Capacity Building Series

Bansho (Board Writing)

Collective Knowledge Production in Ontario Mathematics Classrooms

The Ontario curriculum emphasizes that students learn through problem solving and that problem solving lies at the core of an effective mathematics program (Ministry of Education, 2005, p. 11). In classroom practice, however, questions of application arise. As students solve the lesson problem are they learning mathematics? How is this new learning consolidated? And what do teachers need to know and do in order to develop students' mathematical understanding?

In order to address these questions, this monograph revisits bansho, a powerful instructional strategy for mathematical communication and collective problem-solving (Literacy and Numeracy Secretariat, 2010). Following a brief overview, it outlines how bansho can be used to plan, prepare and implement an effective three-part problem-solving lesson in the mathematics classroom.

Origins in Japanese Mathematics Lessons

Japanese teachers refer to the use and organization of the chalkboard as "bansho" or board writing. Such board writing is derived from and for the development of students' individual and collective mathematical thinking. For bansho, students' solutions and strategies are recorded on a large-size chalkboard or dry-erase board using mathematical expressions – numbers, letters and mathematical symbols, figures, graphs, algorithms and labelled diagrams (Shimizu, 2007; Stigler & Hiebert, 1999; Takahashi, 2006; Yoshida, 2002). Japanese teachers refrain from erasing what they have written on the board because they have strategically selected and recorded mathematical details and annotations that capture the mathematical thinking of the students for the bansho.

Why bansho?

Bansho is an instructional strategy that captures the development of students' individual and collective thinking. Bansho allows students to:

- solve problems in ways that make sense to them

- build understanding of tools, strategies and concepts by listening to, discussing and reflecting on their peers' solutions

- build understanding of concepts through explicit connection-making facilitated by the teacher's board writing

reach every student

Ontario

According to Stigler and Hiebert (1999), Takahashi (2006) and Yoshida (2002), a structured problem-solving lesson generally follows the following sequence: (a) introduction of the problem, (b) understanding and solving the problem, (c) presentation/comparison/ discussion of solutions and (d) conclusion or summary of the lesson. Details of the lesson problem are discussed and recorded at the start of the lesson. During the discussion portion of the lesson, the different methods that students used to solve the problem are presented and compared in order to clarify and justify the merits and limitations of each method. To keep a record of students' solutions and facilitate their participation in these whole-class discussions, teachers depend on the use of the board as a visual aid.

According to Yoshida (2002), bansho has a range of purposes:

- keeping a record of the mathematical details from the lesson discussion
- prompting students to remember what they need to do and think about
- enabling students to see mathematical connections between different parts of the lesson and the progression of the lesson
- providing a visual aid for comparing, contrasting and discussing the mathematical ideas that are represented in students' solutions to the lesson problem
- organizing student thinking for the discovery of new mathematical ideas and for promoting deeper mathematical understanding
- modelling effective organization in order to develop note-taking skills

Adaptation to Ontario Classrooms

Kathy Kubota-Zarivnij (2011) has interpreted and adapted Japanese bansho so that it complements the Ontario curriculum's emphasis on teaching and learning mathematics through problem solving and supports the current exploration of collaborative approaches to knowledge building in the classroom. She explains that bansho is:

- a *mathematics instructional strategy* that makes explicit students' mathematical thinking and provokes students' collective knowledge production through strategically coordinated discussion, organization and mathematical annotation of students' solutions to a lesson problem
- an *assessment for (and as) learning strategy* that enables the teacher and students to discern the range and relationships between mathematical ideas, strategies and models of representation
- a *classroom artifact* that is constructed collectively by the teacher and students in order to display the mathematical relationships derived from students' solutions; it can be organized and used as a mathematics learning landscape or as a mathematics anchor chart
- a *job-embedded professional learning strategy* that develops the teacher's knowledge of mathematics for teaching through the anticipation and construction of a bansho that depicts the breadth, depth and complexity of mathematics elicited throughout a three-part problem-solving lesson

Preparing to Implement Bansho (Board Writing)

Before using bansho in the classroom, there are several components of the lesson that need to be planned in advance. Some suggestions for planning are offered below.

