Capacity Building Series

SPECIAL EDITION #22

Maximizing Student Mathematical Learning in the Early Years

"Deepening of Teacher Understanding + Shifts in Instructional Practice = Impact on Student Learning" – 2010–2011 Early Primary Collaborative Inquiry

Recent research on early mathematics education has uncovered some important findings. Several studies reveal that children bring more mathematical knowledge and experience to school than previously believed. In one study, preschool children were observed frequently engaging in a range of mathematics, including pattern and shape, magnitude, enumeration, spatial relations, classification and dynamic change (Ginsburg et al., 2003, p. 236). According to Ginsburg, children's thinking "is not limited to the concrete and mechanical; it is often complex and abstract" (Ginsburg, 2008, p. 47).

Growing evidence also indicates that early mathematics plays a significant role in later education. From an analysis of six longitudinal studies, Duncan and colleagues found that early mathematics skills were more powerful predictors of later academic achievement in both mathematics and reading than attentional, socioemotional or reading skills (2007, p. 1428). In addition, the differences in mathematical experiences that children receive in their early years "have long-lasting implications for later school achievement, becoming more pronounced during elementary school ... and continuing on into middle school and high school" (Klibanoff, 2006, p. 59).

Such findings raise a critical question:

> How can educators take advantage of the mathematical knowledge and experience that children bring to early primary classrooms?

This monograph, based on the current findings of researchers in early mathematics education, provides some insights into this question and serves as a starting point for conversations about the importance of early mathematics.

Children are splendid little mathematicians ...

"Before the onset of formal schooling, young children do not only memorize … and they do not only employ mechanical skills. They do not operate only on a 'concrete' level. Instead, we can say fairly that young children are splendid little mathematicians. They deal spontaneously and sometimes joyfully with mathematical ideas. This is what real mathematicians do."

(Ginsburg, 2008, p. 55)

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Building on what comes naturally ...

Mathematization is a critical learning process which involves "redescribing, reorganizing, abstracting, generalizing, reflecting upon, and giving language to that which is first understood on an intuitive and informal level."

(Clements & Sarama, 2009, p. 244)

What the research says ...

Two fundamental components play a central role in establishing an environment that is rich in mathematical opportunities. First, it is important for educators to understand the development and characteristics of the early mathematics learner, and second, it is necessary to have a solid understanding of mathematics for teaching.

The Early Mathematics Learner

When children first come to school, they bring inquisitiveness, energy, a wide range of social, intellectual and emotional experiences and an abundance of mathematical knowledge gleaned from their everyday experiences. This is not surprising since studies have shown that mathematical ability is evident in humans as early as infancy when they are able to discern between sets of objects that vary in number (Lipton & Spelke, 2003). Furthermore, mathematical abilities continue to develop into beginning mathematical understandings along a roughly consistent developmental path, with or without adult intervention. Ginsburg and colleagues discovered that when children are given free play opportunities there are no significant differences in the complexity of the exhibited mathematics, regardless of children's cultural or socioeconomic backgrounds (2003, p. 235). Although each child acquires mathematics knowledge through experience, and each child comes to school with a range of prior experience, all children have the potential to productively engage in mathematical activities.

Honouring children's starting points enables educators to build on students' mathematical knowledge with an inquiry-based approach, developing purposeful and meaningful mathematical experiences in the classroom. It is also important to realize that the ways in which young children think in mathematical situations can be quite unique. Educators "must be particularly careful not to assume that children see situations, problems, or solutions as adults do. Instead, good teachers interpret what the child is doing and thinking and attempt to see the situation from the child's point of view" (Clements & Sarama, 2009, p. 4).

Since play is integral to a child's world, it becomes the gateway to engaging in mathematical inquiry. Sarama and Clements suggest that mathematical experiences can be narrowed down into two forms, *play that involves mathematics* and *playing with mathematics itself* (2009, p. 327). Further, it is the adult present during the play who is able to recognize how the children are representing their mathematics knowledge and then build on their understanding through prompting and questioning. Sarama and Clements stress that "the importance of well-planned, free-choice play, appropriate to the ages of the children, should not be underestimated. Such play ... if mathematized contributes to mathematics learning" (2009, p. 329).

Educators also provide experiences in *playing with mathematics itself* by using a repertoire of strategies, including open and parallel tasks that provide differentiation to meet the needs of all students and ensure full participation. Moreover, students do not have to see mathematics as compartmentalized, but instead as it mirrors their life experiences through other subject areas like science and the arts. As such, "high quality instruction in mathematics and high quality free play need not compete for time in the classroom. Engaging in both makes each richer and children benefit in every way" (Sarama & Clements, 2009, p. 331). This equity of opportunity is essential so all students can fully develop their mathematical abilities.

