Designing a Perfume Bottle Level 1, Sample 1

A

Designing a Perfume Bottle

Problem: The Goodsmell perfume producing company has a new line of perfume and is designing a new fancy bottle for it. Because of the expense of the glass required to make the bottle, the surface area must be less than 150cm2. The company also wants the bottle to hold at least 100mL of perfume. The design under consideration is in the shape of a cylinder. Determine the maximum volume possible for a cylindrical bottle that has a total surface area of less than 150 cm2. Determine the volume to the nearest 10mL. Report the dimensions of the bottle and the corresponding surface area and volume.

Solutions:	Radius (cm2)	Height (cm)	Surface Area (cm2)	Volume (mL)
1)	2.3	8.1	150.313	134.632
2)	2.3	7.9	147.423	131.307
3)	2.5	6.8	146.103	133.535
4)	2.4	7.5	149.308	135.734
5)	2	9.5	144.532	119.396
6)	2	8.8	135.734	110.598
7)	2.5	7	149.245	137.463
8)	2.2	8	141.013	121.658
9)	2	7.9	124.423	99.287
10)	4	1.9	148.302	95.517

The process I used to solve the problem was to begin by preparing a table like the one shown above. I typed the radius and the height into its proper column and then began to type the dimensions into the computer. As I got towards the bottom of the table I began to notice that when I kept the height in between 6.5 and 7.9 the surface area was closer to 150cm2.

B

Conclusion: The maximum volume to the nearest 10mL is 10. 053 mL. The dimension rounded to the nearest decimal place was the height 7.9cm rounded to (.8 mL). The total surface area rounded to one decimal place gave me the answer of 35.183 cm2. Below is the formula I used to draw my conclusions. The numbers underlined in my table are the numbers I used to draw my conclusions.

Sa= 2(pie)r2+ 2(pie)r h
= 2(3.141592654) 2^2 + 2(3.141592654)2 (.8mL)
= 2 (3.141592654) 4 + 2(3.141592654)2 (.8mL)
= 25.13cm + 10.053
= 35.183cm^2

V= (3.141592654)r2h
= (3.141592654) 2^2(.8mL)
= (3.141592654) 4 (.8mL)
= 10.053mL

Teacher's Notes

Knowledge/Understanding

– The student demonstrates a limited understanding of the effect on surface area and volume of varying radius and height.

Thinking/Inquiry/Problem Solving

– The student selects values in a systematic manner to a limited degree, selecting only a small number of decimal values for radius and height.
– The student demonstrates limited reasoning in the selection of values.
– The student forms conclusions with limited accuracy (e.g., states the height but not the radius; describes volume inaccurately).

Communication

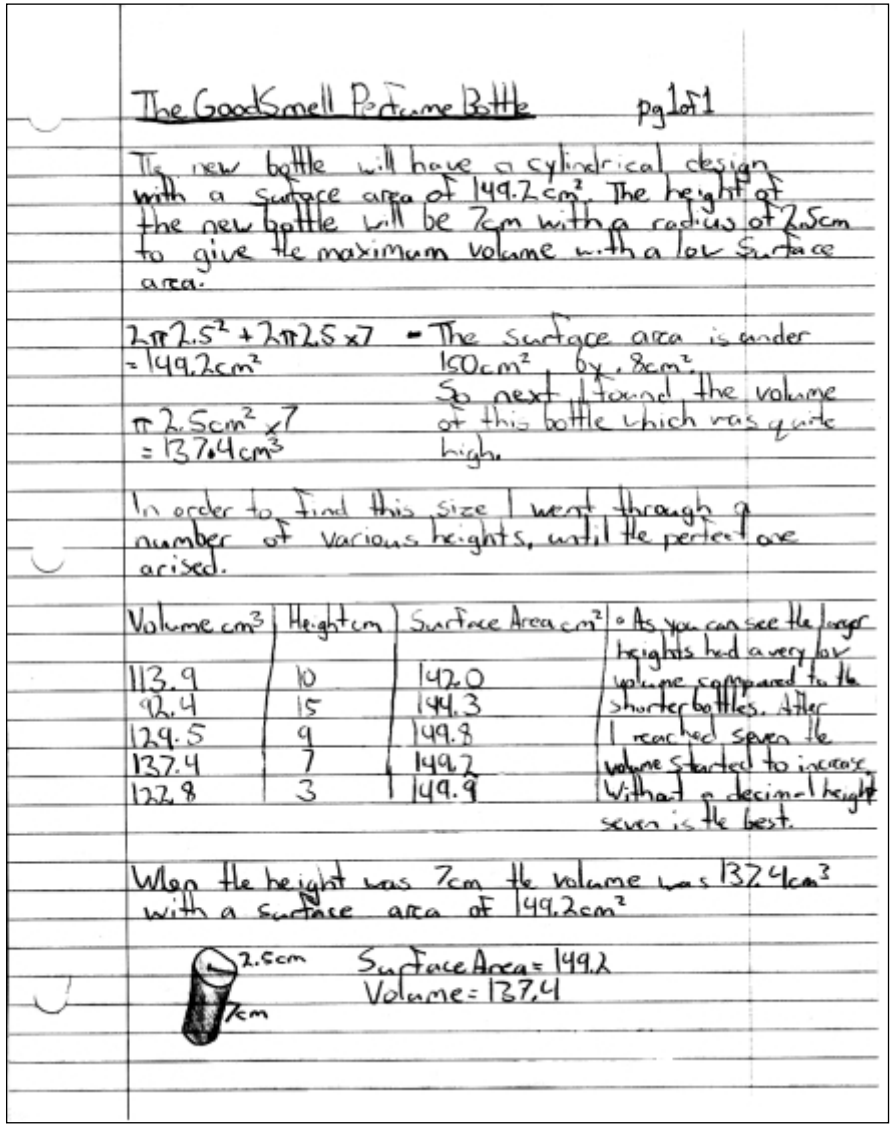
– The student uses units and symbols with limited accuracy (e.g., has units in the chart but does not always use them appropriately in discussion or in the sample calculation).

