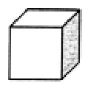
Smothered in Chocolate! Level 2, Sample 1

A

Exemplar Task

Smothered In Chocolate!

Imagine that each individual cube is a piece of toffee.

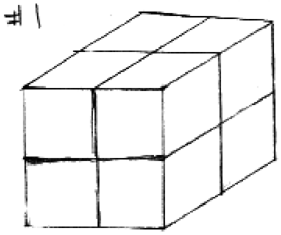


1 piece of toffee

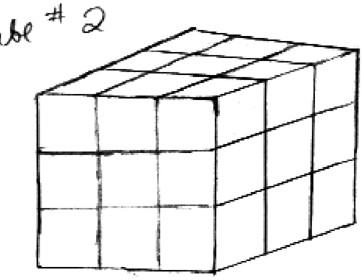
Pieces of toffee are joined together to form cubes.
The cubes will then be completely dipped in chocolate.

1. Build three cubes of different sizes using interlocking cubes. Sketch the three cubes on the isometric paper provided at the end of this package.

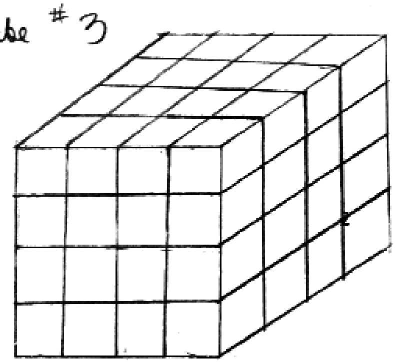
cube #1



cube #2



cube #3



B

2. For each of your three cubes you have just built, your task will be to determine the total number of cubes required for each structure you have built, and that have chocolate on exactly:
   a) 3 faces
   b) 2 faces
   c) 1 face
   d) 0 faces.

   Present your data so that someone looking at the way you organize your work will be able to see how you solved the problem.

cube #1
a) 8
b) 0
c) 0
d) 0

cube #2
a) 8
b) 12
c) 6
d) 1

cube #3
a) 8
b) 16
c) 24
d) 4

C

b) Describe all of the patterns you observe by looking at the data.

By looking at the data I found out that all cubes (made out of cubes) have 8 three faces and no matter how big or small it is, it will be the same.

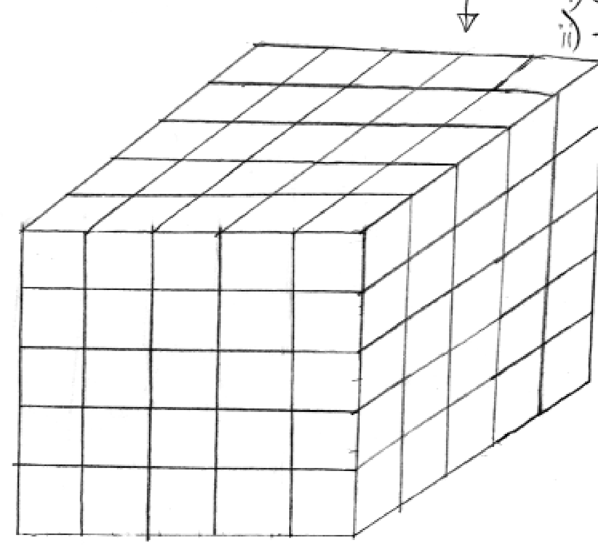
D

3. A number of unit cubes are put together to form a larger cube which was then painted. The larger cube is subsequently taken apart and it is discovered that 27 of the smaller cubes have no paint on them. How big was the original cube and how many of the smaller cubes had paint on only:

i) one face
ii) two faces
iii) three faces

Show how you arrived at your answer.

This is how big the original cube is.



i) one face - 54
ii) two faces - 24
iii) three - 8

E

4. If you had a large cube made from unit cubes such that the dimensions of the large cube are *n x n x n*, how many pieces would be covered in chocolate on exactly three faces, exactly two faces, exactly one face and no faces if the entire cube was covered with chocolate?
Justify your answer.

	i) 3 faces	ii) two faces	iii) one face	iv) no face
a.) 2x2x2	8	0	0	0
b.) 3x3x3	8	8	6	1
c.) 5x5x5	8	24	54	27

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., in question 2, provides three lists to record data, but they contain two errors; in question 3, draws a diagram but does not show or explain the problem-solving method used).

Understanding of Concepts

– The student demonstrates some understanding of algebraic patterns (e.g., in question 2b, identifies the correct pattern for the number of unit cubes with 3 faces covered in chocolate, but does not identify patterns for 0, 1, and 2 covered faces).

