a difficult time making sense of marks split in the way they are in our board. In addition to this, we then churn out ONE overall mark. Coming up with this one final mark is so out of touch with how things were done during the semester that everyone is just simply confused (not to mention reflecting most recent, most consistent performance). Sorry about the rant but it is very troublesome. After close to 10 years of teaching, I seem to become more confused and upset about this every year, rather than becoming better at it. (Participant 24125)

Another challenge that was found in the questionnaire responses was with regard to the elementary requirement to report on strands. Several participants commented on their difficulties with this requirement in response to the final questionnaire item which asked for additional comments. Some examples include:

> Get rid of the reporting requirements on the Report Card. It is absolutely killing the philosophy of good math instruction. (Participant 08364)

> Although the strands are a useful way in which to break math down into different components, the expectation that they be reported on separately (and at least twice each) puts artificial structure onto the course outline. It also looks on the report card as though all strands are equally important -- which they are not. The Ministry needs to recognize this as well and change their expectations for the way in which we report math marks. (Participant 09892)

> For the elementary curriculum the concept of strands, and reporting on many stands each reporting period, needs to be changed. Math needs to be taught in a more linear approach instead of jumping around through the concepts to try to hit on each strand for a report card. (Participant 20010)

The number of participants who included a reference to the challenge of reporting on strands is particularly surprising given that the item asked only for general comments and did not specify assessment as a topic. The frequency of responses from teachers referring to their perceptions of the purpose and difficulty of reporting on strands suggest that this is an area of difficulty for many elementary mathematics teachers.

Concluding comments

A number of connections can be made between the various sources of data with respect to assessment as well as to the issues described in the research literature. One of the dominant messages in the research literature and in the NCTM assessment standards (1995) with regard to effective mathematics assessment is the need for teachers to use a variety of assessment methods. This message is also a central element in the Ontario curriculum documents. Using a variety of assessment methods helps to ensure that all students have an opportunity to demonstrate their learning as well as avoiding a reliance on only paper-and-pencil tasks such as tests and quizzes which tend to emphasize merely knowledge recall and procedural learning (Shepard, 2001). The use of a variety of forms of assessment which both elicit and contribute to developing mathematical understanding and the full range of mathematical processes is critical in mathematics classrooms.

Observations from the CIIM research project suggest that while quizzes and tests are still the most frequently used form of assessment both in getting a sense of students' understanding and in determining students' marks, some teachers are also using other forms of assessment such as performance tasks, homework, observations, and student responses in class. Comments made in the mathematics leader focus groups and in response to the open-ended items on the questionnaire also confirm this as they suggest that while teachers are moving toward using more assessment forms, there is still a strong reliance on tests and quizzes. Some reasons for this are suggested including a lack of time available to develop new forms of assessment, the need for more professional development with regard to newer forms of assessment, teachers' views of mathematics and mathematics teaching and learning, and mixed messages and inconsistent support for new practices. As well, the types of assessments that the teachers are using to get a sense of students' understanding might indicate that further work needs to be done in the use of formative assessment. These observations are consistent with the findings of other research studies (Barnes, Clarke & Stephens, 2000; Manouchehri, 1998; Tierney, 2006; Wiliam et al., 2004).

Some of the other challenges in enhancing teachers' assessment practice that are identified in the research literature seem to be reflected in the observations we have made. Data from the focus groups and questionnaire responses from teachers suggest that some teachers find new assessment ideas challenging and difficult to understand. In some cases, such as the Achievement Chart, teachers seem uncomfortable with its use and struggle with separating assessment tasks into discrete processes or categories. In other cases, such as the requirement to report on strands, the intent of the policy may not be clear to teachers. This seems to result in teachers conforming to mandated policies without necessarily understanding or agreeing with the philosophy behind certain assessment messages.

In general, our observations indicate that teachers' use of new assessment practices are emerging. Our data as well as our review of the research literature suggest that teachers can be further supported through leadership, collaborative work with colleagues, and transparent alignment of curriculum, instructional, and assessment documents and policies (Barnes, Clarke, & Stephens, 2000; Manouchehri, 1998; Wiliam, Lee, Harrison, & Black, 2004). It is also important to recognize that change takes time. Developing new assessment strategies is closely aligned with developing new classroom instructional practices. As teachers continue to develop their expertise in engaging students in mathematical investigations, providing problem solving activities, and offering opportunities for students to develop conceptual understanding of mathematics along with the application of procedures, it is suggested that they also recognize the value of using a variety of types of assessments to capture student understanding (Graue & Smith, 1996; Suurtamm, 2004).

