A

Exemplar Task

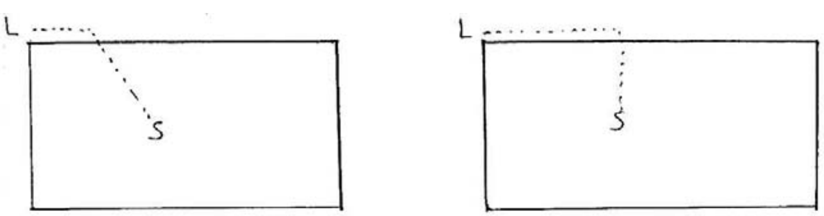
To minimize time that it takes for a lifeguard to rescue a swimmer we must take into consideration the size of the pool, position of the swimmer, the speed of the lifeguards swimming and running and the position of the lifeguard.

Assumptions

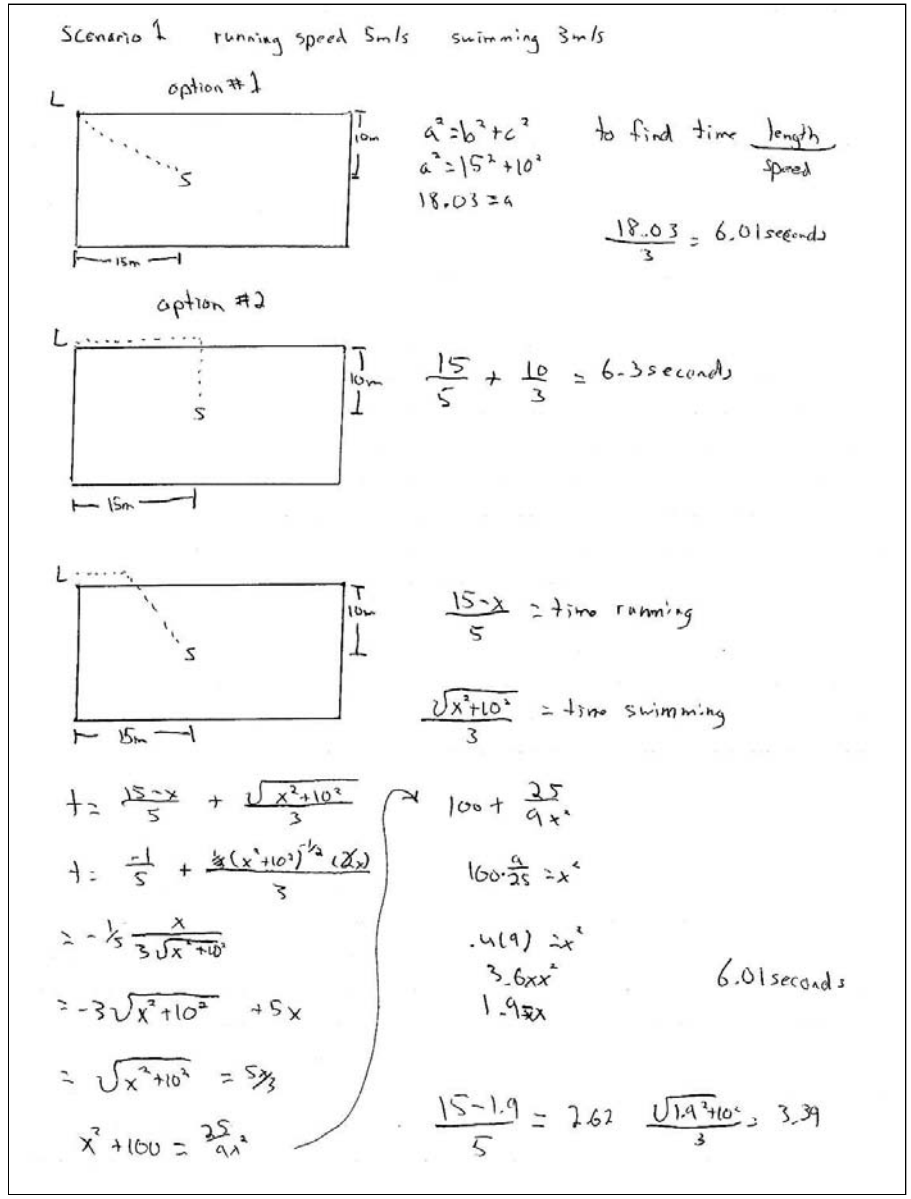
First, we have to assume that the lifeguard runs and swims at a constant speed. Also, the victim is at the surface of the water. We also have to assume there is no chance of the lifeguard slipping on the deck while running. We have to assume that when the lifeguard dives into the pool, they keep a constant speed equal to the speed they swim at.

