Mathematics Exemplar Task
Advanced Functions and Introductory Calculus, Grade 12, University Preparation (MCB4U)

Teacher Package

Title: Lifeguard Procedures Report

Time Requirement: 4–5 periods of 75 minutes each

Expectations Addressed in the Exemplar Task
This task gives students the opportunity to demonstrate achievement of all or part of each of the following selected expectations. Expectations 1, 2, and 4–6 are from the Derivatives and Applications strand of the course. Expectation 3 is from the Underlying Concepts of Calculus strand.

Students will:

1. determine the key features of a mathematical model of an application drawn from the natural or social sciences, using the techniques of differential calculus;
2. pose questions related to an application and answer them by analysing mathematical models, using the techniques of differential calculus;
3. pose problems and formulate hypotheses regarding rates of change within applications drawn from the natural and social sciences;
4. communicate findings clearly and concisely, using an effective integration of essay and mathematical forms;
5. solve optimization problems involving polynomial and rational functions;
6. compare the key features of a mathematical model with the features of the application it represents.

Description of the Task
Present the following scenario and instructions to students:

As a mathematician, you have been hired by the Department of Parks and Recreation to investigate lifeguard procedures. In particular, you are to determine the best route for a lifeguard to take from the edge of a pool to a swimmer in trouble. You will produce a report summarizing your analysis and recommendations. The intent of this report is to provide the Department of Parks and Recreation with safety recommendations that it can implement across the entire region.

Final Product
Each student will produce a project report that includes:

- explicitly stated simplifying assumptions;
- a series of scenarios;
- relevant hypotheses, and confirmation or repudiation of these hypotheses;
- all calculations and analysis;
- appropriate visuals (e.g., diagrams, graphs);
- recommendations with supporting justification.

Assessment and Evaluation
The final draft of the project report will be assessed and evaluated using the task-specific rubric provided.* Introduce the task-specific rubric to the students when you introduce the task. Review the rubric with the students and ensure that each student understands the criteria and the descriptions for achievement at each level. Allow ample time for a thorough reading and discussion of the assessment criteria outlined in the rubric.

Some students may perform below level 1. Although the rubric does not include descriptions of achievement below level 1, the characteristics of these students' work should be reviewed in relation to the criteria outlined in the rubric.

Teacher Instructions

Prior Knowledge and Skills
To complete this task, students are expected to have some experience in, or some knowledge and skills relating to, the following:

- the formula $time = \frac{distance}{speed}$
- writing and testing hypotheses
- interpreting the key features of graphs
- finding local extrema using calculus
- basic rules for derivatives (e.g., sum, power, and chain rules)
- optimization problems using calculus
- the Pythagorean theorem
- the trigonometry of right triangles
- an appropriate structure for writing a report, as detailed in the Final Product section of the task

Accommodations
Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of this performance task.

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The rubric is reproduced on pages 9–10 of this document.

Materials and Resources

- graph paper
- calculators
- graphing calculators, graphing software, or spreadsheets (if available)

Students may use the *The Geometer's Sketchpad®* (if available) to explore the problem. The exploration is not required to complete this performance task, but would enrich students' understanding of the problem and its possible solutions.

Plagiarism

It is important that you discuss copyright issues with your students. Copyright applies to text and visual materials taken from both the Internet and print sources. Plagiarism is defined as "using the work (or part of it) of another person and claiming it as your own".[1]

Task Instructions

Depending on the availability of computers in your school, you may wish to have students do some of the work at home.

Day 1

- Read and discuss the task and the task rubric with the class.
- Brainstorm alternative approaches to the problem with the class. Focus on such criteria as:
  - Does the best path involve minimum distance or minimum time?
  - What is the effect of the different speeds for swimming versus running?
  - Is running and swimming better than other possible approaches (e.g., throwing a rope)?
- Ask students to identify simplifying assumptions that result in a mathematical model of this situation. Among the assumptions involved in the use of constant speeds of running and swimming are:
  - Acceleration at the start of a run, running on a wet deck, and running around corners of the pool have no effect on running speed.
  - The dive portion of the swim has no effect on swimming speed.
  - The lifeguard's route is unobstructed by other swimmers.
- Initiate a brief discussion relating the precision of calculations to physical models (e.g., ask if computing minimum times to several decimal places is useful).
- Have students work through the two scenarios in Appendix A: Practice Worksheet #1. (This worksheet will **not** be submitted as part of the report.) For each scenario, the students need to consider three options:
  - Swim directly to the swimmer.
  - Run to a point directly perpendicular to the swimmer, and then swim.
  - Run to some other point on the edge of the pool, and then swim.