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Professional Development

What does the research literature say?

Current perspectives on mathematics education have moved away from a transmission model of teaching to models that promote a deeper understanding of mathematics. However, realizing these reforms involves fundamental changes to our notions of teaching and learning (Crockett, 2002; Henningsen & Stein, 1997). These models require classrooms that are inquiry-oriented and help students develop mathematical reasoning and the ability to solve complex problems, as well as developing mathematical skills.

However, studies have shown, that despite mounting evidence supporting new instructional methods, it is difficult to change classroom practice (Lachance & Confrey, 2003; Loucks-Horsley & Matsumoto, 1999; Sykes, 1996). In spite of widespread professional development and curriculum resource materials to support a movement to inquiry-oriented mathematics teaching, classroom practices have not substantially changed (Ball, 2003; Frykholm, 1999). The research suggests that teachers tend to teach the way they were taught, and since most teachers experienced transmission approaches as students, these approaches now serve as powerful models for their own teaching (Garet, Porter, Desimone, Birman, & Yoon, 2001). In addition, many teachers have trouble visualizing what reformed classroom practice looks like since there have been few exemplars available to consider (Ball, 1996). Even when teachers are motivated to change their practice, a significant gap may remain between what it is teachers think they are accomplishing and their actual pedagogical practices (Manouchehri & Goodman, 1998). Mathematics teacher development remains a challenge and has been the focus of several recent reports and studies (Ball, 2003; Ball & Even, 2004).

Some would suggest that traditional approaches to professional development are often insufficient to shift mathematics teacher practice with the limitations of such approaches widely described in the literature (e.g., Ball, 1996; Knight, 2002; Little, 1993; Loucks-Horsley & Matsumoto, 1999; Sykes, 1996). Traditional professional development opportunities often take the form of workshops where teachers listen passively to experts' views on general pedagogical approaches or innovations (Boyle, Lamprianou & Boyle, 2005) and receive "teacher proof" resources (Cohen & Ball, 2001; Manouchehri & Goodman, 1998). The sessions are often short, superficial, disconnected from other district initiatives, and do little to build relationships that can sustain innovative approaches (Borko, 2004; Loucks-Horsley, Hewson, Love & Stiles, 1998). The workshop model has been shown to raise teachers' awareness and interest in the topics presented but is often insufficient to promote change in classroom practice by itself (Boyle et al., 2005).

One large-scale study on the characteristics of effective professional development (Birman, Desimone, Porter, & Garet, 2000; Desimone, Porter, Birman, Garet, & Yoon, 2002; Garet et al., 2001) surveyed 1027 mathematics and science teachers who had participated in PD within the previous year. The study concluded that:

> Although some teachers receive high-quality professional development that changes their practice in positive ways, most teachers' professional development consists of activities that do not have the features empirically shown to be related to increasing teacher knowledge and changing teaching practice – long duration with follow-up, participation of groups of teachers from the same school or department, a focus on content, active learning opportunities, and coherence (e.g., consistency with other goals and activities) (Desimone et al, 2002, p. 1267).

Research suggests that meaningful teacher change is a long process (Adey, 2004) and that the effectiveness of PD is shown by teacher engagement, changes in classroom practice, and self-reported growth (Boyle et al., 2005; Garet et al., 2001). As the shortcomings of short-term professional development initiatives are being realized, these initiatives are being replaced by more collaborative and comprehensive models to support teachers as they work with new ideas (Arbaugh, 2003).

New models of effective professional development can take a variety of forms to accomplish different purposes such as developing awareness of new approaches or content, building knowledge of content and practices, translating new knowledge into practice, practicing teaching, and reflection (Loucks-Horsley et al., 1998). Guskey (2003) suggests that effective professional development should increase teachers' content and pedagogical content knowledge, provide sufficient time and resources, and promote collaborative practices. Garet et al. (2001) suggest that effective professional development should improve teachers' content-knowledge, encourage change in teaching practices, define goals for student learning, and improve teachers' knowledge of how students learn and they cite a number of studies to support their argument.

With respect to the format of professional development, several studies suggest that it is important to engage teachers in activities that model the content and pedagogy of reform classrooms since this helps teachers to visualize what the practice looks like in a classroom and how it is different from traditional practice (e.g., Farmer, Gerretson, & Lassak, 2003; Loucks-Horsley et al., 1998; Tirosh & Graeber, 2003). As well, the increased emphasis on teaching for understanding means that teachers themselves need to develop a deep and connected understanding of the mathematical ideas that they teach (Britt et al., 2001; Fennema & Franke, 1992).

