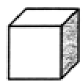
Smothered in Chocolate! Level 3, Sample 1

A

Exemplar Task

Smothered In Chocolate!

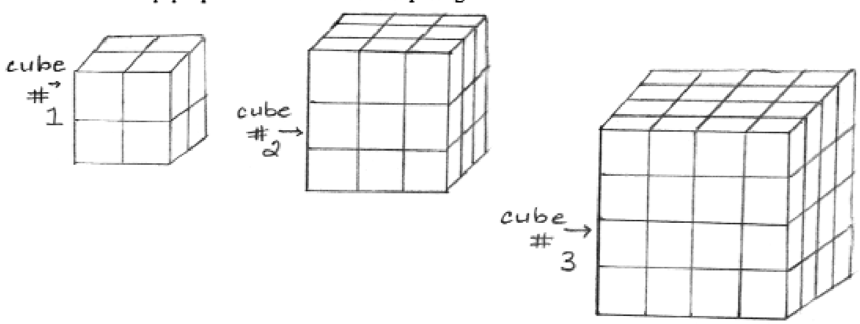
Imagine that each individual cube is a piece of toffee.



1 piece of toffee

Pieces of toffee are joined together to form cubes.
The cubes will then be completely dipped in chocolate.

1. Build three cubes of different sizes using interlocking cubes. Sketch the three cubes on the isometric paper provided at the end of this package.



B

2. For each of your three cubes you have just built, your task will be to determine the total number of cubes required for each structure you have built, and that have chocolate on exactly:
   a) 3 faces
   b) 2 faces
   c) 1 face
   d) 0 faces.

   Present your data so that someone looking at the way you organize your work will be able to see how you solved the problem.

cube # 1 (It took me 8 cubes to create this figure)
a) 3 faces - 8
b) 2 faces - 0
c) 1 face - 0
d) 0 faces - 0

cube # 2 (It took me 27 cubes to create this figure.)
a) 3 faces - 8
b) 2 faces - 12
c) 1 face - 6
d) 0 faces - 1

cube # 3 (It took me 64 cubes to create this figure)
a) 3 faces - 8
b) 2 faces - 24
c) 1 face - 24
d) 0 faces - 8

C

b) Describe all of the patterns you observe by looking at the data.

Cube #1, Cube #2 and cube #3 all have the same number of 3 faces. No matter what size cube, whether it be a 5×5 or a 6×6 there is always 8 - 3 faces.

To see how many cubes you have used for each cube, you can count the number of cubes on the top and then multiply it by the depth of it for example 5×5 would be 25 mulitplied by 5 = 125.

For 2 faces every time you increase the number it goes up by 12. For example 2×2 is 0, 3×3 is 12, 4×4 is 24, 5×5 is 36, 6×6 is 48.

D

3. A number of unit cubes are put together to form a larger cube which was then painted. The larger cube is subsequently taken apart and it is discovered that 27 of the smaller cubes have no paint on them. How big was the original cube and how many of the smaller cubes had paint on only:

i) one face
ii) two faces
iii) three faces

Show how you arrived at your answer.

The Original cube was 5×5×5.

i) one face - 54 - (9 from the top muliplied by 6 because there were 6 sides.)
ii) two faces - 36 (3 cubes with 2 faces on one side at the top multiplied by 12 because there were 12 sides with 3 cubes with 2 face sides.) → part of ii).
iii) three faces - 8

(1 cube with 1 face on one corner at the top multiplied by 8 because there were 8 sides with 1 cube and 3 faces.)

E

4. If you had a large cube made from unit cubes such that the dimensions of the large cube are *n x n x n*, how many pieces would be covered in chocolate on exactly three faces, exactly two faces, exactly one face and no faces if the entire cube was covered with chocolate?
Justify your answer.

3 faces - the number of corners in a cube and that would always no matter how big or small will always be 8.

2 faces - the number of middle cubes on the outside just on one edge and then multiply them by 12.

1 face - the number of block or blocks in the centre of the top multiplied by 6 because there are 6 sides.

0 faces - the number of blocks in the middle of the cube that you cannot see and are never touched. the number of these blocks is bigger when the cube is bigger.

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 4, applies the congruency of the faces to develop a rule).

Understanding of Concepts

– The student demonstrates a general understanding of algebraic patterns (e.g., in question 2, identifies the correct numbers of total cubes and covered and uncovered faces for the three cubes; in question 3, identifies the correct patterns for the number of unit cubes with paint on 3 faces, 2 faces, and 1 face).

