Student Achievement Division

August 2014

WHAT WORKS? Research into Practice

A research-into-practice series produced by a partnership between the Student Achievement Division and the Ontario Association of Deans of Education

Research Monograph #55

Why do we teach growing patterns and what do patterns have to do with algebra?

Exploring the Power of Growing Patterns

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Research Tells Us

- Patterning activities are a powerful tool for introducing young children to sophisticated algebraic concepts.
- Even very young students can develop an understanding of functions.
- For older students, working with linear patterns can serve as an effective introduction to graphs of functions.
- How patterns are presented and discussed is the key to their effective use.

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Patterning activities are pervasive in mathematics textbooks and the Ontario curriculum, K–8. They afford young students the opportunity to develop their algebraic thinking in developmentally appropriate ways.[1,2,3,4] They offer teachers a powerful visual tool for introducing sophisticated algebraic concepts.[5,6,7,8,9] Yet, how patterns are presented and discussed makes a significant difference to the ways in which students use them to think and talk about mathematical structure. This monograph explores how teachers can present and discuss growing patterns to introduce students to functions, an integral part of algebraic thinking.

Two Ways of Thinking About Growing Patterns

Consider a typical growing patterning activity: Jesse (a Grade 6 student) was easily able to describe the pattern in Figure 1 as "Start with three and add two each time." When asked by his teacher how many tiles would be in the next position, he confidently stated, "Well, you have 9 tiles, so 11 for the next position." However, when Jesse was asked to predict the 100th position, the process proved to be too laborious. "That's too big! Maybe… 102 tiles?"

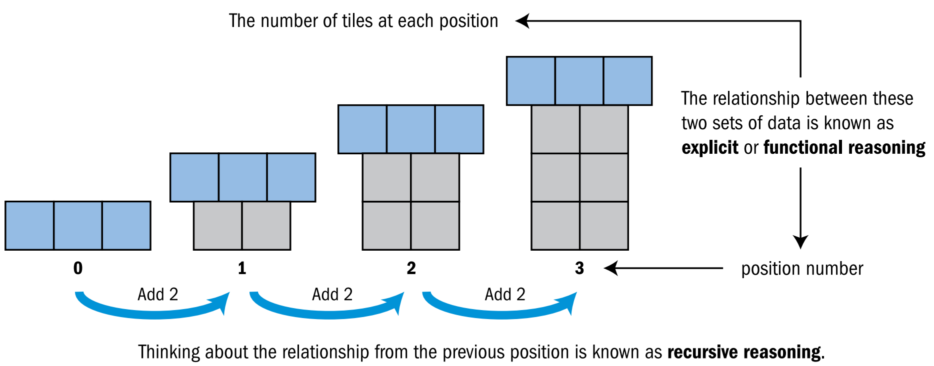

Figure 1: Two Ways of Thinking About Growing Patterns

A Note on Terminology ...

Sometimes "position" is also called "term."

Asking for the 100th position highlights the problem with this method. When students work with patterns like this, they are typically able to describe the pattern, and extend it to the next position, but have difficulty extrapolating much beyond the next position. In order to predict, for example, the 10th position in this pattern, students using this method must first calculate the values for the 4th through 9th positions. If students only think about the change from one position of the pattern to the next, they will have difficulty accurately predicting values for terms far down the sequence.

How Do We Get From Patterns to Functions?

If we focus our instruction on the relationship between two sets of values, rather than on "what comes next," growing patterns become powerful tools for exploring algebraic relationships for all levels of learners. In the following example, the thinking of two students illustrates the advantage of instruction that emphasizes such relationships. When asked to describe the pattern in Figure 1, six-year-old Ava states that it is "growing by 2s." Eleven-year-old Niroshan identifies the pattern rule and justifies his thinking: "It's a times 2 plus 3 pattern, because it's increasing by 2 each time, but the blue tiles stay constant." When asked how many tiles would be in the 10th position, Ava predicts "10 gray tiles and another 10 tiles – 20 gray tiles and 3 blue tiles." Niroshan asserts, "Easy, 10 times 2 is 20 gray and 3 blue." When asked to predict the number of tiles for the 100th position, both students again apply their logic to arrive at the correct answer. Further exploration with Niroshan indicates that he can also work backwards to find the position number from a given number of tiles. When asked, "If there were 163 tiles, what position would they be at?" Niroshan responds, "Let's see … 163 minus 3 is 160, and divide that by 2 … the 80th!" Clearly, both Ava and Niroshan can describe the pattern and make accurate predictions beyond "what comes next."

On the Teacher's Role ...

"With the right kinds of questions, teachers can shift students' attention from focusing on what comes next in a pattern (recursive thinking) to understanding the relationship between two sets of values (explicit thinking)."

How Do We Move From "What Comes Next" to Exploring Functions?

When students are introduced to growing patterns, the instructional emphasis tends to be on additive reasoning. Thus, students often identify the change from the previous position, as Jesse did in the example, when he described the rule as "start with 3 and add 2 each time." This kind of rule limits students to think only about what comes next; it doesn't allow students to a) articulate the mathematical structure of the pattern; or b) accurately predict the number of tiles needed for any position of the pattern. It is more effective to emphasize the relationship between the value of the position number and the tiles.

Patterns offer a way for students to think about the functional relationships between two sets of data. In math, a function is a consistent relationship between two sets of data. Each element of one set is associated with a unique element in the other set, and a function rule describes how the elements in one set are associated with the elements in the other set. For example, in the growing pattern depicted in Figure 1, there is a unique number of tiles associated with each position of the pattern. The figure illustrates the relationship between one set of data (the position number), and another set of data (the number of tiles at each position). Understanding this relationship is known as *explicit* or *functional* reasoning and allows a student to determine the number of tiles for any position.