As well, some suggest that supported fieldwork is integral to changing classroom practice (Borasi, Fonzi, Smith, & Rose, 1999) and that effective and sustainable professional development experiences must involve opportunities for teachers to collaborate with colleagues (e.g., Adey, 2004; Borasi et al., 1999; Garet et al., 2001; Loucks-Horsley et al., 1998). The creation of supportive structures including a culture of professional learning within the school community has been called the "'single most important factor' for successful school improvement" (Harris, 2003, p.380). Such a professional learning community has a shared sense of purpose, opportunities for collaborative work, involvement in decision making, and a sense of collective responsibility (Harris, 2003). In such a professional learning community where the collective emerges from shared activity that arises in response to a meaningful problem, control is decentralized, and this stands in stark contrast to working groups where teachers have been mandated to work together by an external authority for an external agenda (Nickerson & Moriarty, 2005).

What do Ontario math education leaders say?

The following summary of the OAME and OMCA focus group data highlights the themes related to professional development which emerged in the mathematics education leaders' discussions. These themes include the benefits of professional development, different approaches to professional development, and challenges that are faced in providing professional development and implementing new professional learning.

The benefits of professional development

Participants emphasized a number of benefits and purposes associated with PD. For instance, participants suggested that professional development has the potential to shift practice as teachers develop new understandings. PD was also seen as helping to develop and support teacher confidence and comfort regarding the implementation of new teaching practices.

> At the end of my in-services, when we do manipulatives, I find that the teachers are really enthusiastic, and you can see all those light bulbs going off . . . after a little while you can see the teachers getting up there because during the lesson they had even learned a little bit more about the manipulatives and they felt a little bit better about walking around and helping the students. Just from doing that I think that maybe their confidence level went up a little bit. (Brenda, Math Leader Focus Group [MLFG] 6)

Other benefits that were identified included increased dialogue and networking, teachers developing their own mathematical content knowledge, and PD acting as a source of motivation for teachers.

Approaches to professional development

Focus group participants discussed various approaches to PD. When discussing the "train the trainer" model, the mathematics education leaders consistently shared a sense of frustration. Concerns included the fact that key messages were not necessarily being passed on by the PD attendees to other teachers as well as participants questioning the selection of teachers being sent to the PD activities.

> Beth: The "train the trainer" model that we started off with I don't think was very successful because it depended on who they actually were sending to the initial training. Some people thought we better send people who need convincing to change and they were not necessarily the best presenters to come back to.
>
> Dave: And it died. The whole model didn't work very well. (MLFG 1)

However, participants advocated for other approaches to professional development including school-based coaching whereby board resource teachers would visit schools, modelling practices in classrooms and otherwise supporting teachers. This approach exhibited itself differently depending on the context. In a few cases, participants talked about support teams visiting schools and working with teachers for a few days up to two-week periods, cycling back to that same school at a later date. In other cases, resource teachers visited classroom teachers when requested by a particular teacher or did so as a follow-up to extend learning beyond an in-service opportunity.

Modified lesson study was also mentioned whereby teachers were described as planning together, observing each other teach, and then discussing the observations. In the following example, a leader described working with other teachers through lesson study.

> We've been really fortunate, because last spring I was able to take some of my lead teacher days at Grade 7/8 and do a lesson study, like a modified lesson study. Basically, there were five groups of teachers who composed, who created, modified a lesson and they weren't ideal by any stretch, but what it did is when those teachers got in there they got to do that observing of kids that they don't get a chance to do and really focused too. You focus on these two groups of students, you focus on these two. And then the debriefing, and it was huge. That is the biggest thing that I think is making a difference. Now we're just in the midst of a second round with those same teachers. This time, the depth of their planning was way better. (Kerry, MLFG 4)

Similarly, other professional development initiatives involved teachers working together. One participant mentioned teachers meeting to discuss a topic, going back to the classroom to try the new idea, and then coming back to report to the group. The following example describes more of a mandated approach in terms of a board having teachers teaching the Grade 9 Applied course learning to work together on a particular topic.

> All of the groups chose assessment planning [as a topic to focus on] because they want that piece at the end. . . . They're loving it and they're developing these open-ended, some of them formative, but they're looking at one summative, and some formative for this small body of content, and it's really effective. . . And now they're on their own, and the ones that, all except one cluster, they're all buying in, they're all like this is all good. (Kerry, MLFG 4)

Participants identified resources and curriculum interfaces as important to professional development. For instance, one participant noted a learning experience focused on the TIPS documents which moved teachers to explore aspects of their practice in greater detail.

