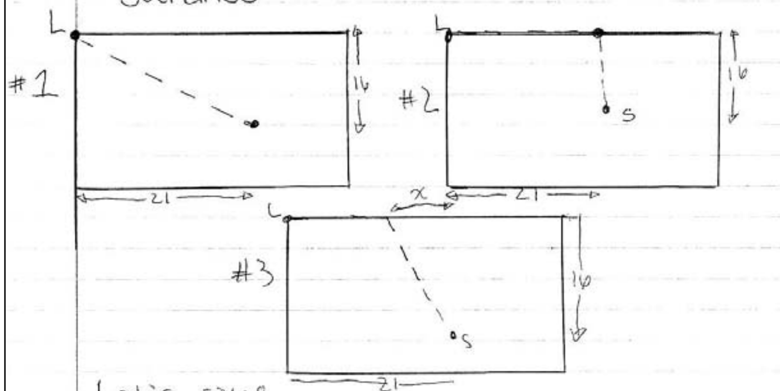
A

Exemplar

Scenarios



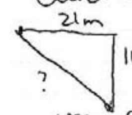
Let's say:
The running speed = 5m/s
The swimming speed = 3m/s

- There is no objects in the course of the swimmer and lifeguard for the rescue.
- The water is calm
- The deck is significantly dry to not diminish the running speed
- There are no hazards preventing the runner from getting to the swimmer
- The life guard is in top physical form
- Swimmers maintain same position

Purpose of these three scenarios is to determine which one minimizes the amount of time taken to get to the swimmer.

B

Scenario #1



use pythagorean theorm

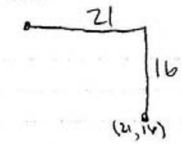
$C^2 = 21^2 + 16^2$

$c = 26.4m$

$T = \frac{26.4m}{3m/s}$

$T = 8.8s$

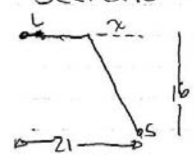
Scenario #2



$T = \frac{21m}{5m/s} + \frac{16m}{3m/s} =$

$T = 7.5s$

Scenario #3



must find equation

$T = \frac{21-x}{5} + \frac{\sqrt{x^2+16^2}}{3} \to (x^2+16^2)^{\frac{1}{2}}$

must find derivative to minimize result

$T' = \frac{(-1)(5) - (21-x)(0)}{25}$

$T' = -\frac{1}{5}$

$T' = \frac{\frac{1}{2}(x^2+16^2)^{-\frac{1}{2}} \cdot 2x(3) - (x^2+16^2)^{\frac{1}{2}}(0)}{9}$

$T' = \frac{x(x^2+16^2)^{-\frac{1}{2}}}{3}$

$T' = -\frac{1}{5} + \frac{x(x^2+16^2)^{-\frac{1}{2}}}{3}$

Now set $T' = 0$ to find x

$\frac{x}{3\sqrt{x^2+16^2}} - \frac{1}{5} = 0$

$\frac{x}{3\sqrt{x^2+16^2}} = \frac{1}{5}$

LOW LEVEL 3

C

$5x = 3\sqrt{x^2+256}$

$\frac{5}{3}x = \sqrt{x^2+256}$

$\frac{25x^2}{9} = x^2+256$

$\frac{16x^2}{9} = 256$

$16x^2 = 2304$

$x = 12$

Now need to sub X into original equation to solve for T

$T = \frac{21-12}{5} + \frac{\sqrt{12^2+256}}{3}$

$T = 8.47s$

End conclusion

Scenario # 1 = 8.8s

scenario #2 = 9.5s

Scenario #3 = 8.5s

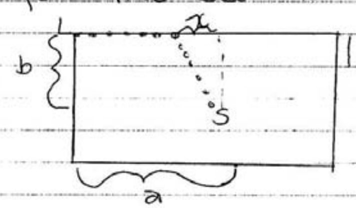
Therefore further analysis of scenario#3 should be completed to determine precise values of swim time, run time and at what angle respectively.

In order to do so we must determine a general formula for the scenario

It being:

General Formula

$T = \frac{a-x}{r} + \frac{\sqrt{x^2+b^2}}{s}$



D

In order to properly use this equation we must find derivative because this will then tell us the minimum time value

$T' = \frac{a-x}{5} + \frac{\sqrt{x^2+b^2}}{3}$

Plug in 5m/s for running and 3m/s for swimming because the is the approx. value for an average ratio of swimming/running for someone like myself.