IDENTIFY THE MATHEMATICAL FOCUS OF THE LESSON

- Establish the mathematics learning goal for the lesson based on grade-specific curriculum.

Bansho in the classroom allows teachers to …

- make explicit the underlying big ideas of mathematics topics
- weave multiple problem-solving strategies into a coherent conceptual framework
- make explicit connections among concrete, symbolic and numeric representations
- acknowledge students' mathematical thinking and differentiate mathematics instruction
- create anchor charts for concept development with their students
- use students' own work to build conceptual understanding and success criteria in mathematics

Bansho promotes professional learning for teachers so that they can …

- create lesson goals that build mathe-matics concepts through engaging with "big ideas"
- build their own schema about student mathematical thinking in action
- deepen their understanding of mathematics concepts for the teaching of mathematics

- Identify the curriculum expectations before and after the focus grade in order to have a sense of the developmental continuum of learning.
- Familiarize yourself with the mathematical concept, in terms of symbols or models of representation and mathematical terms. You will use these symbols and terms to annotate student solutions during the whole-class discussions.

CHOOSE A PROBLEM FOR THE LESSON

- Select a problem that will elicit mathematics aligned with the learning goals.
- Ensure the problem is accessible for all students and supports differentiation (use open-ended and parallel tasks – see, for example, Literacy and Numeracy Secretariat, 2008).

UNDERSTAND AND SOLVE THE PROBLEM

- Ask yourself, "What information do I need to make a plan to solve this problem?"
- Solve the problem yourself in two or three different ways as part of your preparation.

ANTICIPATE EACH PART OF THE LESSON

- Consider ways to activate prior knowledge – students may engage in solving an "activation problem," respond to a prompt or discuss solutions and strategies to a practice question from a prior lesson.
- Make sure each part of the lesson – BEFORE, DURING, AFTER (CONSOLIDATION, HIGHLIGHTS/SUMMARY, PRACTICE) – is mathematically coherent and can be implemented in a timely fashion in order to develop students' conceptual understanding.
- Select instructional strategies that will facilitate students' discussion about how they have solved the problem (e.g., think-pair-share, turn and talk or one stay, the rest stray).
- Anticipate incorrect solutions and typical errors that students may make.

ORGANIZE THE CLASSROOM

- Provide a cleared, large, public writing space (e.g., chalkboard, dry-erase-board, mural or butcher paper) to record the mathematical details generated throughout the lesson.
- Visualize how much of the board space will be used for the different parts of the three-part problem-solving lesson.
- Provide students with ¼ of a piece of square grid chart paper (landscape orientation) and markers to record their solutions (a consistent colour is suggested for student work). The paper needs to be large enough for students to analyze their solutions at the board.
- Use different coloured chalk or markers for annotating student solutions. It is important that the teacher's annotations stand out from student work so that they can easily be read by the students. It may be helpful to colour code the annotations: new math vocabulary (blue), mathematical elaborations (green), questions for clarification (red), symbolic notation (like equivalent numerical expressions) (black).

Some Bansho Tips

- Begin at the far left side of the board and proceed toward the right.
- Envision the use of board space in advance in order to avoid removing elements of the bansho to make space for new ones.
- Consider using a 2-metre piece of mural paper as the background for your bansho (then you can roll it up to use later for classroom use or professional learning).

A Sample Bansho Lesson for the Early Development of Multiplication Concepts (Grade 2)

A bansho might take the following form but keep in mind that every actual bansho will be different.

Starting BANSHO

Getting Started Problem

The Problem of 12

Show as many different ways as you can to model 12. Be sure you can tell how you know there are 12 all together.

The Lesson Problem

Butterfly Problem

Three butterflies landed on a bush.

Then, 4 more butterflies landed.