> I would say TIPS in my particular experience moved people along because first of all, it opened a dialogue between 7, 8 and 9 teachers. And the PD was organized so that they were together and the dialogue started that hadn't been there before, in a way that they were all learning at the same time. I think at least in my Board, it ended up creating task forces of teachers who went out and tried things and came back and worked with other teachers. So there was a place then for them to go watch someone actually teach it with kids, which is very, of course, always more compelling than somebody telling you, you should do it. So that sort of moved us into even a different PD model that had not been done before in 7 to 9. (Joan, MLFG 2)

In addition, resources themselves were noted as shaping teacher learning and practice.

Some common characteristics were evident in the various PD approaches discussed by participants. For instance, many of the approaches were associated with collaboration where support was provided by colleagues (e.g., lesson study and professional learning groups). In addition, many highlighted the importance of direct connections made between professional development and teachers' own classrooms where, for instance, teachers spent time discussing an issue related to their classroom and then went out and tried new ideas.

Another effective characteristic that the math leaders discussed was professional development occurring over a sustained period of time. Participants noted that change takes time and when professional development isn't sustained, teachers revert to old practices. The need for a sustained approach to professional development was raised as an important issue by a few focus groups.

> Steve: I think there needs to be more of a sustained [approach]. "Okay, we'll pull you out for half a day here; we'll get together in another month or two and then go a next step."
>
> Dave: Sustainability and being able to have time to meet on a regular basis. I think that's the key.
>
> Steve: Yes.

Dave: And it's not quite happening enough still.

Beth: No.

Steve: No. (MLFG 1)

Challenges related to professional development

Participants described a number of challenges associated with professional development. These included time constraints, the amount of professional development opportunities available, school administrators and department heads as gate-keepers, teachers' beliefs and perceptions, and the challenge of shifting practice in general.

The participants noted that teachers' time is fairly limited and this restricts the opportunities for teachers to provide mentoring support to colleagues, prepare lessons for their students if they are away for PD sessions, or attend PD when it is offered after-school. Also, it was perceived that teachers feel that implementing new practices takes more class time. New teachers were particularly noted as experiencing somewhat similar struggles due to the time constraints of being a new teacher.

> Dave: When you're getting into your first few years of teaching, there's the time thing again "I've got to plan" and you know, trying to find a good activity or develop a good activity, it's very time consuming. So you know what, you just naturally say "Enough" and "What do I need to do", and then you go back to the traditional stuff. That's what is happening all the time. (MLFG 1)

Relating to issues of time, participants noted that the amount of professional development available to teachers can be a challenge. In particular, the experiences of grades 7 and 8 teachers over the last decade were seen as problematic given that there was an initial lack of support for several years followed by a rapid and overwhelming increase in the number of professional development opportunities.

> A lot of elementary teachers were complaining, "We get no in-service; We get no in-service; We get no in-service" in the last few years. Now, they're saying, "We're getting in-serviced to death; you're taking me out of my class". And before they weren't able to [go to in-service] and, you know, "I don't want to do this outside of my class. I'll do it during a school day but now I don't want to be away from my class". (Dave, MLFG 1)

In some cases, school administrators were noted as obstacles to professional development and the implementation of new professional learning. For instance, one participant talked about how his principal was concerned about the cost of sending teachers to PD opportunities given that they often returned with a list of “things” to purchase. Another participant described an experience whereby a vice-principal challenged the practices of teachers who were implementing what they had learned through recent PD experiences, going so far as to attend a math department meeting to speak against the new practices. Participants also commented that department heads and lead teachers sometimes acted as ‘gatekeepers’ to information and resources meant to be shared with colleagues at their schools.

> Clare: With our secondary, we have Grade 9 Applied professional learning teams right now, and we’ve been fighting our Heads to get it to work . . . we found we had to force the issue past the Department Heads.
>
> Mark: It looks like you’ve got to try different PD models. The traditional thing of going through with the Head as the curriculum leader, sometimes that person becomes the gatekeeper.
>
> Clare: Yes.
>
> Liz: Very often the lock master. (MLFG 4)

Participants identified teachers’ own characteristics and perceptions as a challenge to professional development. For instance, it was noted that some teachers find it hard to take risks and are concerned about struggling in the classroom when trying new practices.

