Lifeguard Procedures Report LOW LEVEL 4

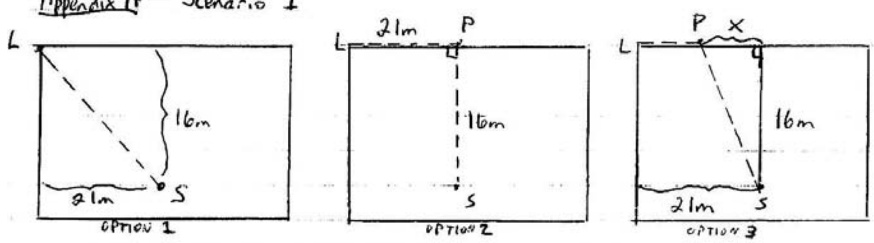
A

Lifeguard Procedures

A Series of Assumptions

Assume such factors as weather, or other extraneous factors will not effect the result.
Assume the swimmer is out of reach of the lifeguard, otherwise the lifeguard could just pull the swimmer out and calculus would be unnecessary.

Appendix A – Scenario 1



Assume running speed is constant at 5m/s.
Assume swimming speed is constant at 3m/s.
L = Lifeguard S = Swimmer. Assume swimmer's position is constant.
My hypothesis is that option 3 is the fastest route.

Option 1

Use Pythagorean Theorum.

$\sqrt{16^2+21^2} = LS$

$LS = 26.4m$ let t = time.

$t = \frac{distance}{speed} = \frac{26.4m}{3m/s} = 8.8s$

B

Option 2

$LP = 21m$ $PS = 16m$

$t_{LP} = \frac{21m}{5m/s}$ $t_{PS} = \frac{16m}{3m/s}$

$= 4.2s$ $= 5.33s$

$t_{TOTAL} = t_{LP} + t_{PS}$
$= 4.2s + 5.33s$
$= 9.53s$

Option 3

Let $t = \frac{21-x}{5} + \frac{\sqrt{x^2+16^2}}{3}$ be the function that represents the time for the lifeguard to reach the swimmer for different values of x.

$t' = -\frac{1}{5} + \frac{1}{6}(x^2+16^2)^{-\frac{1}{2}} \cdot 2x$

$t' = \frac{x}{3\sqrt{x^2+256}} - \frac{1}{5}$

set $t'=0$ to find the value of x that would minimize time.

$0 = \frac{x}{3\sqrt{x^2+256}} - \frac{1}{5}$

$\frac{1}{5} = \frac{x}{3\sqrt{x^2+256}}$

$3\sqrt{x^2+256} = 5x$

$x^2+256 = (\frac{5}{3}x)^2$

$x^2+256 = \frac{25}{9}x^2$

$256 = \frac{16}{9}x^2$

$x^2 = 144$

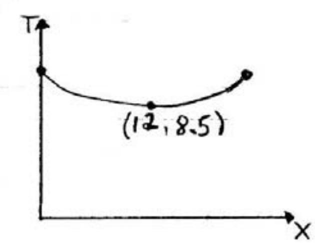
$x = 12$

sub $x=12$ into T.

$T = \frac{21-12}{5} + \frac{\sqrt{144+256}}{3}$

$T = \frac{9}{5} + \frac{\sqrt{400}}{3}$

$T = 8.467s$



C

Therefore for a set swimmer location, a running speed of 5m/s and a swimming speed of 3m/s, option 3 with an x value of 12m is the fastest route to reach the swimmer.

However, the swimmer could be at any location.

Let swimmer be at (a, b), but swimming and running speeds remain constant.

$$t = \frac{a-x}{5} + \frac{\sqrt{x^2+b^2}}{3}$$

using the previous function of t and t' as a model:

$$t' = \frac{x}{3\sqrt{x^2+b^2}} - \frac{1}{5}$$

set $t' = 0$

$$\frac{1}{5} = \frac{x}{3\sqrt{x^2+b^2}}$$

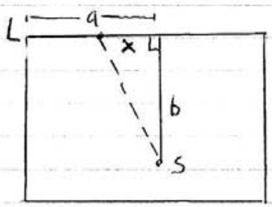
$$3\sqrt{x^2+b^2} = 5x$$

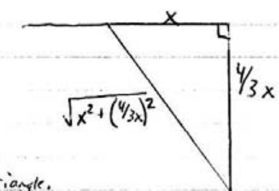
$$x^2+b^2 = \frac{25}{9}x^2$$

$$b^2 = \frac{16}{9}x^2$$

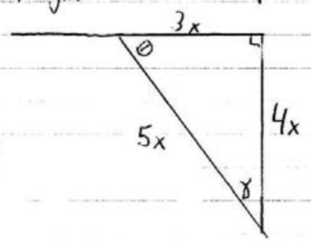
$$b = \frac{4}{3}x$$

$$x = \frac{3}{4}b$$





Multiply each side of the Δ to acheive a 3:4:5 triangle.



$$\sin\theta = 4/5$$

$$\theta = \sin^{-1}(4/5)$$

$$\theta = 53.13^\circ$$

$$180^\circ - 53.13^\circ = 126.87^\circ$$

D

Therefore for any swimmer location, and running speed of 5m/s and a swimming speed of 3m/s, the fastest possible route to the swimmer is for the lifeguard to run until they form an angle of 127° (approx.) with the swimmer, and a point on the edge perpendicular to the swimmer and then swim the rest of the way.