A carefully planned mathematics environment enables the use of manipulatives whether commercial products or found objects, sometimes brought in by the students themselves. Ideally, manipulatives serve as learning tools to help students build their understanding and explain their thinking to others. Research has shown, however, that "manipulatives themselves do not magically carry mathematical understanding. Rather, they provide concrete ways for students to give meaning to new knowledge" (Ontario Ministry of Education, 2003, p. 19). Students need the opportunity to reflect upon their actions with manipulatives, and through discussion, articulate the meaning they generate, so that the link between their representations and the key mathematical ideas is apparent (Clements & Sarama, 2009, p. 274).

Computer manipulatives can sometimes be more powerful than concrete manipulatives. For example, some applications offer flexibility "to explore geometric figures in ways not available with physical shape sets" (Clements & Sarama, 2009, p. 285). However, "effectively integrating technology into the curriculum demands effort, time, commitment and sometimes even a change in one's beliefs" (Clements, 2002, p. 174). In their three-year study of the use of computers in primary classrooms,Yelland and Kilderry found "that most of the tasks the children experienced in early years mathematics classes were unidimensional in their make-up. That is, they focus[ed] on the acquisition of specific skills and then they [were] practised in disembedded contexts" (2010, p. 91). Such tasks were comparable in complexity to traditional pencil and paper mathematics tasks, were teacher directed and anticipated correct answers with narrow solution strategies. In other words, while technology was being used in many classrooms, its potential to promote student thinking was being greatly underutilized. Yelland and Kilderry recommend that technology in primary classes should offer multidimensional mathematical tasks ensuring both student input into the direction of their learning and supporting more varied learning outcomes (2010, p. 101). Such an approach to technology enables students to use powerful yet familiar media to express and extend their learning.

Knowledge of Mathematics for Teaching

Deborah Loewenberg Ball has carried out extensive research on the impact that knowledge of mathematics for teaching has on student achievement. This is the knowledge necessary not only for understanding mathematics but also for being able to impart that understanding to others; it requires the ability to unpack mathematical concepts, "making features of particular content visible to and learnable by students" (Ball, 2008, p. 400). Teachers need to be able to reason through and justify why certain procedures and properties hold true, to talk about how mathematical language is used, to see the connections between mathematical ideas and to understand how they build upon one another. Early mathematical concepts lay the foundation for later learning by providing students with underlying mathematical structures which are built upon over time. In this way, although early math concepts may appear to be basic, they are, in fact, fundamentally important and complex.

Consider the processes involved in counting a simple set of five objects. The child must memorize the first five counting words since there is no established pattern from which to generalize. Each spoken number word can only be associated with one object, yet the objects themselves can be counted in any order, as long as the stability of number order is preserved. For example, the first object on the left could either be counted as one, two, three, four or five, depending on the order in which the child decides to count. The final spoken number does not just relate to the last object, but also to how many objects are in the entire set, also known as the cardinal value (adapted from Ginsburg, 2008, pp. 49–51).

By understanding the intricacies of concepts, like enumeration in the example above, educators can help children achieve mastery and understanding. Knowledge of mathematics for teaching is also needed to, "interpret student thinking and to help [children] take the next step" (Ginsburg, 2008, p. 46). Without such deep understanding, important teachable moments, such as bringing forth concepts of equality and relative magnitude as two children debate over who has more, can be overlooked or lost.

Gaining knowledge for teaching mathematics is by no means an easy task and requires both access to the knowledge and practice in applying it. The investment is worthwhile, however, since the benefits can be great. Studies, such as those carried out by Ball in primary grades, indicate that, "teachers' mathematics knowledge was significantly related to student achievement gains" (2005, p. 371).

What matters most ...

"The teacher's knowledge of mathematics and the skills that the teacher applies in the classroom have the greatest impact on students' learning."

(Ontario Ministry of Education, 2006b, p. 6)

How to Apply YOUR Knowledge of Mathematics for

Teaching to Improve Student Learning

A Starting Point ...

Immerse yourself in the curriculum and supporting documents. Attain a better understanding of the expectations and the seven mathematical processes by reading about the explanations and rationale in the front matter of the *Full-Day Early Learning Kindergarten Program* and Grades 1 to 8 Mathematics curriculum. Look before and beyond the grade you are teaching to see how concepts build upon each other. There is a wealth of resources that can offer extra insight into the mathematics itself and can help to identify and connect the key mathematical concepts. Some of these include the *Guides to Effective Instruction in Mathematics* and the works of Dr. Marian Small, Catherine Twomey Fosnot and John A. Van de Walle. Your professional learning journey will be most effective when you delve into mathematical ideas with colleagues and together inquire about how your understanding impacts your related teaching.

1) Identify and use everyday mathematics knowledge to plan instruction.

