Teacher Package

Mathematics Exemplar Task
Grade 9 Foundations of Mathematics, Applied

Teacher Package

Title: Footprints on the Road

Time requirement: 150–225 minutes
The student task: Part I – Up to 75 minutes
Part II – 150 minutes

Description of the Task

Through small group and whole class discussion, students will generate problems involving the investigation of relationships. Students will identify the data needed for the investigations, collect the data from the whole class, and prepare a class summary data sheet (Part I). Each student will then select one problem, analyse the data, make a conclusion, and submit a written report (Part II).

Part I: Class Discussion (time: 75 minutes)

- In small groups, students generate questions based on the text they read, a scenario involving two friends, Henri and Kim.*
- As a whole class, students identify problems or research arising from a discussion of the questions.
- In small groups, students gather the data identified as necessary for analysing the research problems selected.
- Students prepare a class summary data sheet to facilitate the preparation of their individual final reports.

* *The text of the scenario is reproduced on page 47 of this document.*

Part II: Individual Work (time: 150 minutes)

- Each student analyses a set of data and writes a report.

Note: The time required for Part I will vary according to the extent of students' familiarity with carrying out investigations and describing relationships. Time may also be needed to prepare students for Part II. Allow 150 minutes (75 minutes × 2) for the completion of Part II. Assign homework as appropriate.

Assessment and Evaluation

- You will need to use the task-specific rubric* to assess and evaluate the student work.
- You will need to approve the relationship that a student chooses to investigate. Some relationships will not provide students with as much data as others. See the section "Task Instructions" for ideas of good relationships to investigate.

Expectations Addressed in the Exemplar Task

Relationships

Students will:

1. determine relationships between two variables by collecting and analysing data;
2. compare the graphs of linear and non-linear relations;
3. describe the connections between various representations of relations;
4. pose problems, identify variables, and formulate hypotheses associated with relationships;

(continued)

* *The rubric is reproduced on page 46 of this document.*

5. demonstrate an understanding of some principles of sampling and surveying and apply the principles in designing and carrying out experiments to investigate the relationships between variables;
6. collect data, using appropriate equipment and/or technology;
7. organize and analyse data, using appropriate techniques and technology;
8. describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses;
9. communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms;
10. demonstrate an understanding that straight lines represent linear relations and curves represent non-linear relations.

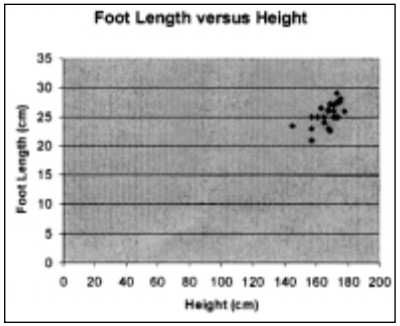
Note that, although all the expectations listed will be addressed through instruction relating to the task, student achievement of expectations 2 and 4 will not be assessed in the final product.

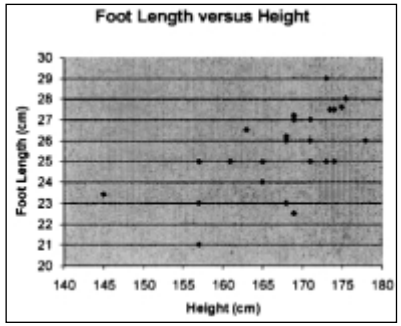
Teacher Instructions

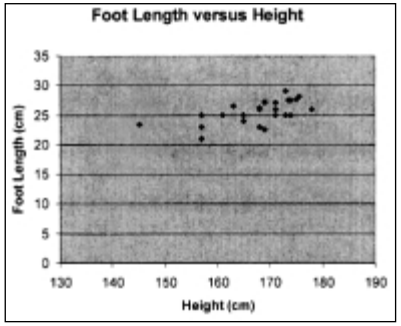
Prior Knowledge and Skills Required

- Have students practise describing relationships by discussing some obvious relationships and the language used to characterize them. For example:
  - Describe the relationship between age and natural hair colour.
  - Describe the relationship between marks and the amount of time spent doing homework and studying.

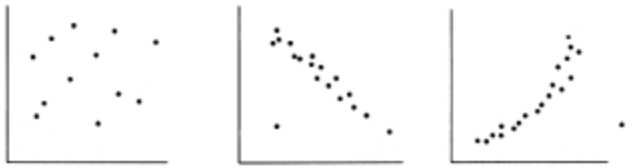
- For the purposes of this exemplar, students will construct graphs by hand. Note also that the choice of scale, or decisions to break the scale on an axis, can have an influence on the appearance of the dispersion. Consider the graphs below, which represent a set of measurements involving foot length and height. The first graph starts both scales at zero; the second breaks the scale on both axes and uses finer scales; the third breaks the scale on the horizontal axis only.







- If students are unfamiliar with interpreting scatter plots, spend some time on simple examples, such as the following:



- The descriptions of the dispersion might make reference to:
  - randomness versus clustering;
  - the nature of the clustering: a line or a curve (linear or non-linear);
  - some description of the line or curve (e.g., points up to the right or points down to the right);
  - the identification of outliers and some explanation for them.

Accommodations

Accommodations that are normally provided in regular classrooms for students with special needs should be provided in the administration of this performance task.

