

Lifeguard Procedures Report L O W L E V E L 1

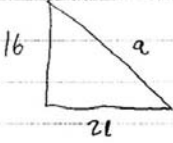
A

Calculus Exemplar

We need to keep in mind that we want to minimize the time, the lifeguard needs to save the swimmer as fast as possible. Therefore we need to find the fastest way (predict Option 3).

Scenario 1

Option 1 – Lifeguard swimming straight to the swimmer, not using land/running

Solu: find time $t = \frac{D}{v}$ → 

$$t = \frac{26.40}{3}$$

$$t = 8.8 \text{ sec}$$

$v = 3 \text{ m/s}$, once the lifeguard will just be on water

Solving with pythagoras

$$a^2 = b^2 + c^2$$

$$a^2 = 657$$

$$a = 26.40 \text{ m}$$

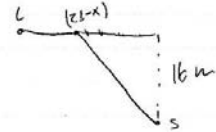
Option 2 – Lifeguard running until reaches a perpendicular angle in relation to the swimmer.

Solu Find the time in land. ① Find time in water.

$t = \frac{D}{v}$	→ $t = 4.2 \text{ sec}$	$t = \frac{16}{3}$	Add both times together
$t = \frac{21}{5}$		$t = 5.33$	4.2 + 5.33
			= 9.5 sec.

B

Option 3: Lifeguard runs until determinate point $(21 - x)$, and swim until the swimmer.



t is equal
time in land + time in water.

Solu:
time in land
 $t = \frac{21-x}{5}$

time in water
 $t = \frac{\text{hypotenuse of the right triangle } (21-x) \text{ and } (16)}}{3}$

To find the hypotenuse
 $a^2 = b^2 + c^2$
 $a^2 = 16^2 + x^2$
 $a = \sqrt{16^2 + x^2}$

$$t = \frac{\sqrt{16^2 + x^2}}{3}$$

$$0 = 4 + \frac{16}{2} + x \cdot 2x \cdot \frac{1}{3}$$

$$0 = 4 + 8 + 2x^2 \cdot \frac{1}{3}$$

$$0 = 4 + 2.667 + 0.666x^2$$

$$0.666x^2 = 6.667$$

$$x = 3.87 \text{ m}$$

$$\therefore t = \frac{21 - 3.87}{5} + \frac{\sqrt{16^2 + (3.87)^2}}{3}$$

$$= 3.426 + 5.487$$

$$= 8.91$$

∴ Option 1 is more suitable

LOW LEVEL 1

C

Scenario 2

(Option 1) Lifeguard swimming straight to the swimmer.

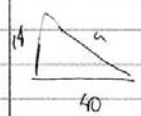
$$t = \frac{D}{V}$$

$$a^2 + b^2 = c^2$$

$$c^2 = 156 + 1600$$

$$c^2 = 1756$$

$$c = 42.38$$

$$t = \frac{42.38}{3} = 14.12 \text{ sec}$$


(Option 2) Lifeguard running until it gets perpendicular to the swimmer.

$$t_1 = \frac{D}{V} \text{ (in land)}$$

$$t_2 = \frac{D}{V} \text{ (in H}_2\text{O)}$$

$$t_1 = \frac{40}{5}$$

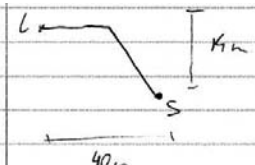
$$t_2 = \frac{14}{3}$$

$$t_1 = 8 \text{ s}$$

$$t_2 = 4.66 \text{ s} \rightarrow 12.66 \text{ sec}$$

D

(Option 3)



$$t(x) = \frac{40-x}{5} + \frac{1}{3} \sqrt{x^2 + 14^2}$$

$$t'(x) = \frac{39}{5} - \frac{1}{3} \frac{x}{\sqrt{x^2 + 14^2}}$$

for t_p : $t'(x) = 0$

$$0 = \frac{7.8}{5} - \frac{x}{3\sqrt{x^2 + 14^2}}$$

$$-7.8 = \frac{x^2 - 2.33x}{6}$$

$$-7.8 = \frac{x^2 + 2.33x}{6}$$

$$\sqrt{5616} = x + 2.33x$$

$$73.658 = 3.33x$$

$$x = 22 \text{ m}$$

$$0 = 7.8 + \frac{x}{6} + \frac{7}{3} \cdot \frac{x}{\sqrt{x^2 + 14^2}}$$

∴ Option 3 is the answer.