1. Canadian Intellectual Property Office, Industry Canada, *A Guide to Copyrights* (Hull, Quebec: Canadian Intellectual Property Office, Industry Canada, 2000) p. 20.

- **Point out that, for the third option in each scenario in Appendix A, *x* is the horizontal distance from the entry point to the point on the edge of the pool perpendicular to the swimmer.**
- For each of the two scenarios, have students predict and then identify the best route.
- Provide students with assistance as necessary for the third option, i.e., in setting up an appropriate equation and applying calculus to find the location of the entry point and the minimum time.

Day 2
- Work with students to complete Appendix B: Practice Worksheet #2. (This worksheet will **not** be submitted as part of the report.)
- Ask students the open-ended question: "Are you sure that you found the route to reach the swimmer in the minimum time?"
- Review the task requirements and the description of the final product with the students.
- Establish any timelines that you consider appropriate for report writing.
- Have students begin their reports.

Day 3
- Have students continue to work on their reports, including all necessary assumptions, calculations, diagrams, and graphs.
- Circulate and provide assistance, where necessary.
- Intervene with short review or clarification sessions, if appropriate.

Day 4
- Have students continue to work on their reports.
- Encourage students to expand on their recommendations (e.g., the edges of the pool deck could be colour-coded to identify where a lifeguard should dive into the pool, depending on the location of the swimmer).
- Remind students to justify their recommendations to the Department of Parks and Recreation.

Day 5
- Have students complete their reports, as outlined in the Final Product section.
- **Allow sufficient time for students to edit and proofread their work before submission.**
- Allow students time to check their reports for completeness.

List of Appendices

Appendix A: Practice Worksheet 1

Appendix B: Practice Worksheet 2

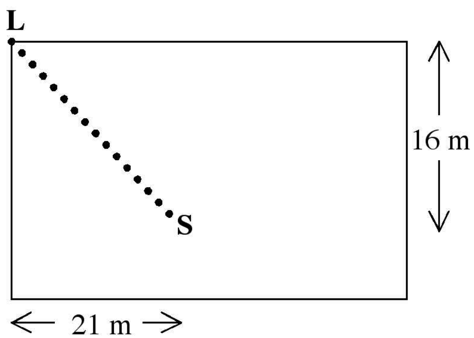
Appendix A: Practice Worksheet 1

Running speed (on edge of pool) = 5 m/s
Swimming speed (in the water) = 3 m/s
Length of pool = 50 m
Width of pool = 20 m
Note that the following diagrams are not to scale.

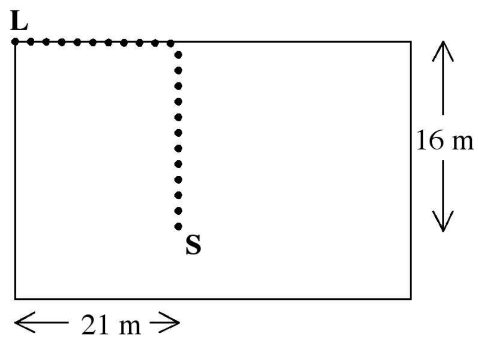
L (Lifeguard)
S (Swimmer)

SCENARIO (1)

