Caterpillar Growth Problem LOW LEVEL 1

A

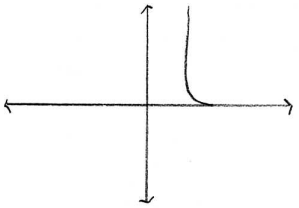
Pleasantville's Caterpillar Problem

Exemplar

Option #1

$y = ca^x$

$y = 80\,000\,(1/2)^{x/4}$



# of Years	Population
0	80 000
1	67 272
2	56 569
3	47 568
4	40 000
5	33 636
6	28 284
7	23 784
8	20 000
9	16 818
10	14 142
11	11 892
12	10 000
13	8409
14	7071.1
15	5946
↓	↓
50	13.811

Pros = The caterpillar population will be cut in half every four years. The chemical spray will begin to act immediately.

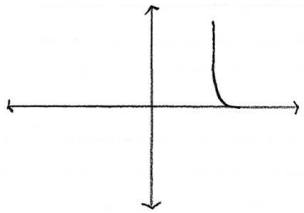
Cons = The possible harm to humans and other organisms not intended to be controlled by the spray is unknown. The spray is going to take over 15 years to eliminate the caterpillers and in that time they would have died out anyways. Also, the chemical spray may be expensive.

B

Option #2

$y = ca^x$

$y = 80\,000\,(1/2)^{x/2}$



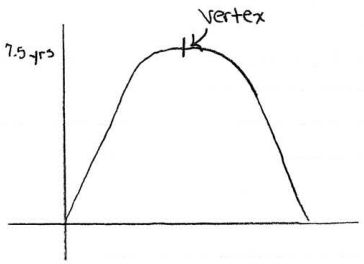
# of Years	Population
0	80 000
1	80 000
2	56 569
3	40 000
4	28 284
5	20 000
6	14 142
7	10 000
8	707.1
9	5000
10	3535.5
11	2500
12	1767.8
13	1250
14	883.88
15	625
↓	↓
50	0.00337

Pros = The introduction of the fly population works faster than the chemical spray. Flys are not known to have any unwanted side affects to humans.

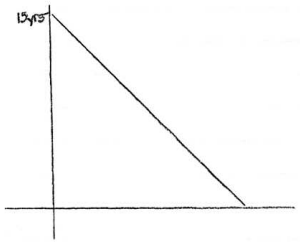
Cons = The fly population may also overpopulate. The flys might harm species or the environment unintentionally. It will take a year to come into affect.

C

Option #3



OR



~ The caterpillar population could reach its maximum population (vertex) and then begin to decline. The environment would not be able to sustain the caterpillar population.

~ The caterpillar population could simply decrease over 15 years until the population is zero. This would be a linear regression.

Pros = Wouldn't cost anything. The caterpillars will die out within 15 years and it is going to take longer than that with the fly population or chemical spray.

Cons = If the caterpillars are left alone they may destroy all of the parks and forests by the time they die out.

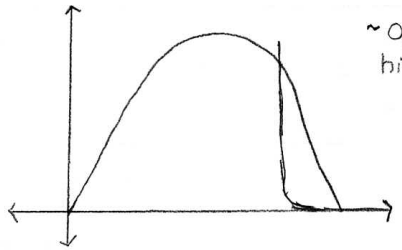
D

Option #4

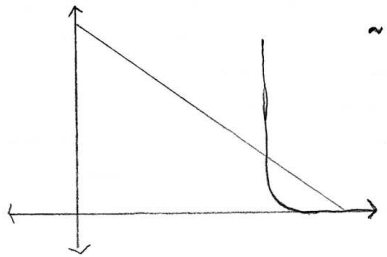
→ Combination of option # 2 and # 3.

Pros = The caterpillar population can be controlled so that they will not completely destroy the parks and forests.

Cons = Although to a lesser degree, a fly population will be introduced. Therefore, it is still a concern that the flys will overpopulate to excess amounts, or end up destroying the vegitation.



~ Option # 2 and 3 if caterpillars hit maximum population.



~ Option # 2 and 3 if the caterpillars steadily decrease over 15 years.

LOW LEVEL 1

E

Exemplar - Course of Action

Through my analysis of the caterpillar population of the city of Pleasantville, I have formulated a course of action that will control the species. Within 15 years the caterpillar population is expected to die out. Although the obvious course of action, I believe that other factors must also contribute to the control of the caterpillars. After 15 years the caterpillars may have destroyed all of Pleasantville's beautiful parks and forests.