Application of Mathematical Procedures

– The student applies mathematical procedures with few errors and/or omissions when analysing the data (e.g., in question 3, applies the patterns discovered in question 2b to find the size of the original cube – 5 x 5 x 5 – and the number of covered faces but does not explain his or her reasoning).
– The student states generalizations and/or algebraic expressions for the *n*th terms that include few errors and/or omissions (e.g., in question 4, explains algebraic expressions accurately to describe the patterns for 1, 2, and 3 covered faces with a partial explanation for 0 faces and no algebraic representations).

Communication of Required Knowledge

– The student uses mathematical language and/or algebraic notation clearly to explain and justify generalizations based on the model (e.g., describes most generalizations in statements without the use of algebraic expressions, as in question 4: "3 faces – the number of corners in a cube and that would always no matter how big or small will always be 8").

Comments/Next Steps

– The student could use charts to list data to help him or her compare data.
– The student should use variables to write or describe algebraic expressions and abstract applications.
– The student needs to use math terminology consistently when communicating discoveries or algebraic expressions.

A

Exemplar Task

Smothered In Chocolate!

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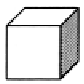

1 piece of toffee

Pieces of toffee are joined together to form cubes.
The cubes will then be completely dipped in chocolate.

1. Build three cubes of different sizes using interlocking cubes. Sketch the three cubes on the isometric paper provided at the end of this package.

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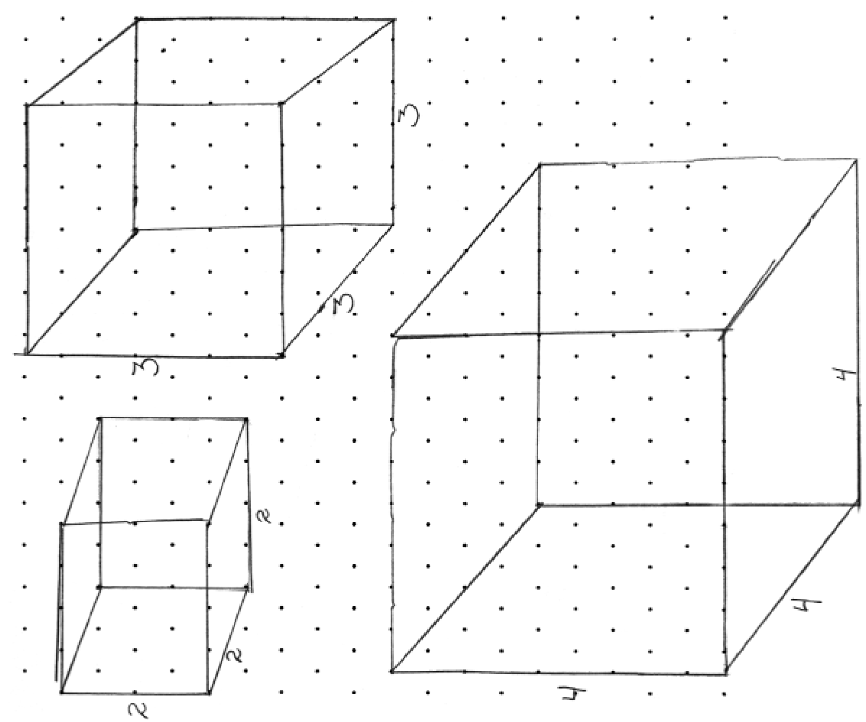

C

2. a) For each of your three cubes you have just built, your task will be to determine the total number of cubes required for each structure you have built, and that have chocolate on exactly:

   i) 3 faces
   ii) 2 faces
   iii) 1 face
   iv) 0 faces.

   Present your data so that someone looking at the way you organize your work will be able to see how you solved the problem.

To determine how many faces are covered and how many cubes needed, simply count how many faces are showing and add the total of number of cubes covered.

2x2x2

# of faces covered	# of cubes covered
3 faces	8
2 faces	0
1 face	0
0 faces	0

8 cubes make up a 2x2x2

3x3x3

# of faces	# of cubes
3 faces	8
2 faces	12
1 face	6
0 faces	1

27 cubes make up a 3x3x3

4x4x4

# of faces	# of cubes
3 faces	8
2 faces	24
1 face	24
0 faces	8

Alltogether

cube	3 faces	2 faces	1 face	0 faces	cubes required
2x2x2	8	0	0	0	8
3x3x3	8	12	6	1	27
4x4x4	8	24	24	8	64

48
+16
64

D

2x2x2
3->8
2->0
1->0
0->0
F->6
E->12
V->8

3x3x3 ->27
3->8
2->12
1->6
0->1
F->6
E->12
V->8

4x4x4 ->
3->8
2->24
1->24
0->8
F->6
E->12
V->8

b) Describe all of the patterns you observe by looking at the data.