Later, 8 more butterflies joined them on the bush.

How many butterflies are on the bush altogether?

Show at least 2 different solutions.

What information are we going to use to solve the problem?
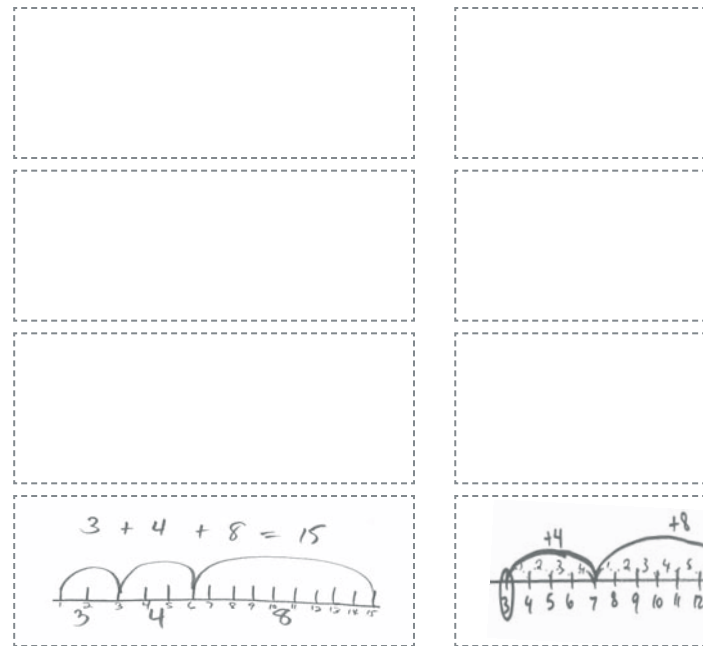
3 butterflies

4 more

8 more

altogether

2 strategies

different tools & strategies

Other students might be invited to consider their work and a

$3 + 4 + 8 = 15$

Counting by ones

Counting on by on

Using the Three-Part Problem-Solving Lesson (45 to 60 minutes)

1. Before – Getting Started

5 to 10 minutes/about $\frac{1}{8}$ of board space
- Activate students' mathematical knowledge and experience using a prompt or problem that directly relates to the mathematics of the lesson problem.
- Record student responses to the prompt/problem in order to highlight key ideas/strategies.
- *Keep in mind:* discussion of the lesson problem may be sufficient to activate students' prior knowledge.

2. During – Working on It

15 to 20 minutes/about $\frac{1}{8}$ of board space
- Introduce the lesson problem (if not already introduced).
- Encourage students to identify the information needed to solve the problem (and record on board).
- Have students record their solutions on chart paper (landscape) using markers so that their work will be visible to the entire class.
- As students work on their solutions, the teacher a) facilitates discussions among students, primarily through questioning and b) observes/records different student solutions in anticipation of the third and final parts of the lesson.

BUTTERFLY PROBLEM LESSON GOAL – Students will learn different strategies for adding whole numbers to 18 and how they relate to "combining equal groups" as a precursor to multiplication.

OVERALL EXPECTATION:

- solve problems involving the addition and subtraction of one- and two-digit whole numbers, using a variety of strategies, and investigate multiplication and division
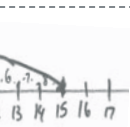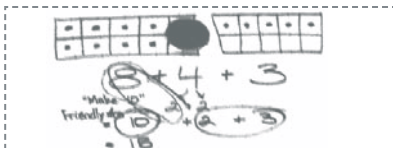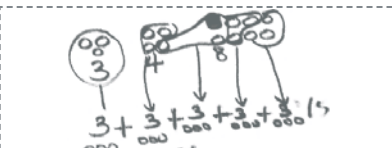
SPECIFIC EXPECTATIONS:

- solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental strategies
- represent and explain, through investigation using concrete materials and drawings, multiplication as the combining of equal groups

– ONE EXAMPLE

Student Solutions

...dd theirs to the bansho in columns depending on the strategy they used to solve the problem.

