Teacher Package

Mathematics Exemplar Task
Grade 8 – Patterning and Algebra
Teacher Package

Title: Smothered in Chocolate!

Time requirement: 150 minutes (total)
- 30 minutes for pre-task 1
- 30 minutes for pre-task 2
- two periods of 45 minutes each for the exemplar task

Description of the Task

This task requires each student to use interlocking cubes to form larger cubes and to investigate the patterns found when large cubes are made from unit cubes. Students then consider what would happen if they were to submerge the cubes they have made into a container of chocolate. The specific task is to determine how many of the unit cubes would have one face, two faces, or three faces covered with chocolate. Students are also given the opportunity to determine the relationships among the vertices, edges, and faces of cubes. They are requested to express their generalizations algebraically and to sketch cubes on isometric paper.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from the Ministry of Education's *Curriculum Unit Planner* (CD-ROM).

Students will:
1. identify, create, and discuss patterns in algebraic terms (8m75);
2. apply and defend patterning strategies in problem-solving situations (8m78);
3. describe and justify a rule in a pattern (8m79);
4. write an algebraic expression for the n^{th} term of a numeric sequence (8m80);
5. find patterns and describe them using words and algebraic expressions (8m81).

Teacher Instructions

Prior Knowledge and Skills Required

Before attempting the task, students should have some knowledge or skills related to the following:
- understanding the concepts of surface area and volume
- identifying the relationship between surface area and volume
- discussing patterns in algebraic terms
- identifying various types of prisms
- understanding and identifying the following features of a cube: edge, face, and vertex

The Rubric*

The rubric provided with this exemplar task is to be used to assess students' work. The rubric is based on the achievement chart given on page 9 of *The Ontario Curriculum, Grades 1–8: Mathematics, 1997.*

Before asking students to do the task outlined in this package, review with them the concept of a rubric.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

Materials and Resources Required

Before students attempt a particular task, provide them with the appropriate materials from among the following:
- a copy of the Student Package (see Appendix 1) for each student
- interlocking cubes (25 per student)
- graph paper
- calculators

Task Instructions

Introductory Activities

The pre-tasks are designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

Pre-task 1

Have students work in groups of four. Give each group ten interlocking cubes. Begin by asking the following questions:
- How many faces are visible on one cube? On two cubes joined together? On four cubes joined together?
- How does the configuration of the connected cubes affect the number of visible faces? (Answers will vary depending on how the cubes are connected.)

The rubric is reproduced on page 65 of this document.

Pre-task 2

Give each group of four students fifty interlocking cubes. Ask students to identify the number of faces, edges, and vertices for one cube, eight unit cubes (2 x 2 x 2) connected to make one cube, and twenty-seven unit cubes (3 x 3 x 3) connected to make a bigger cube. Have students record the data. Then have them represent their data using a table such as the one below. Reproduce this table on the chalkboard or on an overhead.

Cube containing ...	Vertices	Edges	Faces
1 cube			
8 small cubes			
27 small cubes			

After students complete the task, ask them the following questions:

- What data were the same for the one-unit cube? The eight-unit cube? The twenty-seven-unit cube?
- If you were to submerge the largest cube in a container of chocolate, how many of the unit cubes would have chocolate on three sides?
- If you were to build an even larger cube – say, a 5 x 5 x 5 cube – what are some of the patterns you would observe?
- Does Euler's rule hold for each of the cubes you have built?
  (*Euler's Rule:* Faces + Vertices = Edges + 2 or F + V = E + 2)

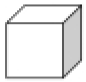
Exemplar Task

1. Distribute a copy of the Student Package to each student.
2. Tell students that they will be working individually and independently to complete the assigned task.
3. Remind students about the rubric and make sure that each student has a copy of it.
4. The problem that the students must solve independently is provided in the worksheets in Appendix 1.

Appendix 1: Student Worksheets

Smothered in Chocolate!

Imagine that each individual cube is a piece of toffee.



1 piece of toffee

Pieces of toffee are joined together to form cubes.
The cubes will then be completely dipped in chocolate.

1. Build three cubes of different sizes using interlocking cubes. Sketch the three cubes on the isometric paper provided at the end of this package.

2. a) For each of your three cubes you have just built, your task will be to determine the total number of cubes required for each structure you have built, and that have chocolate on exactly:
   i) 3 faces
   ii) 2 faces
   iii) 1 face
   iv) 0 faces

Present your data so that someone looking at the way you organize your work will be able to see how you solved the problem.

b) Describe all of the patterns you observe by looking at the data.

3. A number of unit cubes were put together to form a larger cube, which was then painted. The larger cube is subsequently taken apart and it is discovered that 27 of the smaller cubes have no paint on them. How big was the original cube and how many of the smaller cubes had paint on only:

   a) one face?
   b) two faces?
   c) three faces?

Show how you arrived at your answer.

4. If you had a large cube made from unit cubes such that the dimensions of the large cube are $n \times n \times n$, how many pieces would be covered in chocolate on exactly three faces, exactly two faces, exactly one face, and no faces if the entire cube was covered with chocolate?

   Justify your answer.