Hypothesis

Since the lifeguard runs faster than He swims (running speed = 5m/s, swimming speed 3m/s) I think the fastest route would be running on the deck part way and then diving in. example:



B



LEVEL 1

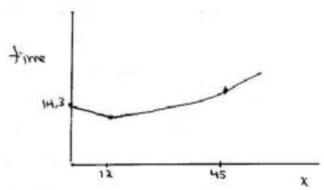
C

Formulas

I had to use Pythagorean theorom to find the answer to option 1. It is $a^2 = b^2 + c^2$ where a = hypotenuse of a right angled triangle and b and c are the other 2 sides.

I also had to use derivative rules to find the answer to option 3.

Graph



the time shows where the lifeguard runs along the edge of the pool until he is perpendicular to the swimmer. Finding the min. t-value, the length of the pool has no relevance to the problem.

$$t = \frac{-5}{25} + \frac{2x}{2(3)\sqrt{x^2+16^2}}$$

$$= -\frac{1}{5} + \frac{x}{3\sqrt{x^2+16^2}}$$

$$= -\frac{1}{5} + \frac{x}{3\sqrt{x^2+256}}$$

$$\frac{1}{5} = \frac{x}{3\sqrt{x^2+256}}$$

$$\frac{3\sqrt{x^2+16^2}}{3} = \frac{5x}{3}$$

$$x^2 + 16^2 = \frac{25}{9}x^2$$

$$256 = \frac{16}{9}x^2$$

$$x = 12$$

Conclusion

My hypothesis was half-correct. The amount of time the lifeguard took in the problem was the same in option # 1 and 3. They both minimized the amount of time it took to rescue the swimmer. The lifeguard should not swim perpendicular to the deck.

Teacher's Notes

Knowledge and Understanding

– The student uses a mathematical model with limited effectiveness. He or she includes a short list of simplifying assumptions, but there is a lack of detail (e.g., it is not stated that the constant swimming speed assumes no obstructions in the pool). The student forms the correct derivative for option 3. However, in two attempts at solving for x, he or she is successful only once. The other attempt contains major mathematical errors. In the successful attempt to calculate x for option 3, the student does not use the resulting value to calculate t.

Thinking

– The student interprets the solutions to the equations with limited effectiveness. He or she states that "Finding the min. t-value, the length of the pool has no relevance to the problem." However, the student does not explain or justify this statement, and he or she appears to confuse the length of the pool with the horizontal distance to the swimmer.

– The student formulates and tests hypotheses with limited effectiveness. The stated hypothesis includes diagrams that show options 2 and 3 from Appendix A, but does not distinguish between them. In deciding that options 1 and 3 "both minimized the amount of time it took to rescue the swimmer", the student relies on calculated times for only one scenario, including an erroneous value for option 3.

Communication

– The student communicates information in diagrams or graphs with limited clarity. In the description of scenario 1, he or she includes diagrams showing the three options. However, the labelling is incomplete and does not show the meanings of a, b, c, and x. There is no diagram that shows the values used in writing the derivative on page C of the report. The student includes a graph of t versus x. However, the domain of the graph, and the value $t = 14.3$ when $x = 0$, are unexplained.

– The student integrates text and mathematical forms with limited effectiveness. The report is laid out in a reasonable order, and some statements are made clearly (e.g., "$\frac{15 - x}{5}$ = time running"). However, the small amount of text on page B contributes to a lack of clarity (e.g., it is never explained that the derivative is being set to 0). On page C of the report, there is no text below the graph to provide a context for the mathematical solution. The report includes other examples of unclear communication (e.g., the failure to distinguish t and t' in writing the derivative, the ambiguous reference to "the time" next to the graph).

Application

– The student uses formulas with limited effectiveness. He or she correctly writes the formula for the Pythagorean theorem and for the time in option 3 of his or her scenario 1. However, in the scenario that follows the graph, the student does not write a formula for the time before starting to write the derivative. He or she also makes no attempt to write a general formula for option 3. The statement "to find time $\frac{\text{length}}{\text{speed}}$" is carelessly expressed, and the student makes no mention of the corresponding formula in the Formulas section of the report. The student substitutes appropriate values into the formulas, but, in considering only one scenario in detail, he or she does not substitute many values.

– The student recommends and justifies a course of action with limited effectiveness. Having concluded erroneously that options 1 and 3 are equally effective, he or she recommends that "The lifeguard should not swim perpendicular to the deck". The report does not include a more detailed recommendation.

Comments

This work is representative of a solid level-1 performance. The student demonstrates a limited degree of achievement of the expectations in all four categories of knowledge and skills.

Next Steps

In order to improve his or her performance, the student needs to:

- include a more extensive list of simplifying assumptions;
- solve all equations correctly;
- interpret solutions to equations more effectively;
- formulate a hypothesis more carefully and test it more thoroughly;
- label diagrams and graphs completely;
- add more text to explain the mathematical reasoning;
- edit and proofread the report to improve clarity and to reduce errors and inconsistencies;
- write a general formula for option 3;
- consider more scenarios;
- make more detailed recommendations about a course of action.