

Lifeguard Procedures Report HIGH LEVEL 1

A

Exemplar Project

Assumptions

- assume constant speeds
- assume two lifeguards
- pool is 50m x 20m
- lifeguards located at two opposite corners
- swimming speed less than running speed

There are 4 possible ways of reaching the swimmer: swim directly, run then swim straight, run then swim on an angle, or throw and reach.

The two following scenarios test which of the options is generally most efficient. Keeping in mind that throwing or reaching prevails if possible.

Scenario One (swimmer at (21, 16))

→ swim directly → run then swim at 90°

$$z^2 = x^2 + y^2$$

$$z^2 = 21^2 + 16^2$$

$$z = 26.4 \text{ s}$$

$$t = \left(\frac{21}{5}\right) + \left(\frac{16}{3}\right)$$

$$= 9.54 \text{ s}$$

→ run then swim on an angle

$$t = \left(\frac{21-x}{5}\right) + \left(\frac{\sqrt{x^2+16^2}}{3}\right)$$

Minimize time, set t' to 0

$$t' = \frac{(-1)(5) - (21-x)(c)}{5^2} + \frac{dy}{dx} \frac{\sqrt{x^2+16^2}}{3}$$

B

$$t' = -1/5 + \frac{dy}{dx} \frac{1}{3}(x^2+16^2)^{1/2}$$

$$t' = -1/5 + 1/3(x^2+16^2)^{1/2}$$

$$t' = -1/5 + 1/6(x^2+16^2)^{-1/2} \cdot 2x$$

$$t' = -1/5 + 1/3x(x^2+16^2)^{-1/2}$$

$$0 = -1/5 + \frac{x}{3\sqrt{x^2+16^2}}$$

$$1/5 = \frac{x}{3\sqrt{x^2+16^2}}$$

$$5x = 3\sqrt{x^2+16^2}$$

Square both sides

$$25x^2 = 9(x^2+16^2)$$

$$25x^2 = 9x^2 + 2304$$

$$16x^2 = 2304$$

$$x^2 = 144$$

$$x = 12$$

$$z^2 = x^2 + y^2$$

$$z^2 = 12^2 + 16^2$$

$$z = 20$$

$$t = \left(\frac{9}{5}\right) + \left(\frac{16}{3}\right)$$

$$= 7.13 \text{ s}$$

For scenario 1 the fastest response time occurs when the life guard runs a distance then swims on an angle.

C

Scenario 2 (swimmer at (40, 14))
 → option 1 → swim directly

$$z^2 = 40^2 + 14^2$$

$$z = \sqrt{40^2 + 14^2}$$

$$z = 42.38$$

$$t = \frac{42.38s}{3 \text{ m/s}}$$

$$= 14.1s$$

→ option 2 → run then swim at 90° angle

$$t = \frac{40}{5} + \frac{14}{3}$$

$$= 12.67s$$

→ Option 3 → run then swim on an angle

$$t = \frac{40-x}{5} + \frac{\sqrt{x^2+14^2}}{3}$$

minimize time by setting $t' = 0$

$$t' = \frac{[-1][5] - [0](40-x)}{5^2} + \frac{dy}{dt} \cdot \frac{1}{3}(\sqrt{x^2+14^2})^{1/2}$$

$$t' = -1/5 + 1/6(\sqrt{x^2+14^2})^{-1/2} \cdot 2x$$

~~$$t' = -1/5 + 1/3x(\sqrt{x^2+14^2})^{-1/2} \cdot 2x$$

$$0 = -1/5 + 2/3x(\sqrt{x^2+14^2})^{-1/2}$$

$$1/5 = 2/3x(\sqrt{x^2+14^2})^{-1/2}$$

$$1/5 = 2/3x(\sqrt{x^2+14^2})^{-1/2}$$

$$1/5 = 2/3x(\sqrt{x^2+14^2})^{-1/2}$$~~

$$t' = -1/5 + \frac{1}{3} \frac{x}{\sqrt{x^2+14^2}}$$

D

$$0 = -1/5 + \frac{x}{3\sqrt{x^2+14^2}}$$

$$1/5 = \frac{x}{3\sqrt{x^2+14^2}}$$

$$\frac{1}{5} = \frac{x}{3\sqrt{x^2+14^2}}$$

$$3\sqrt{x^2+14^2} = 5x$$

$$\sqrt{x^2+14^2} = 5/3 x$$

$$x^2 + 14^2 = 25/9 x^2$$

$$14^2 = 16/9 x^2$$

$$196 = 16/9 x^2$$

$$110.25 = x^2$$

$$10.5 = x$$

$$t = \left(\frac{20.5}{5}\right) + \left(\frac{14}{3}\right)$$

$$= 10.6s$$

For scenario two running then swimming is once again the fastest response time.