You may wish to review the relevant course profile for specific suggestions for accommodations appropriate for students in special education programs.

Materials and Resources Required

- Student instructions
- Scientific calculators
- Graphing materials: pencils, rulers, erasers, graph paper

Rubric

Introduce the task-specific rubric to the students at least one day before the administration of the task. Review the rubric with the students, allowing ample class time to ensure that each student understands the criteria and the descriptions for achievement at each level.

Some students may perform below level 1. It will be important to note the characteristics of their work in relation to the criteria in the assessment rubric and to provide feedback to help them improve.

Task Instructions

Part I : 75 minutes

- Organize students into groups of three or four.
- Hand out the "Student Instructions" page.*
  Have students read the whole page silently. Answer any questions about the overall activity.
- Give students 5 or 10 minutes to work on Part I. Indicate that each student should keep a record of the questions.
- Facilitate a whole group discussion of the questions that have been generated, including the following:
  What is meant by foot length?
  Who is taller, Henri or Kim?
  Is Kim male or female?
  Who is walking faster? Who is taking more steps per minute?
  Is it reasonable that two people who have the same foot length would have different stride lengths?
- As questions are generated, display them on the overhead. (If you are using the board, you might have one student record them all for you.)

  Note: If key questions, such as the second, third and fifth examples, do not come up, suggest them and add them to the list.

** The student instructions are reproduced on page 47 of this document.*

- Have students work in their groups for about 5 or 10 minutes to begin suggesting answers to the questions. Discuss what makes a good hypothesis.
- Facilitate a whole class discussion for 5 or 10 minutes to share some of the hypothesized answers. Many of the answers to the posed questions will depend on mathematical relationships. For example, "Who is taller, Henri or Kim?" assumes a mathematical relationship, perhaps between stride length and height.
- Draw an example such as this from the discussion and rephrase it as a research problem: Is there a relationship between stride length and height in humans? If so, describe it.
- Ask students to identify other research problems that might arise from the discussion:
  Is there a relationship between foot length and height in humans? If so, describe it.
  Is there a relationship between stride length and foot length in humans? If so, describe it.
  Is there a relationship between stride length and leg length in humans? If so, describe it.
  Is there a relationship between stride length and gender? If so, describe it.

  Note: If students have difficulty recognizing the problems for research, you might lead them to likely problems by focusing on the relationships that arose during the discussions above.

- Once a set of research problems has been posed, identify two or three problems that you will allow for use in Part II. Make this decision with student input, but finalize the choice on the basis of your own judgement of which problems would lead to appropriate investigations.

- Good investigations to consider are:
  – foot length and height
  – leg length and height
  – leg length and stride length
  – height and stride length

  Note: Some students may generate questions and research problems that extend the investigation. For example, students might reflect on the predictive value of the depth of the footprint. Gathering data would then be a more extensive and time-consuming activity. As well, depth of footprint might lead to questions about the relationship to a person's weight, which may be a sensitive issue for some students.

  Encourage students to ask wide-ranging questions and to generate interesting research problems, but keep time lines and sensitivity in mind when selecting problems to approve for Part II.

- For each research problem, facilitate a discussion of the points below. Instruct students to take careful notes, because they will be required to explain the discussion as part of the write-up in Part II.

  The following are the discussion points for each research problem:
  – the variables in the relationship, and the data that would be collected to examine the relationship;
    Discuss independent and dependent variables. Note that the classification of variables may not be obvious. For example, is foot length dependent on height, or is height dependent on foot length? This is especially interesting with Grade 9 students, whose growth and height may not have caught up with their foot length. In this discussion, focus on *students' explaining their reasoning*.

  - factors that might affect the validity of the data and that would be considered in gathering a representative sample of data (e.g., gender, age, height);
    Note that the research problems are phrased in terms of human relationships. Discuss the concept of *representative samples* and question whether, on the basis of the variables identified, the class is a representative sample of humanity. Discuss how a representative sample might be identified.
  - factors that might affect accuracy in measuring (e.g., Should shoes be on or off when measuring height? What is meant by foot length? Is the measurement of one stride sufficient?).
- Using the class as a sample, have the students work in small groups to gather the data identified above.
- Prepare a class summary sheet, and have each student record his or her data (without identification), both in his or her notebook and on the class summary sheet.
- Photocopy the summary sheet for use in the next class.

Part II: 150 minutes (75 minutes × 2)

- Ensure that students are now working as individuals.
- Hand out the student instructions for Part II and have students read all of Part II and the assessment rubric.
- Answer any questions and highlight areas that you feel need explanation or emphasis.
- Review and discuss the rubric.
- Hand out the class summary sheet of data.
- Reinforce the importance of clarity and explanation in students' written work.
- Discuss the meaning of each step of the report and the type of answer that would be involved in each step. (See the descriptions from the section "Prior Knowledge and Skills Required".)
- Students work to complete their written reports (Part II).

Follow-up

After students have handed in their written reports, direct a whole class discussion:

- Ask students to share their results on the various research questions.
- Ask students to share their ideas about other instances in which it would be important to identify a relationship between variables.
- Return student attention to the original scenario involving Henri and Kim. Pose some of the original questions generated in Part I and ask students to answer them now, having examined a relationship in some depth.