Application of Mathematical Procedures

– The student applies mathematical procedures with some errors and/or omissions when analysing the data (e.g., in question 3, gives the correct number of cubes with 1 and 3 painted faces but not 2 painted faces; draws the 5 x 5 x 5 cube, but there are no mathematical procedures evident to support the answer of 5 x 5 x 5).

– The student states generalizations and/or algebraic expressions for the *n*th terms that include some errors and/or omissions (e.g., in question 2b, accurately describes the patterns for 3 faces covered in chocolate but does not identify patterns for 0, 1, and 2 chocolate-covered faces; in question 4, uses a chart that contains errors when explaining generalizations).

Communication of Required Knowledge

– The student uses mathematical language and/or algebraic notation with some clarity to explain and justify generalizations based on the model (e.g., in question 2b, notices that, however large the cube made from unit cubes is, the eight corner areas will always be covered: "all cubes (made out of cubes) have 8 three faces and no matter how big or small it is, it will be the same"; in question 4, uses simple charts and diagrams for explanations; provides only simple written explanations).

Comments/Next Steps

- The student needs to use isometric paper to sketch cubes.
- The student should use charts to list data to help him or her identify patterns. All charts should be accurately labelled to ensure easy interpretation of the data.
- The student needs additional practice in identifying and recording patterns.
- The student should retrace his or her steps if he or she is unable to find a pattern.

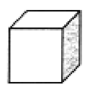
Smothered in Chocolate! Level 2, Sample 2

A

Exemplar Task

Smothered In Chocolate!

Imagine that each individual cube is a piece of toffee.

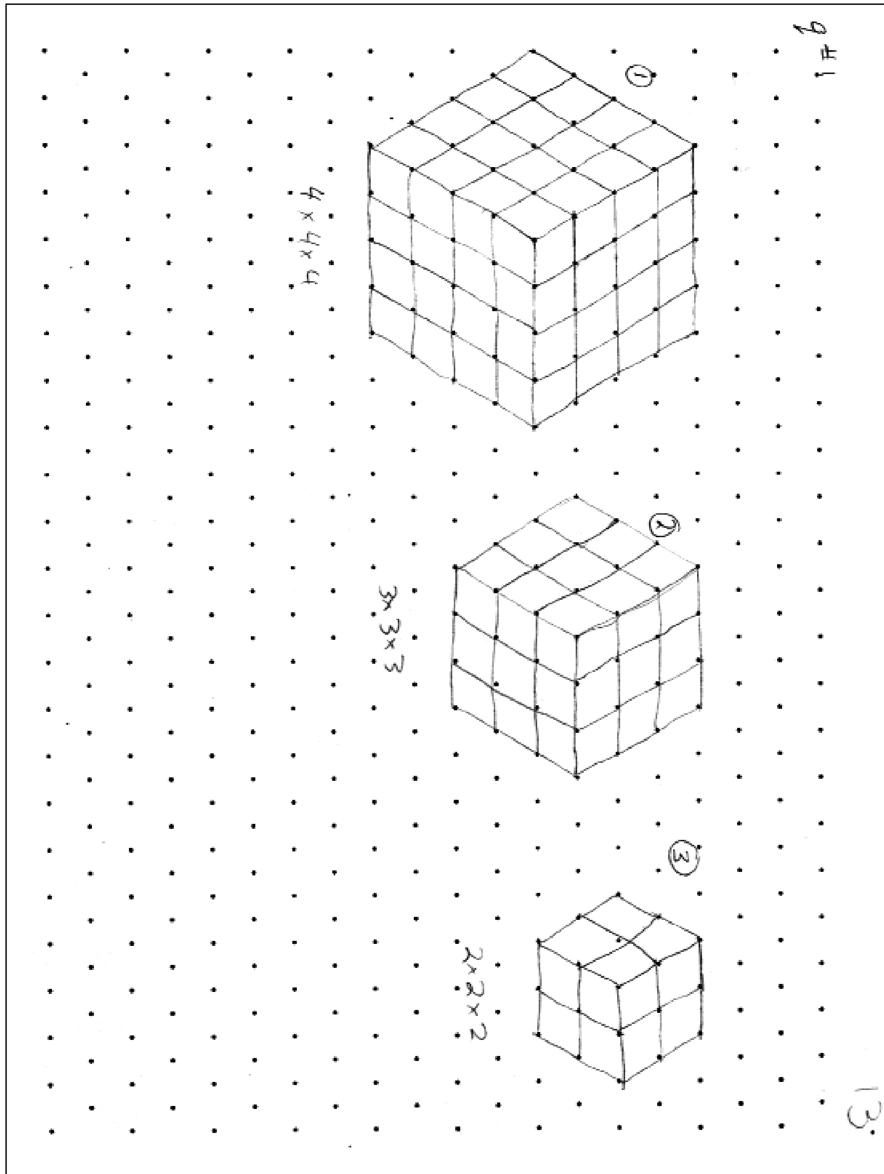


1 piece of toffee

Pieces of toffee are joined together to form cubes.
The cubes will then be completely dipped in chocolate.