$T' = -\frac{1}{5} + \frac{x}{3\sqrt{x^2+b^2}}$

Set equal to zero

$\frac{1}{5} = \frac{x}{3\sqrt{x^2+b^2}}$

$3\sqrt{x^2+b^2} = 5x$

$\sqrt{x^2+b^2} = \frac{5x}{3}$

$x^2+b^2 = \frac{25x^2}{9}$

$b^2 = \frac{25x^2}{9} - x^2$

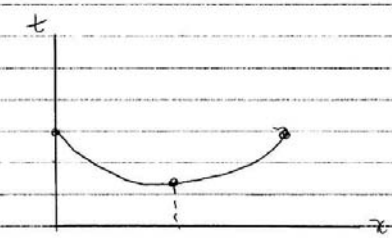
$b^2 = \frac{25x^2}{9} - \frac{9x^2}{9}$

$b^2 = \frac{16x^2}{9}$

$b = \frac{4}{3}x$

$3b = 4x$

$\boxed{x = \frac{3b}{4}}$



This value indicates the fatest way to get to the victim is when x in terms of b is $\frac{3}{4}$ in length. Minimized value! It is evident it is a minimized value by looking at the graph of the derivative above.

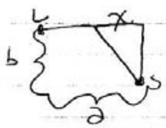
E

The next step:

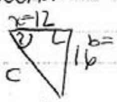
To determine at what angle the lifeguard should jump in at because that again provides information of the fastest way to get to the lifeguard.

This angle is determined when the run speed is 5m/s & swim speed is 3m/s.

We know that x = 3b so therefore this can let us find the angle.



Scenario # 1



use pythagorean theorm

$c^2 = 12^2 + 16^2$

$c = 20$

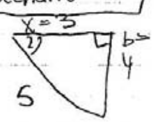
Use sine Law

$\frac{20}{\sin 90} = \frac{16}{\sin bL}$ $\quad \frac{a}{\sin a} = b$

$\sin bL = \sin^{-1}\left(\frac{16 \sin 90}{20}\right)$

$bL = 53°$

$\boxed{\text{Scenario } \# 2}$

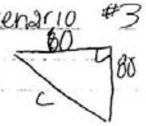


Angle

$\frac{5}{\sin 90} = \frac{4}{\sin bL}$

$bL = 53°$

Scenario #3



$c = 100$

$\frac{100}{\sin 90} = \frac{80}{\sin \angle b}$

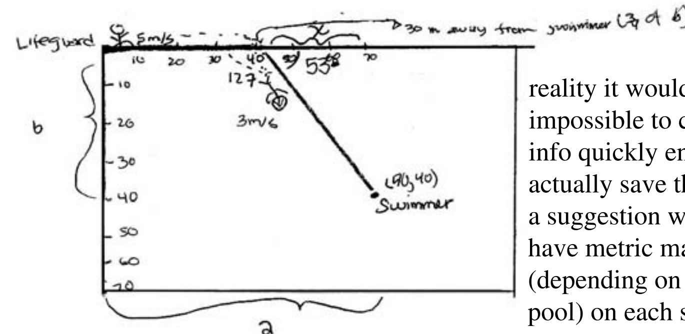
$\angle b = \sin^{-1}\left(\frac{80 \sin 90}{100}\right)$

$\angle b = 53°$

The pattern is at anytime x is $\frac{3}{4}$ of b the angle is at 53°. This saying this is the exact point where the swimmer should jump.

F

With all recommendations considered the swimming speed being 3m/s and the running speed being 5m/s and the swimmer being at a fixed (not moving) speed and no hazards or obstacles preventing the lifeguard to directly and efficeently save the swimmer it is found through a general equation the point at which the lifeguard should jump in is when side b of the pool (vertical length) should be equal to $x = \frac{3}{4}b$; that meaning x is $\frac{3}{4}$ of distance of b (x signifying portion not run, or point at which person should jump in). With this information known it was then possible to determine the exact angle at which the lifeguard should jump in to reach the swimmer the fatest. This was down through using the 'sine law'. With the pattern show the angle value to be 53° in front of the lifeguard from the pool deck.