> Often, the traditional approach is favoured because it's very structured, teacher feels confident, the classroom looks and sounds nice to administrators, and to break out of that mould, to say it's good for kids to do hands on, get up and move and talk […] Teachers are giving up a lot of the comfort zones they have, and so somehow we need to convince them that it's worth it to do that. […] (James, MLFG 5)

It was also noted that some teachers are hesitant to try new practices because of their concern that students may not be able to handle the new practices while other teachers do not see the need for professional development or a shift in practice in the first place.

Some participants talked to the enormity of the shift for some teachers; the challenge of shifting practice given previous experience and the complexity of teaching. The following participant discusses the challenge of teaching using an approach in which teachers may not have learned as a student or used as a teacher early on in their career.

> We're looking at teachers learning new strategies that certainly they probably didn't encounter as a student, they probably didn't do when they first started teaching." (Liz, MLFG 5)

In addition, it was also acknowledged that the shift in practice is complex and that it can be difficult for teachers to understand what a certain practice might look like.

> They're [Teachers are] getting little pieces of how they should be doing things now, but they aren't sure what it should look like at the end. So when they're looking at their kids and what they've accomplished so far and what they know and what they can talk about they think, oh no, is that enough? That's not enough. When I ask them the question, not all of them knew what it was they were supposed to have gotten out of that, etc. That's really complex too in helping them to understand what it is they're supposed to be doing and buy into the approach, because they don't know what's right and wrong or sufficient in terms of what their kids get out of the particular activities. (Jill, MLFG 5)

Some additional challenges identified by participants included concerns regarding students resisting new classroom practices, the pressure of accountability, the lack of resources for teachers to implement what they have learned, the ineffectiveness of some mentors, the discomfort of some teachers to have others in their classrooms, the lack of board expertise in certain areas (e.g., support for a particular piece of software), weaknesses in some pre-service programs in terms of what is taught, and the ineffectiveness of some in-service activities (e.g., travel issues; poor presentation skills).

What do the questionnaire data say?

As part of the CIIM questionnaire, teachers were asked to respond to questions associated with professional development. Most of the questions were selected response questions. One of these questions focused on the extent to which teachers felt that further professional development in particular areas would be helpful to their implementation of the curriculum. A number of other questions that were asked were more general in scope but included components focusing on professional development such as teachers being asked about the extent to which they have time for various activities, including learning new ways of teaching. As well as the selected response questions, there were two constructed response (open-ended) questions, one that specifically dealt with professional development and another that asked for additional comments about any aspect of mathematics curriculum implementation. In this section, we will first report on the themes that emerged from the selected response questions and then focus on the themes emerging from the constructed response questions.

Support for curriculum implementation

As one way of gaining an understanding of the supports currently used by intermediate mathematics teachers in Ontario, participants were asked specifically about supports relating to both planning and implementation of the curriculum. The following question addresses the types of resources teachers use in planning for a specific course.

How often do you use each of the following resources when planning for this class?

Table 1 provides a breakdown of the responses for this item focusing on the *often* and *very often* categories.

Table 1: The degree to which various resources are used for planning.

Resources	**Often or Very Often**
Mathematics textbooks	86%
Ideas from colleagues	78%
Ministry curriculum documents	71%
Other published teacher resource books	57%
Teacher resource books that accompany a textbook	52%
Handouts from professional development workshops	46%
Course profiles	40%
Web resources	40%
Targeted Implementation & Planning Supports (TIPS) for Grades 7-9	34%
Ministry Exemplars	20%
Mathematics education journals	10%

These data suggest that teachers rely heavily on textbooks for planning. However, a large number also identified using ideas from colleagues, as well as the Ministry curriculum documents. Handouts from PD workshops were mentioned by approximately half the participants as being used *often* or *very often*.

The following question is similar to that above but focuses on resources and learning opportunities that have helped the implementation of the curriculum.

To what extent have the following resources or learning opportunities helped your implementation of the mathematics curriculum in this class?

This item had a list of twenty resources or learning opportunities to which to respond. Table 2 provides the results for the top nine choices.