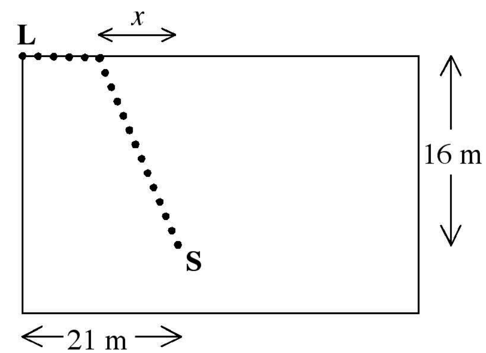
Option 1



Option 2

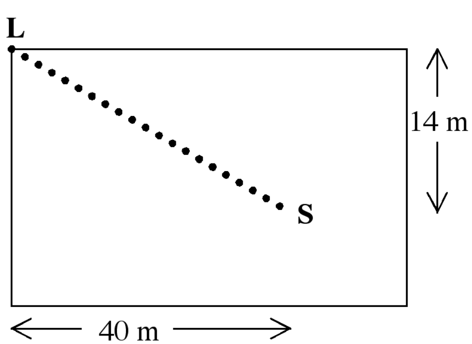


Option 3

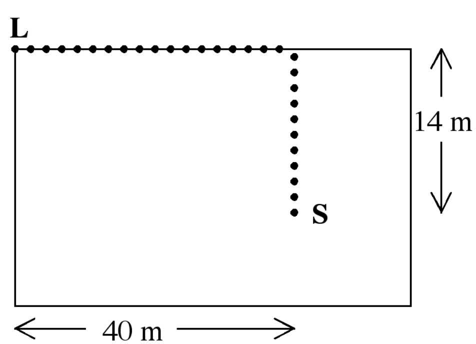


SCENARIO (2)

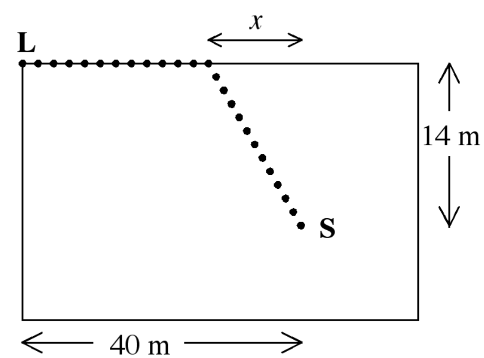
Option 1



Option 2



Option 3

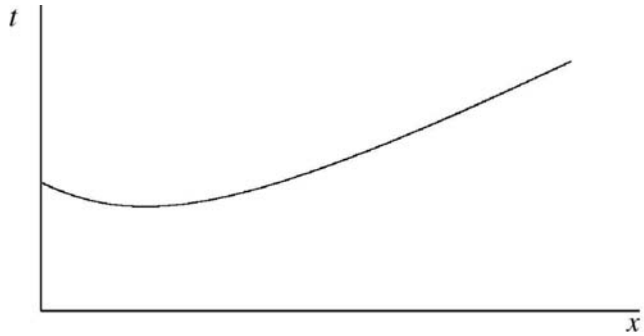


Appendix B: Practice Worksheet 2

Graph 1

Equation: $t = \frac{45 - x}{5} + \frac{\sqrt{x^2 + 16^2}}{3}$

where x is the horizontal distance of the swimming portion (as in Appendix A), and t is the total time to travel the indicated route.



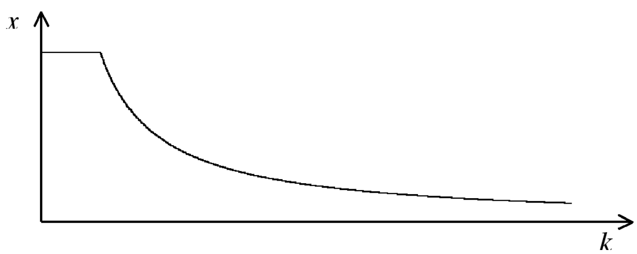
Note: The entire domain of the graph is shown, i.e., the graph does not continue in either direction.

Questions for Graph 1:

1. Compute the t-intercept and explain its physical significance.
2. Why does the graph end at $x = 45$ m?
3. Determine the coordinates of the minimum, and explain their physical significance.
4. Draw and label a diagram to show the swimmer's exact position in the pool.

Graph 2

Note: For this graph, k is the ratio of running speed to swimming speed.



Questions for Graph 2:

1. Why is the graph horizontal between $k = 0$ and $k = 1$?
2. Describe in words the behaviour of the graph as k increases beyond 1.
3. Will the curve ever touch the k-axis? Justify your answer.