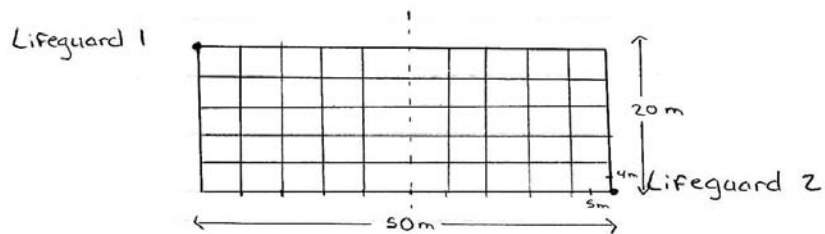
Therefore I will recommend that the lifeguard always runs then swims on an angle when reaching or throwing is not possible.

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E

The following analysis further investigates various response times for various situations and locations in the pool.

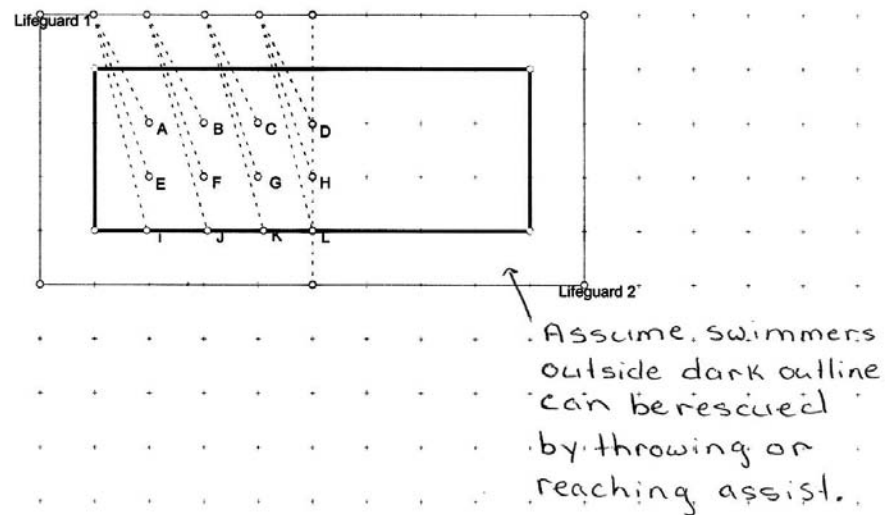
To simplify the responding process for the lifeguard I have divided the pool into a grid and tested 12 potential spots.



For calculations and proof regarding grid see attached sheets from geometers sketch pad. Points are only tested on one side because the other side is guarded by another guard who would follow same principles. Also the angles at which they swim were left constant because I found that changing the angle had a minimum effect compared to the simplicity it added.

Therefore in conclusion I recommend that the Parks and Rec department implement grid markings on the side of the pool. The grid would allow a lifeguard to easily reference the position in the grid and identify their response path.

F



G

A to D are 117 degree angles
 E to H are 109 degree angles
 I to L are 104 degree angles

Lifeguard 1 To A

$$\left(\frac{\text{Lifeguard 1W}}{5}\right) + \left(\frac{\text{WA}}{3}\right) = 0.95 \text{ cm}$$

Lifeguard 1 to B

$$\left(\frac{\text{Lifeguard 1X}}{5}\right) + \left(\frac{\text{XR}}{3}\right) = 1.14 \text{ cm}$$

Lifeguard 1 to C

$$\left(\frac{\text{Lifeguard 1Z}}{5}\right) + \left(\frac{\text{ZC}}{3}\right) = 1.35 \text{ cm}$$

Lifeguard 1 to D

$$\left(\frac{\text{Lifeguard 1AA}}{5}\right) + \left(\frac{\text{AAD}}{3}\right) = 1.55 \text{ cm}$$

Lifeguard 1 to E

$$\left(\frac{\text{Lifeguard 1W}}{5}\right) + \left(\frac{\text{EW}}{3}\right) = 1.25 \text{ cm}$$

Lifeguard 1 to F

$$\left(\frac{\text{Lifeguard 1X}}{5}\right) + \left(\frac{\text{XF}}{3}\right) = 1.45 \text{ cm}$$

Lifeguard 1 to G

$$\left(\frac{\text{Lifeguard 1Z}}{5}\right) + \left(\frac{\text{ZG}}{3}\right) = 1.65 \text{ cm}$$

Lifeguard 1 to H

$$\left(\frac{\text{Lifeguard 1AA}}{5}\right) + \left(\frac{\text{AAH}}{3}\right) = 1.86 \text{ cm}$$

