

**Lifeguard Procedures Report LEVEL 2**

**A**

**Scenario 1**

running speed = 5m/s  
 Swimming speed = 3m/s  
 Hypothesis option 3 is best  
 we wish to minimise the time

$t = t_{land} + t_w$   
 $t = \frac{D_L}{v} + \frac{D_w}{v}$

$D = \frac{21-x}{5} + \frac{\sqrt{16^2+x^2}}{3}$   
 $t = \frac{21-x}{5} + \frac{1}{3}\sqrt{16^2+x^2}$

$t' = \frac{21-x}{5} + \frac{1}{2}(16^2+x^2)^{-1/2}(2x)$   
 $-\frac{1}{5} + \frac{2x}{6\sqrt{x^2+16^2}} \quad t_p = t'(x) = 0$

$\frac{2x}{3\sqrt{x^2+16^2}} = \frac{1}{5} \quad \frac{6x}{3\sqrt{x^2+16^2}} \quad (5x = 3\sqrt{x^2+16^2})^2$   
 $25x^2 = 9(x^2+16^2)$   
 $25x^2 = 9x^2 + 2304$   
 $25-9x^2 = 2304$   
 $16x^2 = \sqrt{2304}$   
 $x = 12 \quad \text{run 9m Swim 23m}$

$0 \leq x \leq 21$   
 as little swim as possible  
 ranbo  
 all swim

$t(0) = 9.5 \text{ seconds}$   
 $t(12) = 8.4 \text{ seconds}$   
 $t(21) = 8.8 \text{ seconds}$

$\frac{21 + \sqrt{16^2}}{3} = 9.5$   
 $1.8 + \frac{\sqrt{400}}{3} = 7.1$   
 $0 + \frac{\sqrt{607}}{3} = 8.8$

$\therefore$  option 3 would be the best

**B**

**Scenario 2**

we wish to minimise the time (t)

$t = \frac{D}{v}$  Total time =  $t = t_{land} + t_{water}$

we know  $t = \frac{D_L}{v} + \frac{D_w}{v}$   
 $v_L = 5 \text{ m/s}$  run  
 $v_w = 3 \text{ m/s}$  swim  
 $D_L = 40-x$   
 $D_w = \sqrt{x^2+14^2}$

$t(x) = \frac{40-x}{5} + \frac{\sqrt{x^2+14^2}}{3}$

$t'(x) = 8 - \frac{1}{5}x + \frac{1}{3}(\frac{1}{2}(x^2+14^2)^{-1/2}(2x))$   
 $t'(x) = 8 - \frac{1}{5}x + \frac{1}{3}(x^2+14^2)^{-1/2}(2x)$   
 $t'(x) = -\frac{1}{5} + \frac{2x}{3\sqrt{x^2+196}}$  turning point  $t'(x) = 0$

$-\frac{1}{5} + \frac{x}{3\sqrt{x^2+196}} = 0 \quad \frac{x}{3\sqrt{x^2+196}} = \frac{1}{5}$  solve for x  
 $(3\sqrt{x^2+196} = 5x)^2$   
 $9(x^2+196) = 25x^2 = 9x^2 + 1764 = 25x^2 - 9x^2$   
 $\sqrt{1764} = 16x^2$   
 $x = 10.5 \quad 10\frac{1}{2}$  run 29.5m Swim 17m

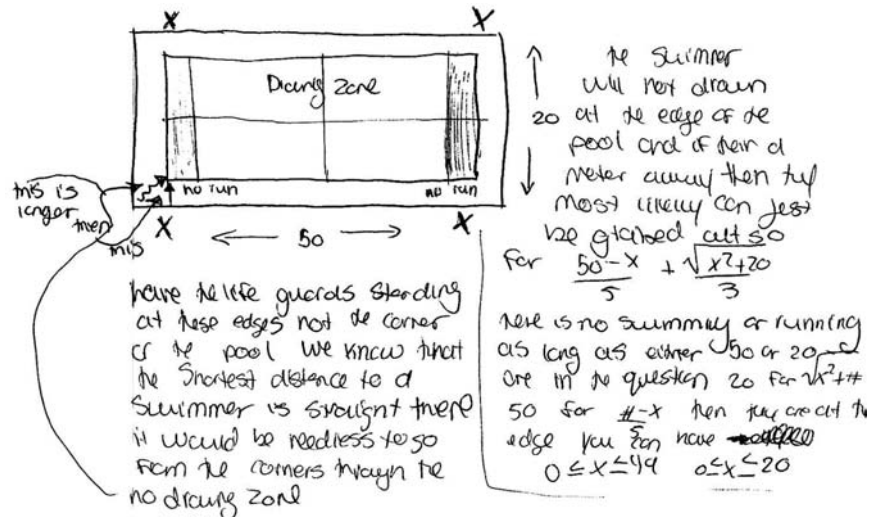
Endpoints Scenario 1 and 2  
 $t(0) = 16.4 \text{ seconds}$   $\frac{40-0}{5} + \frac{\sqrt{0+196}}{3} = 8+8.1 = 16.1$   
 $t(10.5) = 15.9 \text{ seconds}$   $\frac{40-10.5}{5} + \frac{\sqrt{10.5^2+196}}{3} = 5.9 + 10 = 15.9$   
 $t(40) = 24 \text{ seconds}$   $\frac{40-40}{5} + \frac{\sqrt{40^2+196}}{3} = 24.5$

$\therefore$  the option 3 would give you the minimum amount of time.

