

# Teacher Package

## Mathematics Exemplar Task Grade 9 Principles of Mathematics, Academic

### Teacher Package

**Title:** Designing a Perfume Bottle

**Time requirement:** Up to 320 minutes  
Preparation for the task: Up to 110 minutes  
The student task: 210 minutes (70 minutes  $\times$  3)

#### Description of the Task

The Goodsmell perfume producing company has a new line of perfume and is designing a new bottle for it. Because of the expense of the glass required to make the bottle, the surface area must be less than  $150 \text{ cm}^2$ . The company also wants the bottle to hold at least 100 mL of perfume. The design under consideration is in the shape of a cylinder.

Determine the maximum volume possible for a cylindrical bottle that has a total surface area of less than  $150 \text{ cm}^2$ . Determine the volume to the nearest 10 mL. Report the dimensions of the bottle and the corresponding surface area and volume.

#### Final Product

Prepare a written report that includes:

- a clear and complete explanation of the process that you used to solve the problem
- the calculations that you made, presented in an organized fashion

- a rationale for your selection of values (e.g., radius, height)
- **if you used technology, sample calculations of surface area and volume** to demonstrate that you know how to use and manipulate the formulas (e.g., show substitution into formulas as if technology were not used)
- a conclusion that states the maximum volume to the nearest 10 mL, the dimensions rounded to one decimal place, and the total surface area rounded to one decimal place

*Note:* Students do not need to indicate calculator or computer instructions as part of the process of preparing their final reports.

#### Assessment and Evaluation

- You will need to use the task-specific rubric\* to assess and evaluate the student work.
- In order to score the application row of the rubric if the student used technology, evaluate the sample calculations for surface area and volume that the student was asked to provide in the written report.

#### Expectations Addressed in the Exemplar Task

##### Number Sense and Algebra

*Students will:*

1. substitute into and evaluate algebraic expressions involving exponents, to support other topics of the course;
2. communicate solutions to problems in appropriate mathematical forms and justify the reasoning used in solving the problems.

*(continued)*

\* The rubric is reproduced on page 14 of this document.

**Measurement and Geometry***Students will:*

3. determine the optimal values of various measurements through investigations facilitated, where appropriate, by the use of concrete materials, diagrams, and calculators or computer software;
4. solve problems involving the surface area and the volume of three-dimensional objects;
5. identify, through investigation, the effect of varying the dimensions of a rectangular prism or cylinder on the volume or surface area of the object;
6. identify, through investigation, the relationships between the volume and surface area of a given rectangular prism or cylinder;
7. explain the significance of optimal surface area or volume in various applications;
8. solve simple problems, using the formulas for the surface area and the volume of prisms, pyramids, cylinders, cones, and spheres;
9. solve multi-step problems involving the volume and the surface area of prisms, cylinders, pyramids, cones, and spheres.

Note that, although all of the expectations listed will be addressed through instruction relating to the task, student achievement of expectations 7 and 8 will not be assessed in the final product.

**Teacher Instructions****Prior Knowledge and Skills Required**

To complete this task, students are expected to have some knowledge or skills relating to the following:

- the meaning of the concepts of surface area and volume and the units in which surface area and volume are measured;
- the formulas for the surface area and the volume of a cylinder;
- the ability to use the formulas, including the ability to make correct substitutions;
- the effective use of a scientific calculator (e.g., the use of the value for  $\pi$  instead of 3.14);
- rounding to the nearest 10 mL and to one decimal place.

**Accommodations**

Accommodations that are normally provided in regular classrooms for students with special needs should be provided in the administration of this performance task.

You may wish to review the relevant course profile for specific suggestions for accommodations appropriate for students in special education programs.