Lifeguard 1 to I

$$\left(\frac{\text{Lifeguard 1W}}{5}\right) + \left(\frac{\text{IW}}{3}\right) = 1.57 \text{ cm}$$

Lifeguard 1 to J

$$\left(\frac{\text{Lifeguard 1X}}{5}\right) + \left(\frac{\text{XJ}}{3}\right) = 1.78 \text{ cm}$$

Lifeguard 1 to K

$$\left(\frac{\text{Lifeguard 1Z}}{5}\right) + \left(\frac{\text{ZK}}{3}\right) = 1.98 \text{ cm}$$

Lifeguard 1 to L

$$\left(\frac{\text{Lifeguard 1AA}}{5}\right) + \left(\frac{\text{AAL}}{3}\right) = 2.18 \text{ cm}$$

Teacher's Notes

Knowledge and Understanding

- The student uses a mathematical model with some effectiveness. He or she lists simplifying assumptions, but the list is short and lacks detail (e.g., “assume constant speeds” includes unstated assumptions about the lack of obstructions, the speed of the dive, et cetera). The student obtains the correct derivatives for two scenarios, but his or her use of $\frac{dy}{dt}$ in the process suggests a lack of understanding. The use of the quotient rule is not the most efficient method. The student successfully solves the resulting equations, except for carelessly stating in scenario 2 that “ $10.5 = x^2$ ”, instead of $10.5 = x$.

Thinking

- The student interprets the solutions to the equations with limited effectiveness. He or she determines the path that the lifeguard should follow for two scenarios. However, the student makes no attempt to reach a generalization about the value of x , or to calculate the angle between the running and swimming directions. He or she shows three possible angle measures in the data from *The Geometer's Sketchpad*®, but the report does not show how these angle measures are determined. They seem to result from the student's use of a grid. If so, they are based on faulty reasoning and are incorrectly calculated.
- The student formulates and tests hypotheses with limited effectiveness. He or she tests three options for each of two scenarios to show that running then swimming on an angle gives the fastest response time. However, the student calculates incorrect times for this option by using the wrong swimming distances. He or she does not explicitly state a hypothesis before testing the options.

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Communication

- The student communicates information in diagrams or graphs with limited clarity. The only diagram that shows possible routes for a lifeguard is generated using *The Geometer's Sketchpad*®. However, the labelling of the points in this diagram is incomplete, and the accompanying data suggest that this method is used to examine only option 3. The student states that “I have divided the pool into a grid and tested 12 potential spots”, but the diagram of the grid does not indicate these spots. The report does not include a graph.
- The student integrates text and mathematical forms with some effectiveness. Some verbal descriptions help to clarify the mathematical reasoning (e.g., “Minimize time, set t' to 0”, “Square both sides”). However, the report lacks an effective introduction, and the exploration with *The Geometer's Sketchpad*® is not explained and is not clearly integrated into the report.

Application

- The student uses formulas with some effectiveness. He or she generally chooses appropriate formulas for the three options, except for trying to find the time directly from the Pythagorean theorem in scenario 1, option 1. The student substitutes appropriate distances and speeds given in Appendix A. However, he or she does not write a general case for option 3, with distances and speeds as variables.
- The student recommends and justifies a course of action with limited effectiveness. He or she suggests “grid markings on the side of the pool” to allow lifeguards to “identify their response path”. However, the student offers no details of how the grid would be used in practice. The student’s suggestion that swimmers near the edge of the pool “can be rescued by throwing or reaching” is reasonable, but the report does not include a mathematical justification.

Comments

This work is representative of a high level-1 performance. The student demonstrates a limited degree of achievement of the expectations in the Thinking category of knowledge and skills. The student also demonstrates a limited degree of achievement with respect to one criterion in the Communication category and one criterion in the Application category. However, in the Knowledge and Understanding category, and in one criterion in each of the Communication and Application categories, the student demonstrates some degree of achievement of the expectations – i.e., achievement that is more characteristic of level 2.

Next Steps

In order to improve his or her performance, the student needs to:

- include a more detailed list of simplifying assumptions;
- find the derivative more clearly and efficiently;
- edit and proofread the report to eliminate careless errors;
- reach a generalization about the best path for the lifeguard;
- explicitly state a hypothesis and test it effectively;
- use all formulas correctly;
- include fully labelled diagrams of the options for each scenario;
- include a fully labelled graph;
- add an introduction to the report;
- explain the use of *The Geometer's Sketchpad*® more clearly;
- consider a general case for option 3;
- make more detailed recommendations and justify them mathematically.