The 1st pattern I noticed was that in all the cubes, only 8 of all the cubes had 3 faces showing and it's always the corners. I also noticed that by adding 12 to the number of sides with only 2 faces showing, you will get the next cube. I'm not sure if I see a pattern between the cubes with only one side showing, and the same with no cubes showing.

In the 1st cube, I see that each cube has 3 faces showing. The only other pattern I can see is by counting each side as one face instead of counting each invidual cube all cubes have 6 faces, 12 edges and 8 verticies, no matter how large or small the cube is.

Patterns

Pattern #	What it is
1	In each cube, only 8 individual cubes have 3 faces that show.
2	are 8 cubes are the corners
3	by adding 12 to any cube with 2 faces showing, your sum will be the next answer.
4	In the first cube, each individual cube has 3 faces showing.
5	All cubes have 6 faces, 12 edges and 8 vertices.
6	All cubes with 1 face showing are multiples of 6

E

3. A number of unit cubes are put together to form a larger cube which was then painted. The larger cube is subsequently taken apart and it is discovered that 27 of the smaller cubes have no paint on them. How big was the original cube and how many of the smaller cubes had paint on only:

   i) one face
   ii) two faces
   iii) three faces

   Show how you arrived at your answer.

*cube must have more than 27 individual cubes
→27 divided evenly = 3
→put a cube on each side : 5 → width
→27 cubes make a 3×3×3
→if 5 is width, 5 is also length and height because all dimensions must be the same to make a cube.

The original cube is 5×5×5.

Cubes in Paint

faces covered	# of cubes
one face	54
two faces	36
three faces	8

If my patterns are correct, 8 cubes will have 3 faces, 36 will have 2. Because I didn't have a pattern for one face, you must find the number. 5×5×5 = 125. 125 cubes are needed to make a 5×5×5. 27+36+8 = 27+44 = 71. 71 cubes are already being used. 125−71 = 54. 54 cubes have one face with paint.

F

4. If you had a large cube made from unit cubes such that the dimensions of the large cube are $n \times n \times n$, how many pieces would be covered in chocolate on exactly three faces, exactly two faces, exactly one face and no faces if the entire cube was covered with chocolate?
   Justify your answer.

Usually, it depends on what n is equal to. For example:

let n=5 → 5×5×5
3 faces → 8
2 faces → 36
1 face → 54

	3 faces	2 faces	1 face	0 faces
2x2x2 →	8	0	0	0
3x3x3 →	8	12	6	1
4x4x4 →	8	24	24	8
5x5x5 →	8	36	54	27

In a cube, there is always 8 corners, so no matter what the dimensions are, 8 cubes will always have 3 faces showing. For 2 faces, it will always be a multiple of 12. 1 face showing is always a multiple of 6.

Teacher’s Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 3, explains how the answer of 5 x 5 x 5 is determined).

Understanding of Concepts

- The student demonstrates a general understanding of algebraic patterns (e.g., in question 2b, identifies the correct pattern for the number of unit cubes with 3 faces covered in chocolate and provides partial explanations of patterns for unit cubes with 1 and 2 chocolate-covered faces).

Application of Mathematical Procedures

- The student applies mathematical procedures with few errors and/or omissions when analysing the data (e.g., in question 3, applies the patterns discovered in question 2b to find the size of the original cube – 5 x 5 x 5 – and explains how the answer was obtained).
- The student states generalizations and/or algebraic expressions for the *n*th terms that include few errors and/or omissions (e.g., in question 4, provides explanations for the number of cubes with 1 and 2 chocolate-covered faces).

Communication of Required Knowledge

- The student uses mathematical language and/or algebraic notation clearly to explain and justify generalizations based on the model (e.g., describes generalizations in statements without the use of algebraic expressions, as in question 4: “In a cube, there is always 8 corners, so no matter what the dimensions are, 8 cubes will always have 3 faces showing”).

Comments/Next Steps

- The student should use isometric paper correctly to sketch cubes.
- The student needs to use variables to write or describe algebraic expressions and abstract applications.
- The student should use correct mathematical terminology consistently and should complete all explanations of procedures.