Table 2: The degree to which various resources or learning opportunities help implement the curriculum, focusing on *somewhat* or *a lot*

Resources	Somewhat or A lot
Mathematics textbooks	80%
Dialogue with colleagues	70%
Ministry curriculum documents	60%
Teacher resource books that accompany a textbook	53%
Other published teacher resource books	49%
Professional development workshops	46%
Course profiles	32%
Targeted Implementation & Planning Supports (TIPS)	31%
Web resources	30%

We see similar results for this item as for the item presented in the previous table. The resources or learning opportunities that teachers most frequently reported as having helped with the implementation of the curriculum are textbooks, dialogue with colleagues, and Ministry curriculum documents. In some cases, particular resources have been helpful to particular courses. For instance, the TIPS resource was designed specifically for use in Grades 7, 8 and 9 Applied. While in general, 31% of the teachers state that TIPS was somewhat or a lot helpful in the implementation of the curriculum, 39% of the Grade 7, 35% of the Grade 8 and 56% of the Grade 9 Applied teachers state that it is *somewhat* or *very* helpful in implementing the curriculum. The high percentage of Grade 9 Applied teachers might be connected to the fact that for Grade 9 Applied, the TIPS materials provided a plan for the entire course, whereas for Grade 7 and 8, only part of the course was supported.

Since dialogue with colleagues appeared as a useful support for curriculum implementation, it is valuable to look at one of the items in the questionnaire that specifically addresses opportunities for teacher collaboration. Teachers were asked:

How often do you meet with other teachers to discuss and plan mathematics curriculum or teaching approaches for mathematics?

Table 3 presents the summary or all respondents as well as the responses for teachers of Grades 7 and 8 and teachers of Grades 9 and 10.

Table 3: The regularity with which teachers meet to discuss and plan mathematics curriculum or teaching approaches.

	Grade 7 and 8 teachers	Grade 9 and 10 teachers	All respondents
Never	6%	1%	4%
Once or twice a year	32%	13%	23%
Every other month	16%	8%	11%
Once a month	24%	18%	20%
Once a week	13%	20%	17%
Two or three times a week	6%	20%	14%
Almost every day	4%	19%	12%

The table shows that 43% of all respondents report meeting at least once a week. When the data is broken down by panel, 59% of Grade 9 and 10 teachers report meeting at least once a week while 23% of Grade 7 and 8 teachers report the same. However, 38% of teachers of Grades 7 and 8 and 14% of teachers of Grades 9 and 10 identified meeting only once or twice throughout the year or not at all.

Time for professional development activities

One item in the questionnaire focused specifically on the amount of time teachers perceive they have for various activities. Some of those activities are associated with professional development. Teachers were asked:

For this class, to what extent do you feel that you have adequate time for each of the following activities?

Table 4 presents the responses for all activities and is reporting the percentage of teachers who feel they have little or no time for the stated activities.

Table 4: The degree to which there is adequate time for various activities, responses for *not at all* and *a little*.

	Not at all/ A little
To read resource materials	78%
To learn new technologies	71%
To meet with colleagues	67%
To learn new ways of teaching	61%
To work with students experiencing difficulty	59%
To prepare materials and activities	50%
To provide feedback to students	39%
To plan lessons	35%
To learn the mathematics that I will be teaching	33%
To assess students in mathematics	26%
To cover the curriculum	25%

The table demonstrates that teachers often felt as though they had little or no time for activities associated with professional development. Specifically, substantial numbers of teachers indicated that they had little or no time to read resource materials, learn new technologies, meet with colleagues, or learn new ways of teaching.

Further professional development needs

The analysis of the questionnaire data suggests that there is currently a need for professional development activities in a number of areas. Teachers were specifically asked to identify the extent to which they believed that they need further professional development in particular areas.

To what extent do you feel that further professional development in these areas would help support you in implementing the curriculum in this class?

Table 5 presents the responses for the *somewhat* and *a lot* columns for this item for all respondents as well as for each panel.

Table 5: Responses to areas of need for further professional development

	Somewhat or A lot		
	Grade 7 and 8 teachers	**Grade 9 and 10 teachers**	**All respondents**
Teaching through problem solving	84%	75%	78%
Understanding how students learn mathematics	78%	67%	72%
Teaching strategies	76%	66%	71%
Use of manipulatives	79%	57%	67%
Assessment in mathematics	79%	55%	66%
Using group work in mathematics	71%	52%	61%
Facilitating investigations	67%	54%	60%
Use of other computer software (other than GSP)	72%	42%	56%
Use of Geometer's Sketchpad®	68%	40%	53%
Use of graphing calculators	52%	42%	46%
Content knowledge for teaching	48%	27%	37%
Other (please specify in comments section on the next page)	30%	24%	27%

This table provides a summary of the areas where teachers felt they needed professional development. The areas of greatest need are identified as teaching through problem solving, understanding how students learn mathematics, and teaching strategies. Looking across panels, the table shows that in all areas, more teachers of Grades 7 and 8 expressed an interest in further professional development than did teachers of Grades 9 and 10. In some areas, there are very specific needs for the different panels.