**Materials and Resources Required**

- Student instructions
- Use of a computer lab, a set of graphing calculators, or a set of scientific calculators

### Preparation

The following are suggestions and instructions about the students' use of technology in completing the task:

- The exemplar task lends itself to the use of a spreadsheet, and spreadsheet use should be encouraged whenever possible.
- If the use of a spreadsheet is not feasible, then students should use scientific calculators, and it is recommended that students work in pairs to share the calculation load.
- If students work in pairs, only one solution per pair should be handed in.
- Regardless of the calculation method used, students must report their calculations.
- If students use technology in completing work that is to be assessed externally, it can be difficult to ascertain whether they understand how to use and manipulate formulas. For this reason, students are asked to provide sample calculations of surface area and volume so that the teacher can see whether students are able to substitute appropriately.

The following are comments about a sample solution of the exemplar task, which has been included in the Teacher Package **for the use of the teacher only**:

- A sample solution for the performance task is provided as the last section of this Teacher Package.
- The sample solution does not represent a model for student answers. It is only meant to show how a student might approach the problem.
- The sample solution was completed by means of a spreadsheet.
- The sample solution represents one strategy for finding a solution; students may use different or more sophisticated strategies.
- To familiarize yourself with the rubric for the exemplar task, you may wish to use the rubric to assess the model solution.

### Please do *not* share the sample solution with students.

Note: Students may provide answers within an acceptable range, depending on the investigation, line of reasoning, or strategy they pursue, or the value of they use.

### Rubric

Introduce the task-specific rubric to the students at least one day before the administration of the task. Review the rubric with the students, allowing ample class time to ensure that each student understands the criteria and the descriptions for achievement at each level.

Some students may perform below level 1. It will be important to note the characteristics of their work in relation to the criteria in the assessment rubric and to provide feedback to help them improve.

### Task Instructions

#### Pre-task Activities: Up to 110 minutes

If you have not yet covered surface area and volume of a cylinder:

- review the meaning of the concepts of surface area and volume and the units in which surface area and volume are measured;
- introduce the information for the surface area and volume of a cylinder;
- demonstrate the use of formulas, including correct substitution, and the effective use of a scientific calculator.

If you have not yet covered optimal value of surface area and volume:

- try a sample problem (e.g., determine the minimum surface area for a cylinder having a volume of 300 mL);
- let students work in pairs, attempting to solve the problem;

- discuss the methods students used to solve the problem (encourage the discussion of many different methods) and the conclusions that they reached;
- discuss the importance of a systematic and efficient method for examining values;
- hand out the assessment rubric that will be used with the actual exemplar task;
- discuss the assessment rubric in conjunction with the students’ solutions to the sample problem just completed.

**The Task: 210 minutes**

- Hand out the student package.
- Read through with students the section “Description of the Exemplar Task”.
- Have the students read the task silently.
- Answer any questions they may have about the activity.
- Remind students about the rubric, and ensure that each student has a copy of it.
- Set students to work on the task, allowing 210 minutes for completion.

**Assessment Instructions**

- Review the rubric with students at the beginning of each day.
- Remind students who have used technology to include in their report the sample calculations for surface area and volume.

**SAMPLE SOLUTION**

**FOR TEACHER USE ONLY!!**

**The Search for Maximum Volume With Total Surface Area Less Than 150 cm<sup>2</sup>**

| r (cm) | h (cm) | TSA (cm <sup>2</sup> ) | V (mL) |
|--------|--------|------------------------|--------|
| 1.00   | 31.83  | 206.28                 | 100.00 |
| 2.00   | 7.96   | 125.13                 |        |
| 3.00   | 3.54   | 123.22                 | ***    |
| 4.00   | 1.99   | 150.53                 |        |
| 5.00   | 1.27   | 197.08                 |        |
| 6.00   | 0.88   | 259.53                 |        |
| 7.00   | 0.65   | 336.45                 |        |
| 8.00   | 0.50   | 427.12                 |        |
| 9.00   | 0.39   | 531.16                 |        |
| 10.00  | 0.32   | 648.32                 |        |