There are some exceptions. If the swimmer is really close to the lifeguard, no running is needed. If the swimmer is already perpendicular, or at an angle of less than 127°, then there is no sense in running away. Just swim the whole way.

Also, some lifeguards have different running and swimming speeds.

$$t = \frac{a-x}{r} + \frac{\sqrt{x^2+b^2}}{s}$$

$$t' = \frac{x}{s\sqrt{x^2+b^2}} - \frac{1}{r}$$

set $t' = 0$

$$\frac{1}{r} = \frac{x}{s\sqrt{x^2+b^2}}$$

$$s\sqrt{x^2+b^2} = rx$$

$$x^2+b^2 = \frac{r^2x^2}{s^2}$$

$$s^2x^2+s^2b^2 = r^2x^2$$

$$s^2b^2 = x^2(r^2-s^2)$$

$$x^2 = \frac{s^2b^2}{r^2-s^2}$$

$$x = \frac{sb}{\sqrt{r^2-s^2}}$$

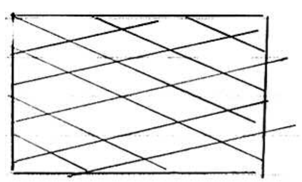
$\therefore r > s$. This makes sense because one's running speed is always greater than one's swimming speed.

LOW LEVEL 4

E

However, taking into account the different running and swimming speeds of each individual lifeguard in every situation would be far too complex to effectively implement at the pool.

Therefore, assuming that 5 m/s and 3m/s are reasonable running and swimming speeds, respectively, the best solution is to always run to the 127° angle mark and then jump in and swim. Overall, it is up to the lifeguard's discretion. I suggest the pool have lines painted on the deck, and across the bottom of the pool.



* diagram not to scale.

Thus, the lifeguard could estimate at which line corresponds to the drowning guy, and jump in at that line.

Teacher's Notes

Knowledge and Understanding

– The student uses a mathematical model with considerable effectiveness. He or she includes simplifying assumptions, but they are too brief and too vague (e.g., "weather, or other extraneous factors will not effect the result"). The student forms the derivative and solves the resulting equations accurately and efficiently.

Thinking

– The student interprets the solutions to the equations with a high degree of effectiveness. For the speeds given in Appendix A, the student shows that $x = \frac{3}{4}b$ and that the angle between the running and swimming directions is 127°. The student realizes that this angle may not apply for some locations of the lifeguard (e.g., "If the swimmer is already perpendicular, or at an angle of less than 127°, then there is no sense in running away. Just swim the whole way").

– The student formulates and tests hypotheses with a high degree of effectiveness. In showing the options for scenario 1, he or she states the hypothesis that "option 3 is the fastest route." The student confirms this hypothesis for scenario 1. The student also considers all scenarios by writing the derivative and solving the resulting equation for the general case, with distances and speeds as variables.

Communication

– The student communicates information in diagrams or graphs with considerable clarity. He or she includes clearly labelled diagrams showing the three options for scenario 1, as well as a general case for option 3. The student also includes a diagram of the pool markings described in the recommendations, but it lacks details, such as distances, angles, and lifeguard location(s). The report includes a graph showing data obtained for scenario 1. The coordinates of the minimum point are labelled, but those of the endpoints are not.

– The student integrates text and mathematical forms with a high degree of effectiveness. The report flows well, and the text makes it very easy to follow the student's mathematical thinking (e.g., "set t′ = 0 to find the value of x that would minimize time"). The report lacks an effective introduction, but overall the writing is clear.

Application

– The student uses formulas with a high degree of effectiveness. He or she chooses appropriate formulas when considering the three options, and correctly writes general formulas with distances, and then with distances and speeds, as variables. The student substitutes appropriate values into the formulas, i.e., distance and speed values from Appendix A. He or she correctly uses a trigonometric formula to find the measure of the angle between the running and swimming directions.

– The student recommends and justifies a course of action with considerable effectiveness. He or she suggests painting lines on the pool to show the 127° angle. The student considers the issue of different speeds from those given in Appendix A, but he or she decides that "taking into account the different running and swimming speeds of each individual lifeguard in every situation would be far too complex to effectively implement at the pool". The statement that "Overall, it is up to the lifeguard's discretion" is vague.

Comments

This work is representative of a low level-4 performance. The student demonstrates a high degree of achievement of the expectations in the Thinking category of knowledge and skills. The student also demonstrates a high degree of achievement with respect to one criterion in the Communication category and one criterion in the Application category. However, in the Knowledge and Understanding category and in one criterion in each of the Communication and Application categories, the student demonstrates a considerable degree of achievement – i.e., achievement that is more characteristic of level 3.

Next Steps

In order to improve his or her performance the student needs to:

- include a more detailed list of simplifying assumptions;
- label all diagrams and graphs fully;
- add an effective introduction to the report;
- allow for different speeds in the recommendations.