One of the last items on the questionnaire was an open-ended question focusing specifically on professional development. Teachers were asked to respond to the following:

> **Please describe a professional development experience that has positively influenced the way you teach mathematics.**

Of the 1096 teachers who participated in the survey, 757 responded to this item.

In the analysis process for this item, a random sample of participant responses were used to develop a coding system of main themes for the responses. Following this, all responses were initially coded using these themes and any additional themes that were also identified through the coding process. The responses were then coded a second time using the established codes. The main themes identified through data analysis were used to structure this section of the report.

Benefits of professional development

Some teachers described the benefits to particular PD opportunities. For instance, certain participants discussed their learning in terms of gaining knowledge about a topic, while in other instances, teachers wrote about learning how to implement a particular resource, such as manipulatives or software. Some talked about gaining new mathematical knowledge. Still other teachers wrote about gaining new perspectives or ways of thinking about an aspect of teaching that had an impact on their practice.

> Exposure to algebra tiles in 1999 led to the use for Grade 9 and 10 mathematics to teach adding, subtracting, multiplying and factoring - also completing the square. And now I can't see teaching the concepts without the tiles. (14658, Q42)

Other benefits included gaining confidence and comfort with aspects of their teaching practice, and becoming renewed or motivated to try new ideas. Some teachers made links between teacher PD and the benefits for students. This included both the impact of teacher PD on student levels of engagement and enjoyment in mathematics classes as well as on student understanding of mathematical concepts.

> I took an ABQ Mathematics course through [name of university] this winter. The instructor's name was [name of instructor]. He was an awesome teacher with a lot of knowledge and resources to share. His passion for the subject inspired me. This course made me realize how important manipulatives and struggling through problems are to learning math. I feel the course brought me up to date in the area of grade 7 and 8 mathematics. This course also gave me the confidence to try new things in my mathematics classes. Now, if my school would just get a functioning computer lab the sky's the limit. (17405, Q42)

Some respondents suggested that the benefit of PD was in creating the opportunity for networking and collaboration among teachers. Teachers also wrote about receiving resources and ideas from colleagues and/or presenters at PD opportunities.

> Board level workshops led by our math consultant have been very helpful. It has been awesome to attend free (as conferences cost the teacher personally) workshops offered during the workday with other colleagues from local schools. Our consultant is very articulate, models the approaches she believes in, and provides us with resources to support her lessons. We certainly can't leave and say "yeah, but..." (12044, Q42)

Topics of professional development

In responses, teachers often discussed the topic of their professional development experience. While general teaching practices were mentioned most often, more specific aspects of teaching were also discussed. Manipulatives was a very common topic while others included technology, TIPS, learning theories, the curriculum, assessment, transition, literacy, textbooks, resources, and classroom management.

Some teachers referred to issues connected to elementary to secondary transition. These included such things as developing a deeper understanding of curriculum across panels, learning how to better prepare students for high school, sharing cross-panel teaching experiences, sharing student information, as well as sharing teaching strategies. The following comment illustrates one such learning opportunity.

> This year I have been part of a professional learning group that has combined members of both the elementary and secondary panel. This group has allowed us to see how the two levels of curriculum are joined. As well, it has allowed us to begin to examine how to better prepare students entering high school. (01655, Q42)

Professional development approaches

Respondents identified a number of approaches or models for professional development that positively influenced the way they taught mathematics. Workshops and collaboration among teachers were the most common approaches discussed. Conferences, leadership roles, courses, PRIME, reflection on teaching practice, classroom observation, lesson study, summer institutes, and Leading Math Success were also mentioned. A few participants made reference to other opportunities such as reading professional literature or marking EQAO tests.

Activities involving collaboration were mentioned by approximately one third of the individuals who responded to this item. Teachers wrote about collaboration occurring in both more formal contexts, such as organized lesson studies, courses or book clubs, as well as informal contexts, such as through mentor support or discussion among colleagues. The comments below illustrate how, for some teachers, collaboration was seen as their most valuable PD experience.

> I think the biggest key is being able to have a good math department with good colleagues so that you can bounce ideas off of each other. (15110, Q42)

> I have found that time spent with colleagues is most valuable with a free exchange of ideas over a longer period of time. Spending our lunch hours together talking about math questions and problems is one of the most valuable uses of our time. (22794, Q42)

> The best workshops that I have attended and that have improved my teaching of mathematics are the ones in which math teachers worked together to share ideas and work towards a common goal such as planning a unit. (03371, Q42)

Teachers noted a wide variety of leadership roles they considered to be part of their professional development. For instance, a number of individuals wrote about the value of being the math lead at their school. Other respondents wrote about PD experiences in terms of being on school, board, or association committees and councils as well as leading or running workshops and conferences. Still others wrote about writing or reviewing documents such as the curriculum, TIPS, and textbooks.

