Lifeguard Procedures Report HIGH LEVEL 2

A

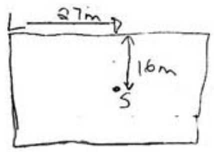
Exemplar Task

1. Mathematical model

$$\frac{a-x}{b} + \frac{\sqrt{c^2+x^2}}{d}$$

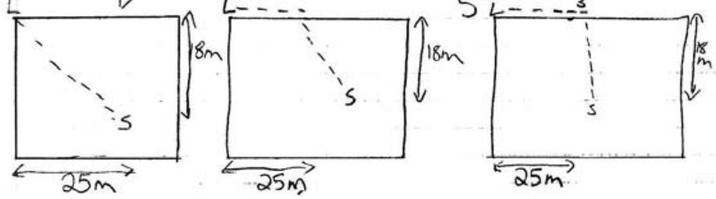
a = is the horizontal distance from the lifeguard to the swimmer in m.
b = is the speed of a in m/s
c = is the vertical distance from the lifeguard to the swimmer in m.
d = is the speed of c in m/s

Example



Running speed = 5m/s
Swimming speed = 3m/s

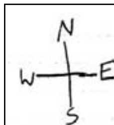
The equation would be $t = \frac{27-x}{5} + \frac{\sqrt{16+x^2}}{3}$



In these diagrams, assumptions that are made are that the running and swimming speed are constant. Also the lifeguard does not dive into the pool, because the speed is constant. Wind, weather and people in the pool are not factors. No time is spent by turning direction, because the speed is constant.

Assumptions
- It is assumed that the lifeguard is positioned at the top left of the swimming pool at all times.
- The lifeguard has a constant running speed of 5m/s, and a swimming speed of 3m/s
- The dimensions of the pool are 50m by 40m
- The weather in all scenerios are same.
- It is assumed that there are no other swimmers in the pool, except for the one drowning.
- The lifeguard doesn't dive into the pool, they enter the pool in one motion

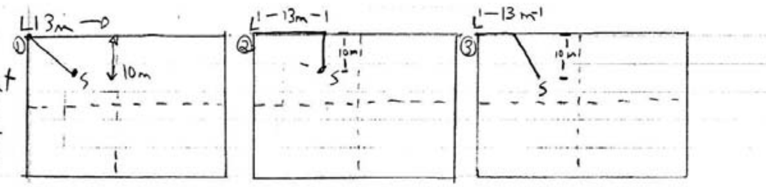
B



Scenerio #1

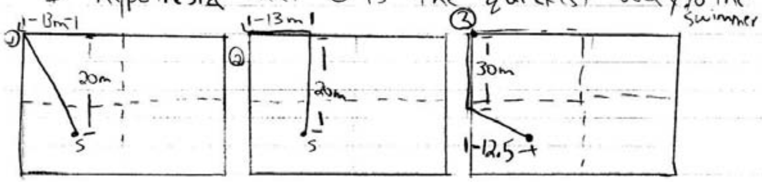
Quadrant

Q1	Q3
Q2	Q4



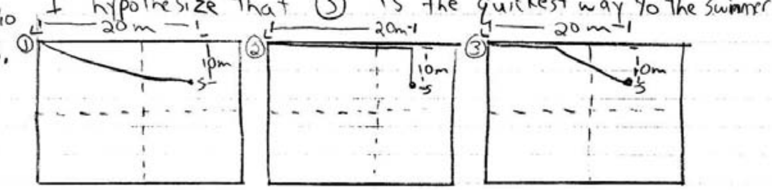
I hypothesize that ③ is the quickest way to the swimmer

Scenerio #2.



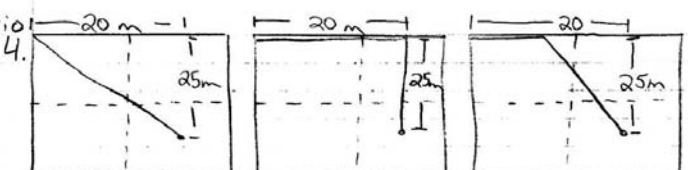
I hypothesize that ③ is the quickest way to the swimmer

Scenerio #3.



I hypothesze that ③ is the quickest way to the swimmer.

Scenerio #4.