LEVEL 2

C

It was conclusive that the most optimal time efficient rescue would be achieved (for both scenarios) by combining both swimming and running. However, this was the case because of the values given for the running/swimming ratio if the speeds were different then so would the distances run and swam. Also to begin to use these calculations we have to make some assumptions we have to assume the running and swimming speeds are not only constant but the same for all guards. These speeds would have to be predetermined and matched everytime there was a rescue. To use the form of calculation we also have to assume that no other swimmers in the pool would get in the way of the guard and that when the guard reached the swimmer (who would have to stay in one spot) that the swimmer wouldn't interfere with the rescue. So path would have to be constant. Quite obviously life guards are people not calculators and you can't have a guard on the side of the pool doing calculus, so one way to loosely integrate calculus logic into life guarding would be to make sure the guards have an equal ability to both run and swim at efficient speeds. Strategic placement will also be key and an understanding in each life guard the concept of minimising the time through running and swimming. The first thing we can do is look at the set up of the pool



D

So you could have 4 guards on duty each having their own area and just have them know that as the swimmer's position increases vertically in respect to them the running distance decreases and swimming distance increases and when the swimmer's position increases horizontally in respect to them the distance run increases. Of course this rate of change in respect to the swimmer's distance to the guard occurs because running speed is faster than swimming speed.

## LEVEL 2

**Teacher's Notes****Knowledge and Understanding**

- The student uses a mathematical model with some effectiveness. He or she writes the correct derivatives for scenarios 1 and 2, but makes several incorrect statements in the process. The student solves the resulting equations correctly for  $x$ , but this process also includes incorrect statements (e.g., " $16x^2 = \frac{\sqrt{2304}}{16}$ "). Some reasonable simplifying assumptions are included in the middle of the long paragraph on page C of the report, but more assumptions would be appropriate.

**Thinking**

- The student interprets the solutions to the equations with some effectiveness. He or she determines that option 3 is the best route for scenarios 1 and 2. However, the student does not reach a generalization about the value of  $x$ , and does not calculate the angle between the running and swimming directions.
- The student formulates and tests hypotheses with some effectiveness. He or she tests three options for each of two scenarios to show that running then swimming on an angle gives the fastest response time. However, the student makes calculation errors in finding all three time values for scenario 2. In the analysis of scenario 1, the student states "Hypothesis option 3 is best", but there is no indication that this hypothesis applies to both scenarios.

**Communication**

- The student communicates information in diagrams or graphs with some clarity. In keeping with the use of just one formula to find the times for the three options, the student shows only one diagram for each scenario. However, the diagram for scenario 1 is not clearly labelled (e.g., the meaning of the " $21 - x$ " label is unclear). A diagram of the pool is included with the recommendations, but it does little to clarify them (e.g., the meaning of the "no run" labels is unclear). The report does not include a graph.

- The student integrates text and mathematical form with some effectiveness. The small amounts of text included in the mathematical analysis help to clarify the process (e.g., "we wish to minimize the time"), but more explanation is needed. The flow of the report is poor in places (e.g., the simplifying assumptions follow the mathematical analysis that is based on them). The report includes sections for scenarios 1 and 2, but the more descriptive material is not organized into sections to help readability (e.g., there is no separate section for the assumptions). The report does not include an introduction.

**Application**

- The student uses formulas with some effectiveness. For each scenario, he or she uses one formula to find the times for all three options. Using a simpler formula, of the form  $t = \frac{d}{v}$ , would be more efficient for options 1 and 2. In scenario 1, the student carelessly writes " $D = \dots$ " in the formula for finding the time. The student does not write a general case, with distances and speeds as variables. He or she uses appropriate distances and speeds from Appendix A.
- The student recommends and justifies a course of action with some effectiveness. He or she recommends the placement of the lifeguards (e.g., "you could have 4 guards on duty each having their own area"). However, some statements are vague (e.g., "make sure the guards have an equal ability to both run and swim at efficient speeds"). The student's handwriting is difficult to read, and some recommendations are difficult to understand. The student does not clearly relate the recommendations to the mathematical analysis in the report.

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## LEVEL 2

### Comments

This work is representative of a solid level-2 performance. The student demonstrates some degree of achievement of the expectations in all four categories of knowledge and skills.

### Next Steps

In order to improve his or her performance, the student needs to:

- include a more extensive list of simplifying assumptions;
- improve the processes for finding the derivative and solving the resulting equations;
- reach generalizations about the preferred route;
- determine the angle between the running and swimming directions;
- edit and proofread the report to eliminate mathematical, grammatical, and typographical errors;
- clearly label all diagrams;
- include a clearly labelled graph;
- include more text to explain the mathematical analysis;
- organize the report into well-defined sections, and include an introduction;
- write a formula for a general case, with distances and speeds as variables;
- express recommendations more clearly;
- use the mathematical analysis to justify the recommendations.