Application

– The student demonstrates limited skill in the application of the formula (e.g., the surface area calculation contains the value 0.8, which was likely meant to be 8.0, rounded from 7.9).

Comments/Next Steps

– The student's use of decimal values for radius and height shows insight, but radius and height should be investigated in a more systematic manner, specifically by fixing volume or surface area.
– The student could check for accuracy when selecting values to substitute into formulas.
– The student could demonstrate more reasoning in the selection of values.



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The new bottle will have a cylindrical design with a surface area of 149.2 cm². The height of the new bottle will be 7cm with a radius of 2.5cm to give the maximum volume with a low surface area.

$2\pi 2.5^2 + 2\pi 2.5 \times 7$ = 149.2 cm²

$\pi 2.5 cm^2 \times 7$ = 137.4 cm³

The surface area is under 150 cm² by .8 cm². So next I found the volume of this bottle which was quite high.

In order to find this size I went through a number of various heights, until the perfect one arised.

Volume cm³	Height cm	Surface Area cm²
113.9	10	142.0
92.4	15	144.3
129.5	9	149.8
137.4	7	149.2
122.8	3	149.9

As you can see the longer heights had a very low volume compared to the shorter bottles. After I reached seven the volume started to increase. Without a decimal height seven is the best.

When the height was 7cm the volume was 137.4cm³ with a surface area of 149.2cm²

Surface Area = 149.2
Volume = 137.4

Teacher's Notes

Knowledge/Understanding

- The student demonstrates a limited understanding of the effect on surface area and volume of varying dimensions (e.g., by fixing the radius at 2.5 cm and then using a limited selection of heights).

Thinking/Inquiry/Problem Solving

- The student selects values in a systematic manner only to a limited degree (e.g., the radius is fixed but the height selections are random).
- The student forms conclusions with limited accuracy.
- The student demonstrates limited reasoning in the selection of values (e.g., focuses on surface area without regard to maximizing the value of the volume).

Communication

- The student presents calculations with limited clarity (e.g., shows the calculation steps but does not identify the formulas in the sample calculation).
- The student's use of language and symbols has limited accuracy.
- In the student's explanation, the radius appears to have been set at 2.5 cm; however, in the chart, calculations based on that radius do not work out. For example, for radius 2.5 cm and height 9 cm:

  $V = \pi r^2 h$ $\doteq$ 176.7 cm³ $\neq$ 129.5 cm³ (student solution)

  $SA = 2\pi r^2 + 2\pi rh$ $\doteq$ 180.6 cm² $\neq$ 149.8 cm² (student solution)

Application

- The student demonstrates limited skill in the application of the formula (e.g., does not provide the general form of the formula but makes the correct substitutions in the samples).

Comments/Next Steps

- The student could improve the overall solution by using a more systematic approach involving both surface area and volume with a more in-depth investigation.
- The student could clearly identify concluding statements with a title or add the word "conclusion" in the statements.