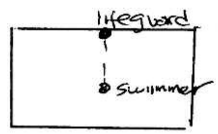
NOTE: in reality it would be virtually impossible to calculate this info quickly enough to actually save the swimmer so a suggestion would be to have metric markings (depending on what size pool) on each side of the pool where the lifeguard can quickly eye up how far the swimmer is down the pool and across the pool (x, y) and then use the calculation of $x = \frac{3}{4}b$ and at 53° and then simply go for the save. This can be used anywhere. All lifeguards should be tested for their run/swim speed before being hired and then be given the information that I have calculated to make the most minimized in time save.

LOW LEVEL 3

G

Keeping in a logical mindset if the lifeguard were to be a superior swimmer they should clearly jump in where they stand and swim the whole way if no objects are preventing them and if a lifeguard were to be a far superior rumer they should almost definitely take the option #2 being run perpendicular to the swimmer then jump in. The metric markings could still be beneficial in these scenarios as well to know directly where the swimmer is located in the pool.

Also if the lifeguard is directly in front of the swimmer to begin with it would be sensless to use the calculated information, and go backwards but rather just jump in immediately right there.



I hope you find this information useful!!

Teacher's Notes

Knowledge and Understanding

– The student uses a mathematical model with some effectiveness. He or she lists several reasonable simplifying assumptions. The student writes the correct derivative. However, he or she makes errors in the format by finding each part of the derivative separately, without explanation. The report therefore contains incorrect statements, such as "$T' = -\frac{1}{5}$". The student uses the quotient rule, which is not the most efficient method for finding the derivative.

Thinking

– The student interprets the solutions to the equations with considerable effectiveness. He or she uses the speeds given in Appendix A to show that "$x = \frac{3b}{4}$" for the "fatest way to get to the victim", and that the related angle is 53°. However, using three scenarios to determine the angle from the ratio of x to b is inefficient.

– The student formulates and tests hypotheses with some effectiveness. He or she tests the three options from scenario 1 in Appendix A to show that option 3 requires the least time. The student also writes the derivative and solves the resulting equation for a general case with the distances, but not the speeds, as variables. Thus he or she considers all scenarios for the given speeds, but not for other speeds. Before testing the options, the student does not explicitly state a hypothesis.

Communication

– The student communicates information in diagrams or graphs with considerable clarity. Most diagrams are clearly labelled with variables identified. However, the units of length are missing from some diagrams (e.g., the diagram included with the recommendations). The graph of t versus x is not labelled in a way that clearly links it to the student's work.

– The student integrates text and mathematical forms with some effectiveness. The text interspersed with the solutions to equations shows the student's thinking. However, the report lacks an effective introduction. Grammatical and typographical errors create a significant lack of clarity in places (e.g., the use of "T′" in place of *T* at the top of page D; the reference to the graph as "the graph of the derivative"). The student also seems to confuse the terms *scenario* and *option* from Appendix A, referring to the three options in one scenario as "these three scenarios".

Application

– The student uses formulas with considerable effectiveness. He or she chooses appropriate formulas when considering the three options in scenario 1 from Appendix A. The student writes a general formula for the time. He or she substitutes appropriate values from Appendix A into the formulas. The student uses the law of sines correctly to find the 53° angle between the running and swimming directions, but this law is not the most efficient choice for right triangles.

– The student recommends and justifies a course of action with considerable effectiveness. He or she suggests "metric markings … on each side of the pool where the lifeguard can quickly eye up how far the swimmer is down the pool and across the pool". The lifeguard would then "use the calculation of $x = \frac{3}{4}b$" to decide where to dive into the pool. The student also suggests that "All lifeguards should be tested for their run/swim speed", but there is no indication of how speeds other than those in Appendix A would affect the course of action. The suggestion that option 2 should be used if the lifeguard is a good runner is not appropriate.

Comments

This work is representative of a low level-3 performance. The student demonstrates a considerable degree of achievement of the expectations in the Application category of knowledge and skills. The student also demonstrates a considerable degree of achievement with respect to one criterion in the Thinking category and one criterion in the Communication category. However, in the Knowledge and Understanding category, and in one criterion in each of the Thinking and Communication categories, the student demonstrates some degree of achievement – i.e., achievement that is more characteristic of level 2.

Next Steps

In order to improve his or her performance, the student needs to:

- find the derivative more efficiently, and show the steps accurately;
- find the angle between the running and swimming directions more efficiently;
- explicitly state a hypothesis;
- label all diagrams fully;
- link the graph to his or her work more clearly;
- include an introduction in the report;
- edit and proofread the report to eliminate errors and unclear wording;
- consider the effect of varying the running and swimming speeds on the recommended course of action.