> I currently am part of our board team that is re-writing the long range plans for the 3 new math textbooks in Ontario. This dialogue with colleagues over the last 1/2 year has been excellent in terms of assessment, solid teaching practices and resource development. (9087, Q42)

Some teachers stated that participating in courses was a positive professional development experience. Many of these related to mathematics ABQs, AQs, and Honours Specialists while a few respondents mentioned B.Ed., graduate, or other courses and professional qualifications (e.g., guidance).

> Taking my Honours Specialist in Mathematics changed the way I teach mathematics. It's not about getting the answer - it's everything that happens from the moment a student reads a problem to the moment just before he/she gets the answer. I just learned that I want to inspire students to think, reason and problem solve - and to use as many tools that are necessary to reach their result. Whether they use graphing calculators, manipulatives, computers or just a paper and pencil - the process of solving the problem is what makes mathematics real. (21850, Q42)

Reading and reflection on one's own practice were noted by some respondents as professional development. Participants also wrote about classroom observation as an opportunity for learning. Sometimes this involved informal observation such as sitting in on a colleague's class while others wrote about formal observation being organized at the school or board level.

> One school I have previously taught in required all teachers to observe other colleagues as they teach (a minimum of 2 times per year). This was great to go and see other approaches, methods of motivation, and classroom management techniques. (03017, Q42)

Challenges to professional development

While participants were asked to provide information about a professional development activity that had positively influenced the way they taught mathematics, some respondents also discussed challenges relating to PD. The most common challenge related to teachers' lack of time. This included teachers talking about the lack of time to participate in PD, time to collaborate and support colleagues, as well as time to absorb, consolidate, and implement new learning.

Another reported challenge was the lack of access to manipulatives and technology within schools. Teachers noted that lack of access to these resources limited both teachers' motivation to begin the process of engaging with and using such resources as well as limiting the implementation of new strategies gained from PD sessions. Additional challenges included a sense of isolation noted by some respondents who did not have colleagues to collaborate with and learn from given the small size of their school or being the only intermediate mathematics teacher in the school. Costs associated with attending professional development were mentioned as a barrier by a few respondents.

Concluding Comments

A number of observations and connections can be made from the various sources of data. The data from the leaders in mathematics education reflects the evidence presented in the research literature that suggests that engaging teachers in new ways of teaching mathematics is a challenge. However, leaders in mathematics education as well as many teachers responding to the survey are able to describe particularly effective professional development opportunities that have helped teachers make positive changes to their classroom practice. Many of these professional development opportunities have addressed teachers' understanding of mathematics or how students learn mathematics, as well as developing teachers' confidence in teaching mathematics.

Teachers' responses to the questionnaire also suggest that there is a desire for further professional development in areas connected to new ways of teaching mathematics such as assessment, the use of technology and manipulatives, facilitating problem solving and investigations, and understanding how students learn mathematics. The data also suggest that teachers engage in a variety of professional development models that include workshops, lesson study, leadership opportunities and visiting colleague's classrooms. However, there are specific elements of professional development that emerge as particularly important in supporting changes in classroom practice. We would like to draw your attention to two which emerged repeatedly from the multiple sources of data.

The first pertains to the importance of dialogue with colleagues and opportunities for collegial collaboration. Through informal collaboration in regular meetings with other teachers in one's school, or through collaboration occurring in other settings such as lesson study or resource development, teachers and math education leaders reported that they learned much from their peers and valued the opportunity to do so. The second element pertains to the effectiveness of those professional development initiatives that were connected to classroom practice and sustained over time. The participants reported that opportunities to engage in new ideas with colleagues, try them out in classrooms, and re-visit them in further discussions was responsible for renewed energy and insight. These findings are in keeping with the research literature in which peer interactions among teachers sustained over time, have a positive impact on their understanding of mathematics and their teaching practices (Crockett, 2002; Manoucherhi, 2002). The kinds of changes teachers are being asked to undertake to accomplish this are not simple and require a substantive re-orientation of their basic beliefs about the world in general, and mathematics education in particular. Such a re-orientation requires time and ongoing and iterative cycles of professional engagement (Adey, 2004; Borasi et al., 1999; Garet et al., 2001; Loucks-Horsley et al., 1998).

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