I hypothesize that ③ is the quickest way to the swimmer

C

Calculations

Scenerio 1

③ $y = \frac{(13-x)}{5} + \frac{\sqrt{10^2+x^2}}{3}$

$= \frac{13-x}{5^2} + \frac{\sqrt{100+x^2}}{3^2}$

der $y' = \frac{(-1)(5)-(0)(13-x)}{25} + \frac{\frac{1}{2}(100+x^2)^{-\frac{1}{2}}(2x)(3)-\sqrt{100+x^2}(0)}{9}$

$= \frac{-5}{25} + \frac{x}{3(100+x^2)^{\frac{1}{2}}}$

$\frac{1}{5} = \frac{x}{3\sqrt{100+x^2}}$

$5x = 3\sqrt{100+x^2}$

$25x^2 = 9(100+x^2)$

$25x^2 = 9x^2 + 900$

$16x^2 = 900$

$x^2 = 56.25$

$x = \pm 7.5$

$y(7.5) = \frac{(13-(7.5))}{5} + \frac{\sqrt{10^2+(7.5)^2}}{3}$

$= 1.1 + 4.16$

$t = 5.26s$

∴ It takes 5.26 sec for the lifeguard to get to the swimmer in scenerio 1, option 3.

Time on Land $= \frac{d}{v} = \frac{7.5}{5} = 1.5$ sec

5.26 sec $- 1.5$ sec $= 3.76$sec

$d = vt = (3)(3.76) = 11.28$ m in the water

water

D

Scenerio 2

③ $t = \frac{30-x}{5} + \frac{\sqrt{12.5^2+x^2}}{3}$

$= \frac{30-x}{5} + \sqrt{156.25+x^2}$

$t' = \frac{-1}{5} + \frac{x}{3(156.25+x^2)^{\frac{1}{2}}}$

$\frac{1}{5} = \frac{x}{3\sqrt{156.25+x^2}}$

$5x = 3\sqrt{156.25+x^2}$

$25x^2 = 9(156.25+x^2)$

$25x^2 = 9x^2 + 1406.25$

$16x^2 = 1406.25$

$x = 9.4$m

$t = (9.4) = \frac{30-9.4}{5} + \frac{\sqrt{12.5^2+9.4^2}}{3}$

$= 4.12 + 5.21$

$= 9.33s$

∴ It takes 9.33sec for the lifeguard to get to the swimmer in Scenerio 2, option 3.

Scenerio 3

③ $y = \frac{(20-x)}{5} + \frac{\sqrt{10^2+x^2}}{3}$

$y = \frac{20-x}{5^2} + \frac{\sqrt{100+x^2}}{3^2}$

$y' = \frac{(-1)(5)-(20-x)(0)}{25} + \frac{\frac{1}{2}(100+x^2)^{-\frac{1}{2}}(2x)(3)-\sqrt{100+x^2}(0)}{9}$

$= \frac{-5}{25} + \frac{x}{3(100+x^2)^{\frac{1}{2}}}$

$\frac{1}{5} = \frac{x}{3\sqrt{100+x^2}}$

$5x = 3\sqrt{100+x^2}$

$25x^2 = 9(100+x^2)$

$25x^2 = 9x^2 + 900$

$16x^2 = 900$

$x^2 = 56.25$

$x = \pm 7.5$

$t(7.5) = \frac{(20-7.5)}{5} + \frac{\sqrt{10^2+7.5^2}}{3}$

$= 2.5 + 4.16$

$t = 6.66$ sec

∴ It takes 6.7 sec for the lifeguard to get to the swimmer in scenerio 3, option 3.

HIGH LEVEL 2

E

Scenerio 4

(3) $y = \frac{(20-x)}{5} + \frac{\sqrt{25^2+x^2}}{3}$

$y = \frac{20-x}{5} + \frac{\sqrt{625+x^2}}{3}$

$y' = \frac{(-1)(5)-(20-x)(0)}{25} + \frac{\frac{1}{2}(625+x^2)^{-\frac{1}{2}}(2x)(3)-\sqrt{625+x^2}(0)}{9}$

$= \frac{-5}{25} + \frac{x}{3(625+x^2)^{\frac{1}{2}}}$

$\frac{1}{5} = \frac{x}{3\sqrt{625+x^2}}$

$5x = 3\sqrt{625+x^2}$

$25x^2 = 9(625+x^2)$

$25x^2 = 9x^2 + 5625$

$16x^2 = 5625$

$x^2 = 351.56$

$x = \pm 18.75$

$t(18.75) = \frac{(20-18.75)}{5} + \frac{\sqrt{25^2+18.75^2}}{3}$

$= .25 + 10.416$

$t = 10.66$

$\therefore$ It takes 10.66 sec for the lifeguard to get to the swimmer in Scenerio 4, option 3.

F

Report

Hypothesis: It is hypothesized that if the lifeguard runs a certain reccommend meters and then swims the rest of the reccomended meters, then Option 3 for all the 4 quadrants will be the fastest method for the lifeguard to rescue the swimmer.