...es from 3

Adding by regrouping to make 5s and 10s

Adding by regrouping to make groups of 3

Highlight and Summary

We can regroup quantities into equal sized groups so we can skip count or use multiplication.

Practice Problem(s)

3. AFTER – CONSOLIDATION/HIGHLIGHTS/PRACTICE

AFTER (Consolidation)
20 to 25 minutes/about $\frac{1}{2}$ of the board

- Select two or more solutions for class analysis and discussion in a sequence based on mathematical relationships between the solutions and the lesson learning goal.
- Have students (authors) explain and discuss their solutions with their classmates.
- Facilitate student work by asking probing questions.
- During whole-class discussion, organize (and reorganize) solutions to show mathematical elaboration from one solution to the next and progression toward lesson learning goal.
- Mathematically annotate (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

AFTER (Highlights/Summary)
5 minutes/about $\frac{1}{8}$ of the board

- Revisit the student solutions for key ideas, strategies and models of representation that are related to the lesson learning goal.
- List key ideas, strategies and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

AFTER (Practice)
5 to 10 minutes/about $\frac{1}{8}$ of the board

- Choose two or three problems that are similar to the lesson problem for students to solve individually and in pairs (problems may vary by number, problem contexts or what is unknown/needs to be solved needs to be solved).

Sample Solutions to the Butterfly Problem

SOLUTION A

SOLUTION B

SOLUTION C

SOLUTION D

Preparing for Consolidation

How do you make sense of the mathematics that students are communicating orally, in their models (e.g., concrete materials, technology) and in their written forms (e.g., pictorial, symbolic notation, explanatory descriptions)? You may wish to consider the following questions as you look at and listen to students' mathematical thinking:

WHAT MATHEMATICS IS EVIDENT IN STUDENTS' COMMUNICATION (ORAL, WRITTEN, MODELLED)?

- **Solution A** -> 3+4+8 is regrouped to 5 equal groups of 3 = 3+3+3+3+3
- **Solution B** -> starts counting by ones from the number, 3 for 4 counts, then continues counting by ones from 7 for 8 counts, ending at 15
- **Solution C** -> counting by ones, starting at 1, not starting at 0
- **Solution D** -> reorders the numbers to start with the largest number, 8, and decomposes 4 to be 2+2, then regroups numbers to make tens

WHAT MATHEMATICAL LANGUAGE SHOULD WE USE TO ARTICULATE THE MATHEMATICS WE SEE AND HEAR FROM STUDENTS (i.e., MATHEMATICAL ACTIONS, CONCEPTS, STRATEGIES, MODELS OF REPRESENTATION)?

- addition mental strategies – count by 1s, counting on from the first number, counting on from the larger number, joining or combining quantities (addition), regrouping to make 5s and 10s, repeated addition, doubles, doubles 1 (or 2), doubles minus 1 (or 2)
- equal groups – 3 groups of 5 = 15 is the same as 5+5+5 = 15 and 5 x 3 = 15
- size of group (multiplicand, 3); number of groups (multiplier, 5); product (result of multiplication, 15)
- number expressions that are equal or equivalent (5+5+5 = 5x3; 15 = 5+5+5; 5x3 = 15)

WHAT MATHEMATICAL CONNECTIONS CAN BE DISCERNED BETWEEN STUDENTS' DIFFERENT SOLUTIONS? HOW ARE THE SOLUTIONS MATHEMATICALLY RELATED?

- **Solutions C and B** – both solutions are counting by ones; solution C starts from 1 (should be 0), solution B starts from the first number, 3
- **Solutions C, B, and A** – solution A shows a regrouping of numbers to make equal groups of 3 which can be shown on the number lines in solutions B and C by drawing jumps of 3
- **Solutions C, B, and D** – solution D shows starting with the largest number, 8, decomposing 4 to 2+2, so 2 can be combined with 8 to make 10 and the other 2 combined with 3 to make 5; this regrouping can be drawn on a number line used in solutions B and C

Selecting Student Solutions – As students are solving the lesson problem, the teacher chooses two to four different solutions that can be knitted together to develop the students' understanding of the lesson learning goal.