| r (cm) | h (cm) | TSA (cm <sup>2</sup> ) | V (mL) |
|--------|--------|------------------------|--------|
| 1.00   | 63.66  | 406.28                 | 200.00 |
| 2.00   | 15.92  | 225.13                 |        |
| 3.00   | 7.07   | 189.88                 |        |
| 4.00   | 3.98   | 200.53                 |        |
| 5.00   | 2.55   | 237.08                 |        |
| 6.00   | 1.77   | 292.86                 |        |
| 7.00   | 1.30   | 365.02                 |        |
| 8.00   | 0.99   | 452.12                 |        |
| 9.00   | 0.79   | 553.38                 |        |
| 10.00  | 0.64   | 668.32                 |        |

| r (cm) | h (cm) | TSA (cm <sup>2</sup> ) | V (mL) |
|--------|--------|------------------------|--------|
| 1.00   | 47.75  | 306.28                 | 150.00 |
| 2.00   | 11.94  | 175.13                 |        |
| 3.00   | 5.31   | 156.55                 |        |
| 4.00   | 2.98   | 175.53                 |        |
| 5.00   | 1.91   | 217.08                 |        |
| 6.00   | 1.33   | 276.19                 |        |
| 7.00   | 0.97   | 350.73                 |        |
| 8.00   | 0.75   | 439.62                 |        |
| 9.00   | 0.59   | 542.27                 |        |
| 10.00  | 0.48   | 658.32                 |        |

| r (cm) | h (cm) | TSA (cm <sup>2</sup> ) | V (mL) |
|--------|--------|------------------------|--------|
| 1.00   | 41.38  | 266.28                 | 130.00 |
| 2.00   | 10.35  | 155.13                 |        |
| 3.00   | 4.60   | 143.22                 | *****  |
| 4.00   | 2.59   | 165.53                 |        |
| 5.00   | 1.66   | 209.08                 |        |
| 6.00   | 1.15   | 269.53                 |        |
| 7.00   | 0.84   | 345.02                 |        |
| 8.00   | 0.65   | 434.62                 |        |
| 9.00   | 0.51   | 537.83                 |        |
| 10.00  | 0.41   | 654.32                 |        |

| r (cm) | h (cm) | TSA (cm <sup>2</sup> ) | V (mL)          |
|--------|--------|------------------------|-----------------|
| 1.00   | 44.56  | 286.28                 | 140.00          |
| 2.00   | 11.14  | 165.13                 |                 |
| 3.00   | 4.95   | 149.88                 | This one works! |
| 4.00   | 2.79   | 170.53                 |                 |
| 5.00   | 1.78   | 213.08                 |                 |
| 6.00   | 1.24   | 272.86                 |                 |
| 7.00   | 0.91   | 347.88                 |                 |
| 8.00   | 0.70   | 437.12                 |                 |
| 9.00   | 0.55   | 540.05                 |                 |
| 10.00  | 0.45   | 656.32                 |                 |

**My problem solving process:**

I set the volume at 100 mL because the question said that this was the least it could be. Then I chose r values from 1 to 10. I rearranged the volume formula so that I could calculate h for each value of r. Then I substituted each r and h into the total surface area formula. I looked to make sure that a surface area of less than 150 cm<sup>2</sup> was possible.

Then I repeated this for volume = 200 mL. This time, a surface area less than 150 cm<sup>2</sup> was not possible. So then I repeated the calculation for volume = 150 mL. Still not possible.

After that, I tried 130 mL, which worked. And then I tried 140 mL. It also worked. I was supposed to find the maximum volume to the nearest 10 mL. So it must be 140 mL.

**Sample calculations:**

for r = 3 cm, h = 4.95 cm

$$\begin{aligned}
 \text{Surface Area} &= 2\pi r^2 + 2\pi r h & \text{Volume} &= \pi r^2 h \\
 &= 2\pi(3)^2 + 2\pi(3)(4.95) & &= \pi(3)^2(4.95) \\
 &= 56.5 + 93.3 & &= 139.96 - \text{rounded to } 140 \text{ mL} \\
 &= 149.8 \text{ cm}^2 & &
 \end{aligned}$$

**My conclusion is:**

A maximum volume of 140 mL occurs when r = 3.0 cm and h = 5.0 cm (rounded from 4.95 cm). The surface area is 149.8 cm<sup>2</sup>.