Knowledgeable educators begin planning by carefully observing children at play or engaged in other activities in order to identify their everyday mathematics. Next, they accurately interpret the mathematics underlying the behaviours and how it fits into the key mathematical concepts and curricula. Once identified, educators can create activities which allow assimilation of new concepts into the children's prior knowledge (Ginsburg, 2008, p. 59). As educators observe student problem-solving, they can document what children say, do and represent in order to make both planned and "in-the-moment" decisions about how to respond, challenge and extend student thinking.

2) Encourage and foster "math talk."

Klibanoff and colleagues discovered that teacher-facilitated "math talk" in the early years significantly increased children's growth in understanding of mathematical concepts (2006, p. 59). Knowledgeable educators recognize that although young children may have a beginning understanding of mathematical concepts they often lack the language to communicate their ideas. By modelling and fostering math talk throughout the day and across various subject areas, educators can provide the math language that allows students to articulate their ideas. It is also important to encourage talk among students as they explain, question and discuss their strategies while co-operatively solving problems. In order to facilitate mathematical thinking rather than direct it, knowledgeable educators recognize when student thinking is developing or stalled. If it is developing, the educator observes but leaves the students to work through their thinking (Sarama & Clements, 2009, p. 325). If it is stalled, probing questions can be asked that provoke thinking about alternate ways to perceive the problem.

After students have worked through solving a problem, educators facilitate consolidation time (either with individual students or with small groups or large groups) in order to allow students to talk about their thinking. This consolidation time is sometimes referred to as the third part of the three-part lesson in mathematics. As educators value a variety of strategies and solutions, they guide students to make connections between them, to recognize how the thinking relates to the key mathematical concepts and to make further conjectures and generalizations.

Suzanne Chapin proposes five effective talk moves which help to create meaningful mathematics discussions. *Revoicing* is one move that is particularly useful when a student's explanation is confusing or hard for others to understand. The teacher repeats all or some of what the child

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said and then asks for clarification, which in turn provokes the child to clarify and offer further explanation. This also gives the educator an opportunity to embed mathematics vocabulary so the child can further explain his/her thinking (2009, p. 14).

3) Facilitate experiences that allow for mathematization of everyday knowledge.

Knowledgeable educators help students transform their everyday mathematics into a more formalized understanding that can be transferred and applied to other situations. Several researchers refer to this as "mathematization" which requires students to abstract, represent and elaborate on informal experiences and create models of their everyday activities (Clements & Sarama, 2009, p. 244). The educator can play an integral role by making meaningful connections between the mathematical strands, the real world and other disciplines, and most importantly, "between the intuitive informal mathematics that students have learned through their own experiences and the mathematics they are learning in school" (Ontario Ministry of Education, 2003, p. 14). For example, as a child naturally creates and extends a pattern while making a necklace, the educator can effectively pose questions that provoke the student not only to describe the pattern, but also to make predictions and generalizations.

4) Model and nurture positive attitudes, self-efficacy and engagement.

As educators gain the mathematical knowledge for teaching, they become more capable – and confident – in helping students extend and formalize their understanding of mathematical concepts. This can contribute to students' development of positive attitudes toward mathematics and an increase in their sense of self-efficacy. Self-efficacy, which is an individual's belief in whether he or she can succeed at a particular activity, plays an integral role in student success. Bruce and Ross discovered that "increases in teacher efficacy led to increases in student efficacy and outcome expectancy and to student achievement" (2010, p. 10). In turn, strong student self-efficacy can contribute to greater enthusiasm and engagement in mathematics (Ross, 2007, p. 52).

Five productive talk moves ...

1. **Revoicing** – Repeating what students have said and then asking for clarification
   So you're saying it's an odd number?
2. **Repeating** – Asking students to restate someone else's reasoning
   Can you repeat what he just said in your own words?
3. **Reasoning** – Asking students to apply their own reasoning to someone else's reasoning
   Do you agree or disagree and why?
4. **Adding on** – Prompting students for further participation
   Would someone like to add something more to that?
5. **Waiting** – Using wait time
   Take your time … We'll Wait …

(Chapin, O'Connor & Anderson, 2009, p. 13)

Some Practical Tips for Creating a Mathematics-Rich Environment

Once educators have a good understanding of the child and have developed their own mathematical knowledge for teaching, they can create situations which capitalize on children's everyday mathematical knowledge. Researchers have identified five common core characteristics of early learning environments that support effective mathematical pedagogy and foster positive attitudes and beliefs about mathematics (Clements & Sarama, 2009, p. 259). The characteristics, which have been taken and/or adapted from this research, are described below.

The "Voices from the Field" belong to participants in the the Early Primary Collaborative Inquiry (EPCI), a ministry-supported network for primary educators, K to 2. Their examples and reflections, drawn from new professional learning and classroom experience, illustrate each characteristic in greater depth.