Selection or sorting criteria emphasize the notion of mathematical elaborations; that is, how one solution relates to, builds on and/or leads to the mathematics inherent in other solutions. Examples of such sorting criteria include: type of mathematical method or strategy (e.g., counting by ones, joining numbers to make a new number, regrouping numbers to make fives and tens, doubles 1 more than or 1 less than); relationship between representations of a concept and alternative and standard algorithms; equivalent numeric expressions.
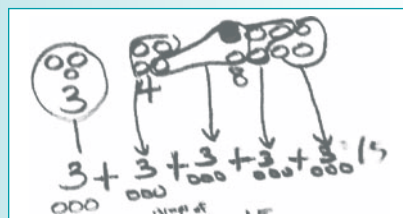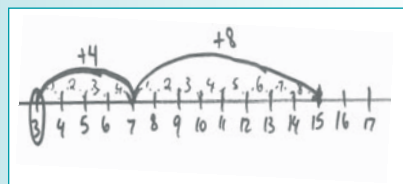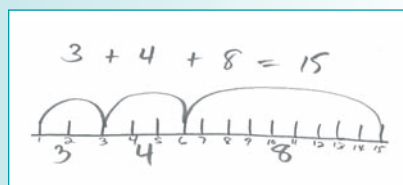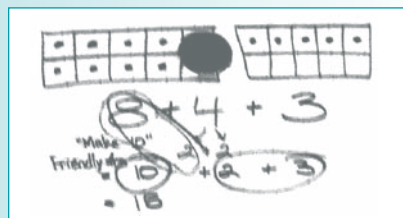
Deciding on the Sequence of Student Solutions – Rather than having students randomly volunteer to share their solutions, strategically sequence the solutions so that mathematical understanding can be developed. This does not mean that student solutions are organized by levels of achievement. Instead, generally, solutions that are conceptually based are shared first, followed by those solutions that include more efficient strategies or algorithms, followed by solutions that show generalizations. To help you to decide which solution should be shared first, second, and third, ask yourself these questions:

WHAT MATHEMATICS (i.e., CONCEPT, ALGORITHM, STRATEGY, MODEL OF REPRESENTATION) ARE THE STUDENTS USING IN THEIR SOLUTION? HOW DOES THE MATHEMATICS IN THE SOLUTION RELATE TO THE MATHEMATICS LESSON LEARNING GOAL?

- **Solution A** -> repeated addition of 3 or adding 5 groups of 3
- **Solution B** -> counting by ones, starting at the first number, 3, on a number line
- **Solution C** -> counting by ones, starting at 1 (error, should be 0), on a number line
- **Solution D** -> adding, starting at the larger number, 8, and regrouping to make 5s and 10s

WHICH SOLUTIONS ARE CONCEPTUALLY-BASED? WHICH SOLUTIONS HAVE AN EFFICIENT METHOD OR ALGORITHM? WHICH SOLUTIONS INCLUDE A MATHEMATICAL GENERALIZATION?

- **Solutions B and C** are conceptually based but are the least efficient as they focus on counting by ones, while solution D shows flexibility with numbers through regrouping to make friendly numbers, 5s and 10s;
- **Solution A** shows a more efficient strategy of regrouping numbers to be of equal group size which has the greatest potential for efficiency and generalization (from addition to multiplication)

HOW ARE THE SOLUTIONS RELATED TO ONE ANOTHER, MATHEMATICALLY? HOW ARE THE SOLUTIONS RELATED TO THE MATHEMATICS LEARNING GOAL OF THE LESSON?

