

# Designing a Perfume Bottle

## The Task

This task required students to:

- determine the maximum volume possible for a cylindrical bottle having a total surface area of less than  $150 \text{ cm}^2$ ;
- determine the volume of the bottle to the nearest 10 mL;
- report the dimensions of the bottle and the corresponding surface area and volume.

For the specific problem as it was posed to students, see page 37 of this document.

Students recorded the results of their investigation in a report that included an explanation of the process; a rationale for the selection of values (e.g., radius, height); a demonstrated knowledge of how to use and manipulate the formulas for surface area and volume; and a conclusion that stated the maximum volume to the nearest 10 mL, the dimensions rounded to one decimal place, and the total surface area rounded to one decimal place.

## Expectations

This task gave students the opportunity to demonstrate achievement of the following selected expectations from two strands – Number Sense and Algebra, and Measurement and Geometry.

## Number Sense and Algebra

*Students will:*

1. substitute into and evaluate algebraic expressions involving exponents, to support other topics of the course;
2. communicate solutions to problems in appropriate mathematical forms and justify the reasoning used in solving the problems.

## Measurement and Geometry

*Students will:*

3. determine the optimal values of various measurements through investigations facilitated, where appropriate, by the use of concrete materials, diagrams, and calculators or computer software;
4. solve problems involving the surface area and the volume of three-dimensional objects;
5. identify, through investigation, the effect of varying the dimensions of a rectangular prism or cylinder on the volume or surface area of the object;
6. identify, through investigation, the relationships between the volume and surface area of a given rectangular prism or cylinder;
7. explain the significance of optimal surface area or volume in various applications;

8. solve simple problems, using the formulas for the surface area and the volume of prisms, pyramids, cylinders, cones, and spheres;
9. solve multi-step problems involving the volume and the surface area of prisms, cylinders, pyramids, cones, and spheres.

*For information on the process used to prepare students for the task and on the materials and equipment required, see the Teacher Package reproduced on pages 37–41 of this document.*

### **Prior Knowledge and Skills**

To complete this task, students were expected to have some knowledge or skills relating to the following:

- the meaning of the concepts of surface area and volume and the units in which surface area and volume are measured
- the formulas for the surface area and the volume of a cylinder
- the ability to use the formulas, including the ability to make correct substitutions
- the effective use of a scientific calculator (e.g., the use of the value for  $\pi$  instead of 3.14)
- rounding to the nearest 10 mL and to one decimal place

## Task Rubric – Designing a Perfume Bottle

Expectations*	Criteria	Level 1	Level 2	Level 3	Level 4
<b>Knowledge / Understanding</b>					
<b>The student:</b>					
5	– demonstrates an understanding of the effect on surface area and volume of varying dimensions	– demonstrates a limited understanding of the effect on surface area and volume of varying dimensions	– demonstrates some understanding of the effect on surface area and volume of varying dimensions	– demonstrates considerable understanding of the effect on surface area and volume of varying dimensions	– demonstrates a thorough understanding of the effect on surface area and volume of varying dimensions
<b>Thinking / Inquiry / Problem Solving</b>					
<b>The student:</b>					
2, 3, 5, 6,	– selects values in the problem-solving process systematically – demonstrates reasoning in the selection of values – forms accurate conclusions	– selects values in a systematic manner to a limited degree – demonstrates limited reasoning in the selection of values – forms conclusions with limited accuracy	– selects values in a somewhat systematic manner – demonstrates some reasoning in the selection of values – forms conclusions with some accuracy	– selects values in a generally systematic manner – demonstrates considerable reasoning in the selection of values – forms conclusions with considerable accuracy	– selects values in a highly systematic manner – demonstrates a high degree of reasoning in the selection of values – forms conclusions with a high degree of accuracy
<b>Communication</b>					
<b>The student:</b>					
2	– clearly identifies calculations and presents them in a logical, organized way – uses mathematical language, symbols, and units accurately	– identifies and presents calculations with limited clarity – uses language, symbols, and units with limited accuracy	– identifies and presents calculations with some clarity – uses language, symbols, and units with some accuracy	– identifies and presents calculations with considerable clarity – uses language, symbols, and units with considerable accuracy	– identifies and presents calculations with a high degree of clarity – uses language, symbols, and units with a high degree of accuracy
<b>Application</b>					
<b>The student:</b>					
1, 3, 4, 9	– demonstrates skill in the application of concepts and procedures (the formula) when determining the optimal size of the perfume bottle	– demonstrates limited skill in the application of the formula	– demonstrates some skill in the application of the formula	– demonstrates considerable skill in the application of the formula	– demonstrates a high degree of skill in the application of the formula

\* The expectations that correspond to the numbers given in this chart are listed on page 12. Note that, although all of the expectations listed there were addressed through instruction relating to the task, student achievement of expectations 7 and 8 was not assessed in the final product.

*Note:* A student whose overall achievement at the end of a course is below level 1 (that is, below 50%) will not obtain a credit for the course.