Growing patterns also offer a concrete way for students to explore the notion of generalization in the form of a pattern rule. Niroshan, for example, found the generalized pattern rule, which allowed him to accurately predict the number of tiles for any position of the pattern.

Research has demonstrated that even very young students can develop an understanding of functions.[10,11,12,13] Ava saw the pattern as both groups of two tiles growing by 2's and as 2 columns of tiles, with each column composed of a number of tiles equal to the value of the position number. She also remembered that the 3 blue tiles stay the same (the constant).

For older students, working with linear patterns can serve as an effective introduction to graphs of functions.[14,15] The position numbers of the growing patterns map onto values along the *x*-axis. The total number of tiles in each position is represented by values along the *y*-axis. The *y*-axis also represents the number of tiles that would be at the 0 position of a growing pattern (the value of the constant), which is graphed as the *y*-intercept. (Figure 2 illustrates such growing patterns, as well as how different multipliers affect the trend line of the graph.)

A helpful resource for teachers ...

For more examples of graphs and a comprehensive lesson sequence see Ruth Beatty and Catherine D. Bruce, *From Patterns to Algebra: Lessons for Exploring Linear Relationships* (Nelson Education Canada)

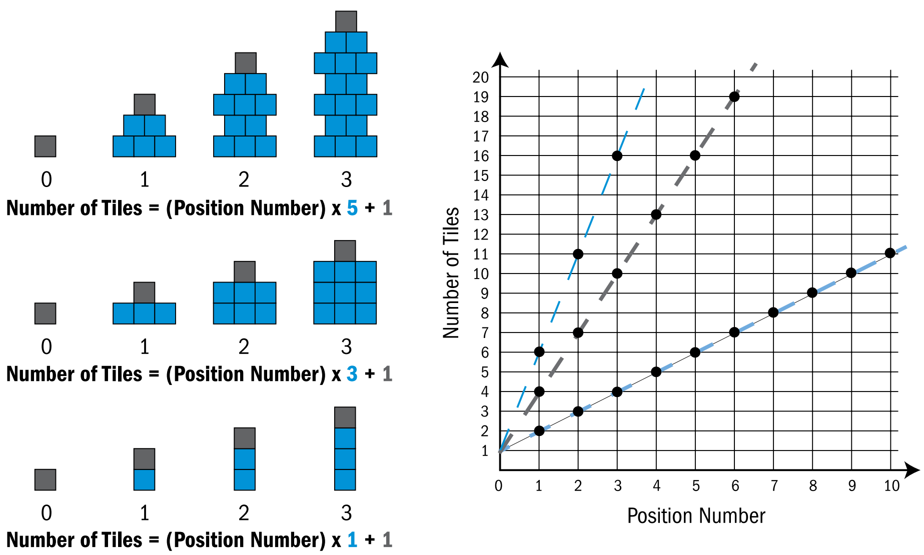

Figure 2. Multiple Representations of Three Pattern Rules

Implications for Practice

How Can We Make Working With Patterns More Powerful?

- Understand the concept of growing patterns as a teacher and have a list of questions and prompts at your fingertips. Resources such as CLIPS, *Paying Attention to Algebraic Reasoning* and *From Patterns to Algebra* can further your own thinking and provide you with ways to extend student thinking as well.
- When introducing growing patterns, make explicit the connection between the position number and the number of tiles. In class, recreate Figure 1. Emphasize that, at each subsequent position, there is one set of 3 blue tiles, but an increasing number of groups of 2 gray tiles. Note that the position number tells how many "groups of" there are, while the pattern rule tells how many tiles are in each group. The constant part of the rule tells how many tiles stay the same.

- Instead of focusing on what comes next, ask students for near predictions (e.g., how many tiles there will be at the 10th position) and far predictions (e.g., how many tiles there will be at the 100th position). Also ask for the rule that will allow an accurate prediction for any position of the pattern. When students are only asked, “What comes next?” it reinforces a focus on thinking recursively.
- All students, from Kindergarten to Grade 8, should be encouraged to physically construct patterns using square tiles and position number cards. This builds an understanding of the relationship between the position number and number of items at each position. The process of building a pattern core (like the “group of 2” gray tiles in Figure 1) that is then replicated at each position according to the value of the position supports students’ multiplicative thinking beyond memorizing multiplication facts.
- Provide opportunities for students to make connections among different representations. For example, ask how changing the number of tiles in the “groups of” (the multiplier) in the pattern will affect the pattern rule and the trend line of the graph? Invite students to test their hypotheses by plotting examples on a graph. Through this exploration, students learn that the value of the multiplier is responsible for the rate of growth of the pattern and, therefore, the steepness of the trend lines (See Figure 2). Ask students to repeat this exercise, using the same multiplier but different constants. Compare and discuss the results.

In Sum

Working with growing patterns allows students to develop a strong understanding of functions. With the right kinds of questions, teachers can shift students’ attention from focusing on what comes next in a pattern (recursive thinking) to understanding the relationship between two sets of values (explicit thinking). By emphasizing the connection between position number and number of tiles and prompting students to make predictions for positions further down the sequence, teachers can support even very young students to make this conceptual shift.

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ISSN 1913-1097 What Works? Research Into Practice (Print)
ISSN 1913-1100 What Works? Research Into Practice (Online)