- Discussion order for the solutions is **C**, **B**, **D**, **A** which shows mathematical development from counting by ones, to adding by regrouping to make 5s and 10s, to adding by regrouping to make equal groups of 3. This is a precursor to multiplication solutions which show different strategies for adding whole numbers to 18 and how they relate to "combining equal groups."

Consolidation in the Classroom

CO-ORDINATION OF WHOLE-CLASS DISCUSSION AND ANALYSIS OF STUDENT SOLUTIONS

The third part of a problem-solving lesson is the most critical part because new, intentional learning is collectively developed, made explicit and practised.

Teacher-selected solutions are explained by students in a sequence that has been constructed by the teacher to scaffold learning as students work toward the lesson learning goal. While some students are explaining their solutions, others are listening, making comments and asking questions to clarify and expand on mathematical ideas. All of this discussion is recorded on and around the students' solutions, using precise, concise and explicit mathematical language. Such mathematical language or mathematical annotations (e.g., labelled diagrams, symbolic notation, concise and precise explanations) makes explicit students' mathematical thinking. Also, students' ideas become formalized when annotated with formal mathematical terms and notation (symbols, numerals). In fact, when mathematical connections between students' responses are made evident, students can see how solutions unfold from and into one another.

Mathematical annotations include ...

- precise mathematical notation
- mathematical language
- mathematical representation
- the mathematical elaboration of student thinking

Possible Mathematical Annotations for the Butterfly Problem

- **addition mental strategies** – count by 1s, counting on from the first number, counting on from the larger number, joining or combining quantities (addition), regrouping to make 5s and 10s, repeated addition, doubles, doubles 1 (or 2) more than, doubles 1 (or 2) less than

- **equal groups** (e.g., using counters to show that 3 groups of 2 = 6 is the same as 2+2+2 = 6 and 2 x 3 = 6

- **size of group** (multiplicand, 3); number of groups (multiplier, 2); product (result of multiplication, 6)

- **number expressions that are equal** (2+2+2 = 2 x 3; 6 = 2+2+2; 2 x 3 = 6)

Highlights/Summary

RECOUNTING KEY MATHEMATICAL IDEAS AND STRATEGIES RELATED TO THE LEARNING GOAL OF THE LESSON

During the learning, a wealth of mathematical notations and diagrams have been generated to make students' thinking explicit. For some students, the focus of the lesson is obscured by such abundant detail. Have the students refer back to the mathematical annotations recorded on and around the solutions presented, in order to identify and describe two or three key ideas and/or strategies. These two or three key ideas and/or strategies are elaborations of the lesson learning goal. The teacher records student ideas and/or strategies in an itemized list to enhance readability.

Practice

SOLVING A PROBLEM THAT IS SIMILAR TO THE LESSON PROBLEM IN ORDER TO PRACTISE APPLYING NEW IDEAS AND STRATEGIES

One or two practice problems are recorded on the board for students to work on in pairs or individually. These practice problems are similar in structure to the lesson problem with a slight variation, such as different numbers or different problem context. Have students record two different solutions to one of the practice problems on the board. When done as an exit pass, this can serve as assessment-for-learning information about individual students' understanding of the problem.

Embedding Bansho in School Culture

Bansho is an effective, research-informed strategy that can be used to enhance both student and teacher learning of mathematics. By incorporating bansho into collaborative co-planning and co-teaching sessions, teachers, working as partners or in groups, can effectively learn about mathematics for teaching. When these sessions occur within communities of practice that meet regularly over time, learning for both teachers and students is significantly enhanced (Kubota-Zarivnij, 2011; Fleming, 2011).

Learn about Bansho

LNS webcasts depict Ontario teachers learning to use bansho ...

- *Making Mathematics Accessible for All Students*

- *Coaching for Student Success in Mathematics*

- *Investigating High Yield Strategies for Improving Mathematics Instruction and Students' Learning*

- *Through the Eye of the Learner: From Student Work to Teacher Practice*

http://www.curriculum.org/ secretariat/archive.shtml

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