1. Use problems that have meaning for children (both practical and mathematical).	
Context	Voices from the Field
By taking an inquiry stance, pose a balance of educator-initiated and child-initiated problems to individuals, small or large groups. Draw from students' intuitive mathematics and real-life experiences. Relate problems to the curriculum expectations and the mathematical processes. Challenge students with problems that are developmentally appropriate yet do not underestimate their abilities.	"We began to value the mathematical processes rather than the final product and correct answer. We encouraged students to reason their way to a solution and the teacher's role shifted to facilitator." "Our team began to pay more attention to what the students were genuinely interested in while they were playing and came up with real life problems to extend their thinking and to make things more purposeful." "How we question students has changed. We see students as much more capable problem solvers."

2. Expect that children will invent, explain and offer critiques of their own solution strategies within a social context.	
Context	Voices from the Field
Provide opportunities for students to collaborate as they engage in mathematical activities so they can articulate, discuss and question each other's thinking. Foster and encourage math talk and facilitate consolidation time with individuals, small or large groups. Value and honour a variety of strategies and solutions. Offer students regular opportunities to revise their thinking.	"We began to incorporate time for students to reflect and explain their thinking to other students as they solved these problems. This began to develop a collaborative culture in the classroom." "Sometimes, we as educators, might assume that students do not have an understanding of mathematical concepts, when really, they just did not have the mathematical language to explain their thinking." "One practice that I have changed as a result of this inquiry is to bring students back together to share their solutions and thinking with their peers. This has made a big difference in the climate and culture in my classroom. It's a safer place to share our thinking."

3. Provide opportunities for both creative invention and practice.	
Context Intentionally plan experiences based on observations of children engaged in activities. Provide a variety of materials so children can work through their strategies and make their thinking visible. Once a concept is acquired, provide practice experiences to consolidate learning. Practice is not meant to be rote or mechanical in nature; it occurs "through mathematical investigations that take place through free exploration, focused exploration, and guided activity" (Ontario Ministry of Education, 2010, p. 94). Sarama and Clements refer to this as mathematical play, or "playing with the math itself" (2009, p. 327).	**Voices from the Field** "We used observations of children's actions to understand and assess their reasoning." "Focused observations helped to rethink the ability of many of the students. They proved to be more capable than expected." "I have learned that paper and pencil tasks do not provide good, accurate assessment." "How materials are presented makes as much difference as the choice of materials that are presented. The way that students use the materials is crucial to their learning."
4. Encourage and support children by providing carefully scaffolded opportunities which allow them to deepen their understanding, use meaningful and elegant solution strategies and confidently engage in the mathematics.	
Context Provide opportunities for students to mathematize their informal mathematics knowledge. Help students reflect on their strategies and representations. Nurture positive attitudes, self-efficacy, engagement and perseverance.	**Voices from the Field** "We found out that really choosing questions carefully had an impact on students' ability to deepen their understanding and express their ideas." "We found that scaffolding in mathematics is more about the support we give children to solve a problem than about chunking problems into more manageable pieces." "Unequivocally, the most noticeable impact on learning from involving students in inquiry was an increase in student engagement … students were on task, discussing the math during investigations, involved in problem solving, joyful and excited about what they were doing, and receptive to new learning from their peers." "The three-part lesson gives children control and independence. They were participating and looking forward to being part of the process."
5. Help children see connections between various types of knowledge and topics, with the goal of having each child build a well-structured, coherent knowledge of mathematics.	
Context Integrate mathematics activities into other subjects, like the arts and science, and make connections between the key mathematical ideas and the real world.	**Voices from the Field** "A conscious effort was made to provide opportunities to reflect on student learning related to big ideas." "We need to relate our problems to the real world and help the children make connections to the validity and usefulness of mathematics in their lives. Problem solving helps to develop curiosity, confidence and an open-mindedness that will help them to solve unfamiliar problems through their life and in the work force."

"Math can be seamlessly integrated into children's ongoing play and activities. But this usually requires a knowledgeable adult who creates a supportive environment and provides challenges, suggestions, tasks, and language."

(Sarama & Clements, 2009, pp. 332–333)

Summing It Up

Research reveals that young children come to school with a wealth of everyday mathematical knowledge that, if effectively built upon, can positively impact their future educational achievement. Knowledge about early mathematics learners and the mathematics for teaching can help educators create a rich environment and guide students to attain strong conceptual understandings, positive attitudes and self-efficacy. The research is also an excellent starting point for initiating conversations, at all levels of education, about the significance of early mathematics and how we can help *all* of our "splendid little mathematicians" develop the confidence and competence necessary to adeptly engage in mathematics throughout their lives.

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