Because of this, I propose to not only let the caterpillar population die out, but to also introduce a fly population to reduce the caterpillar species. Although the fly population will take a year to come into affect, it is still more effective than chemical spray "X".

The fly population also poses some risks. Although flies are not known to cause damaging side affects such as the chemical spray, they do have the potential to overpopulate. This in itself would cause another insect infestation that could harm the vegetation and wildlife of Pleasantville, and would need to be controlled.

With this in mind, I propose to introduce a small fly population to the city of Pleasantville. I have determined a whole fly population is not needed. Enough flies to control the caterpillars should be introduced so that the caterpillars do not completely destroy the parks and forests. Within 15 years the caterpillars will become extinct in Pleasantville and the fly population will have controlled the caterpillars enough so that they do not destroy all of the beautiful parks and forests. This also lessens the chance of the flies overpopulating and causing another insect infestation.

In conclusion, I believe that a small fly population should be introduced to the city of Pleasantville. Within 15 years, the caterpillars should die out, and the flies will have prevented the caterpillars from causing irreversible damage to Pleasantville's beautiful parks and forests. Hopefully there will be no undesirable side affects to my course of action, such as the overpopulation of the flies, or not enough flies to control the caterpillars appetite for Pleasantville's parks and forests. By combining option #2 and option #3 I hope to have assured the preservation of Pleasantville's beautiful forests and parks for countless future generations to enjoy.

Teacher's Notes

Knowledge and Understanding

– The student uses mathematical models with limited accuracy. For option 1, the student's population values are accurate, including the initial caterpillar population of 80 000. However, for option 2, he or she does not realize that the population increases to 160 000 by the time the flies take effect. The table and the equation are consistent for option 1, but not for option 2. For options 1 and 2, the sketches of the graphs do not accurately represent the tables of values or the equations. For each of options 3 and 4, the student provides sketches of two different graphs, but he or she does not support them with tables or equations.

Thinking

– The student analyses the advantages and disadvantages of the four options with limited effectiveness. He or she describes "Pros" and "Cons" for each option and makes some valid points (e.g., "The possible harm to humans and other orgainisms" for option 1; "the caterpillars… may destroy all of the parks and forests by the time they die out" for option 3). However, some of the information is not original (e.g., the only "Pros" indicated for option 1 are given in the task) or is questionable (e.g., "The introduction of the fly population works faster than the chemical spray").

Communication

– The student communicates information in tables and graphs with very limited clarity. He or she includes tables that show population decreases for options 1 and 2. No tables are provided for options 3 and 4. All graphs are small sketches that include no titles or scales. The graphs for options 1, 2, and 4 include no labels. For option 3, the only labels are times (i.e., "7.5 yrs", "15yrs"), but they are on the wrong axis.

– The student integrates text and mathematical forms with limited effectiveness. There are brief descriptions of the graphs for options 3 and 4. However, for options 1 and 2, the student makes no reference to the equations, tables, or graphs in the text, aside from stating that "The spray is going to take over 15 years to eliminate the caterpillers" for option 1. The report lacks an introduction.

Application

– The student formulates mathematical models with limited effectiveness. The equation for option 1 is correct, but the equation for option 2 is not. The report does not include a mathematical model for the initial exponential growth in the caterpillar population. The student does not provide quantitative models for options 3 and 4. He or she describes option 4 as a "Combination of option # 2 and # 3". However, the graphs for option 4 repeat the graphs for options 2 and 3, rather than showing a new combination of these two options.
– The student recommends and justifies a course of action with limited effectiveness. He or she proposes a combination of options 2 and 3. The stage at which the flies are to be introduced is not clearly indicated, so it is unclear if this combination is the same as that shown in the graphs for option 4. The idea of introducing "a small fly population" because "a whole fly population is not needed" is not well supported.

Comments

This work is representative of a low level-1 performance. The student demonstrates a limited degree of achievement of the expectations in the Knowledge and Understanding, Thinking, and Application categories of knowledge and skills. The student also demonstrates a limited degree of achievement with respect to one criterion in the Communication category. However, with respect to the other criterion in the Communication category, the student demonstrates a very limited degree of achievement.

Next Steps

In order to improve his or her performance, the student needs to:

- calculate all population data accurately;
- include all necessary tables, graphs, and equations;
- ensure consistency between the table, graph, and equation(s) for each option;
- analyse the advantages and disadvantages of the four options more fully;
- plot accurate, fully labelled graphs;
- use more text to make meaningful references to graphs, tables, and equations;
- include an introduction to the report;
- formulate appropriate mathematical models for all four options and for the initial exponential growth in the caterpillar population;
- recommend a clearer course of action, and provide a detailed justification.