1. Build three cubes of different sizes using interlocking cubes. Sketch the three cubes on the isometric paper provided at the end of this package.

B



C

2. a) For each of your three cubes you have just built, your task will be to determine the total number of cubes required for each structure you have built, and that have chocolate on exactly:

i) 3 faces
ii) 2 faces
iii) 1 face
iv) 0 faces.

Present your data so that someone looking at the way you organize your work will be able to see how you solved the problem.

4x4x4 cube

8 cubes have 3 faces covered in chocolate these cubes are the 8 corner ones
16 cubes have 2 faces covered in chocolate these are the cubes between the corner's.
24 cubes have chocolate on 1 face they are the ones in the middle of the sides.
16 cubes have no chocolate on them I calculated this by counting all the cubes and taking away the ones with chocolate on 3 faces, 2 faces and 1 face.

3x3x3 cube

8 cubes have chocolate on 3 sides there are the corners
13 cubes have chocolate on 2 sides these are the ones between the corners.
6 cubes have chocolate on 1 face these are the cubes in the center of each side.
1 cube has No chocolate and it's the one in the very center

2x2x2 cube.

this cube only has 8 cubes in it so therefor all of the cubes have chocolate on 3 sides.

D

b) Describe all of the patterns you observe by looking at the data.

I think that the only recagnisable patern is that no mater how big or small the cube is there will always be 8 little cubes with 3 sides showing and these cubes are always the corner cubes.

E

3. A number of unit cubes are put together to form a larger cube which was then painted. The larger cube is subsequently taken apart and it is discovered that 27 of the smaller cubes have no paint on them. How big was the original cube and how many of the smaller cubes had paint on only:

   i) one face : 54
   ii) two faces
   iii) three faces 8

   Show how you arrived at your answer.

The origonal cube was 5x5x5.
how I calculated this was that
I taulk a cube made of 27 unit
cubes then bilt a case around it.

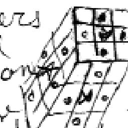
There were 54 cubes with paint
on one face.

24 unit cubes had paint on
2 sides and 8 unit cubes have
paint on 3 sides and these are
the corner cubes.

F

4. If you had a large cube made from unit cubes such that the dimensions of the large cube are *n x n x n*, how many pieces would be covered in chocolate on exactly three faces, exactly two faces, exactly one face and no faces if the entire cube was covered with chocolate?
   Justify your answer.

No matter what n is equal to you will
always have 8 unit cubes with paint on them
they are the corner cubes. Then you add
up any unit cubes that are on the sides
between the corners the ones with
dots on them and that is how
many have paint on two faces then
you count up the rest on the out-
side the ones with stars then you know
how many have paint on one face then
all the cubes left in the center are inside
of the cube have no paint on them any
where



Teacher's Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., in question 1, correctly draws three cubes; in question 2, describes many of the correct numbers of unit cubes with 0, 1, 2, and 3 faces covered in chocolate for two of the cubes, and the correct number of cubes with 3 chocolate-covered faces for the 2 x 2 x 2 cube; in question 3, states that he or she built a model "case around it" to find the original cube).

Understanding of Concepts

- The student demonstrates some understanding of algebraic patterns (e.g., in question 2b, identifies and describes only the correct pattern for determining the number of unit cubes with 3 faces covered in chocolate).

Application of Mathematical Procedures

- The student applies mathematical procedures with some errors and/or omissions when analysing the data (e.g., in question 3, finds the correct number of paint-covered faces on the unit cubes in a 5 x 5 x 5 cube without finding the total number of unit cubes used to create the larger cubes; provides some correct answers but does not show any calculations).
- The student states generalizations and/or algebraic expressions for the *n*th terms that include some errors and/or omissions (e.g., in question 4, attempts to explain all his or her conclusions, but provides simplistic and incomplete statements).

Communication of Required Knowledge

- The student uses mathematical language and/or algebraic notation with some clarity to explain and justify generalizations based on the model (e.g., in question 2b, describes only one pattern: "I think that the only recagnisable patern is that no mater how big or small the cube is there will always be 8 little cubes with 3 sides showing and these cubes are alway's the carner cubes").

Comments/Next Steps

- The student should use charts and/or systematic lists to help him or her identify patterns.
- The student should use variables to write or describe algebraic expressions.
- The student needs to use mathematical procedures (describing, generalizing, and translating into algebraic expressions to the *n*th terms) to justify his or her assertions.
- The student should retrace his or her steps if he or she is unable to find a pattern.