Recommendations for
the Department of Parks and Recreation

Quadrant 1
If there were to be a drowning victim in quadrant 1, the fastest method for the lifeguard to reach the victim would be for the lifeguard to run 5.5m jump in the pool and swim to the drowning victim. That would minimize the amount of time it takes to rescue the victim.

Quadrant 2
To minimize time to save a drowning victim in Quadrant 2, the lifeguard should run 20.6m then jump in the pool and swim to the drowning victim.

Quadrant 3
To minimize time to save a drowning victim in Quadrant 3, the lifeguard should run 12.5m then jump in the pool and swim to the drowning victim.

Quadrant 4
To minimize time to save a drowning victim in quadrant 4, the lifeguard should run 1.25m then jump in the pool and swim to the drowning victim.

Recommendations

1. The pool should be marked off at different spots along the sides. An idea could be every 5 m, it is marked off so the lifeguards knows where to jump into the pool.

2. The pool should be divided into 4 quadrants, and then the lifeguard will know the approximate spot to jump in, and to know how long it will approximatly take to get to the swimmer.

Teacher's Notes

Knowledge and Understanding

- The student uses a mathematical model with considerable effectiveness. He or she includes a list of simplifying assumptions, but they are not described very clearly (e.g., "the lifeguard does not dive into the pool, because the speed is constant"). The student correctly forms the derivative and solves the resulting equation. However, the use of the quotient rule is not the most efficient method. The student shows two solutions when finding square roots (e.g., $x = \pm 7.5$), indicating an awareness of extraneous roots.

Thinking

- The student interprets the solutions to the equations with some effectiveness. He or she defines a path that the lifeguard should follow for four scenarios. However, the student does not explore the possibility of a relationship between the value of x and the location of the swimmer, and does not find the angle between the running and swimming directions. The calculated values shown on page C of the report, beginning with "Time on Land", are incorrect. They are based on a misinterpretation of the meaning of x.
- The student formulates and tests hypotheses with some effectiveness. He or she hypothesizes that option 3 from Appendix A is the "quickest way to the swimmer". The student shows all three options in the diagrams for four scenarios. However, he or she tests only option 3, i.e., the testing is incomplete.

Communication

- The student communicates information in diagrams or graphs with some clarity. He or she labels distances clearly in most diagrams, but omits the units of length in a few cases. Some distance labels in the diagrams of the four scenarios appear to be inconsistent with each other and with the shape of the pool. The student does not show the meaning of the variable x in the diagrams for option 3 of each scenario. The report does not include a graph.
- The student integrates text and mathematical forms with some effectiveness. The mathematical reasoning is clear in the main, but verbal descriptions would clarify it further (e.g., the student sets the derivative to 0, but does not state that he or she is doing so). The report lacks an introduction, and some material is out of order (e.g., the Hypothesis section follows the mathematical analysis). Some verbal statements are unclear or repetitive (e.g., "b = is the speed of a in m/s"; the student writes the same hypothesis four times). The report includes a number of misleading equations, because the student writes radical signs carelessly (e.g., "$\sqrt{100} + x^2$", instead of $\sqrt{100 + x^2}$).

Application

- The student uses formulas with some effectiveness. He or she uses an appropriate formula when considering option 3, and substitutes appropriate distance and speed values. However, in writing the general case at the start of the report, the student omits the time as a variable. Because the student does not perform calculations for options 1 and 2, it is not possible to tell if he or she would have used an appropriate formula in these cases.
- The student recommends and justifies a course of action with some effectiveness. Marking distances along the sides of the pool is a reasonable suggestion. However, the student recommends that the pool should be divided into quadrants and that the lifeguard should run the same distance for all locations of the swimmer in each quadrant of the pool. For most locations of the swimmer, this recommendation would not give optimal results. The student does not consider the effect of varying the running or swimming speeds on the recommended course of action.

HIGH LEVEL 2

Comments

This work is representative of a high level-2 performance. The student demonstrates some degree of achievement of the expectations in the Thinking, Communication, and Application categories of knowledge and skills. However in the Knowledge and Understanding category, the student demonstrates a considerable degree of achievement – i.e., achievement that is more characteristic of level 3.

Next Steps

In order to improve his or her performance, the student needs to:

- express simplifying assumptions more clearly;
- find the derivative more efficiently;
- use the solutions to the equations to look for general relationships;
- test hypotheses fully;
- label diagrams fully and clearly;
- include a labelled graph;
- add text to explain mathematical reasoning in more detail;
- write mathematical equations more carefully;
- edit and proofread the report to eliminate errors and repetition, and to improve the clarity and flow;
- make more detailed recommendations, allowing for different locations of the swimmer and different running and swimming speeds of the lifeguard.