$$t = \frac{40 - 22}{5} + \frac{\sqrt{(22)^2 + 14^2}}{3}$$

$$t = 6.578 + \frac{\sqrt{50.5521 + 196}}{3}$$

$$t = 6.578 + \frac{\sqrt{246.5521}}{3}$$

$$t = 6.578 + 6.233$$

$$t = 12.81 \text{ sec}$$

Teacher's Notes**Knowledge and Understanding**

- The student uses a mathematical model with very limited effectiveness. He or she states no simplifying assumptions. The student forms only part of each derivative accurately and makes major errors in solving for x (e.g., writing $(256 + x^2)^{\frac{1}{2}}$ as $\sqrt{256 + x^2}$, simplifying $\frac{1}{2}\sqrt{256 + x^2}$ to $\frac{16}{2} + x$).

Thinking

- The student interprets the solutions to the equations with very limited effectiveness. He or she examines two scenarios and finds the time taken for each of the three options in each case. Because the values of x are incorrect, the values of t are incorrect for option 3. The student makes no attempt to reach any generalizations about the preferred route.
- The student formulates and tests hypotheses with limited effectiveness. Before testing the options, he or she very briefly states a hypothesis, i.e., “we need to find the fastest way (predict Option 3)”. The student examines only the two scenarios given in Appendix A, and does no further testing despite the apparent conflict in the preferred routes for these two scenarios.

Communication

- The student communicates information in diagrams or graphs with limited clarity. He or she includes diagrams for some options, but not for others. The diagrams for option 3 are poorly drawn and are not clearly labelled (e.g., for option 3 of scenario 1, the meaning of “ $(21 - x)$ ” is unclear; for option 3 of scenario 2, x is not labelled at all). The report does not include a graph.
- The student integrates text and mathematical forms with limited effectiveness. He or she includes some descriptions to clarify the mathematical reasoning, but the writing is difficult to read and contains numerous careless

errors (e.g., “Solving with pitagoras”). Because the report lacks some important parts (e.g., generalizations, recommendations), it is impossible to assess how effectively they would be communicated or integrated.

Application

- The student uses formulas with limited effectiveness. He or she chooses appropriate formulas when considering options 1 and 2. However, the Pythagorean formula is carelessly written as “ $c^2 = b^2 + c^2$ ” in one case. For option 3, the student writes separate formulas for the running and swimming times, but he or she does not combine them into a single formula before writing the derivative. The student substitutes the distance and speed values given in Appendix A, but he or she does not consider a general case for option 3, with distances and speeds as variables.
- The student recommends and justifies a course of action with very limited effectiveness. Aside from suggesting a specific option for each examined scenario, he or she does not recommend a course of action.

Comments

This work is representative of a low level-1 performance. The student demonstrates a limited degree of achievement of the expectations in the Communication category of knowledge and skills. The student also demonstrates a limited degree of achievement with respect to one criterion in the Thinking category and one criterion in the Application category. However, with respect to the Knowledge and Understanding category and one criterion in each of the Thinking and Application categories, the student demonstrates a very limited degree of achievement.

LOW LEVEL 1

Next Steps

In order to improve his or her performance, the student needs to:

- include a list of simplifying assumptions;
- find derivatives accurately and solve resulting equations correctly;
- reach generalizations about the preferred route;
- consider more scenarios to test the hypothesis more thoroughly;
- include more diagrams;
- carefully draw and label all diagrams;
- include a graph;
- write more legibly;
- edit and proofread the report to eliminate errors;
- produce a report that is complete;
- write a complete formula for option 3 before finding the derivative;
- consider a general case for option 3;
- recommend and justify an appropriate course of action.