Teacher Package

Mathematics Exemplar Task
Grade 8 – Geometry and Spatial Sense

Teacher Package

Title: Exploring the Pythagorean Theorem

Time requirement: 175 minutes (total)
- 40 minutes for pre-task 1
- 45 minutes for pre-task 2
- two periods of 45 minutes each for the exemplar task

Description of the Task

This task requires each student to draw or construct as many non-congruent squares as possible on either five by five geopaper or a five by five geoboard, and to determine the area of each of the squares. Each student is then asked to construct a right-angled triangle, to draw a square on each side of the triangle, and to determine the area of each of the squares formed on the sides of the triangle. Students will then compare the sizes of the squares drawn on the sides of the right-angled triangles and will state the relationship among the squares – the Pythagorean theorem. Finally, they will be asked to construct semicircles on the sides of a right-angled triangle and to determine whether the area of the semicircle on the hypotenuse is equal to the combined areas of the semicircles on the other two sides.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from the Ministry of Education's *Curriculum Unit Planner* (CD-ROM).

Students will:

1. investigate geometric mathematical theories to solve problems (8m59);
2. use mathematical language effectively to describe geometric concepts, reasoning, and investigations (8m60);
3. investigate the Pythagorean relationship using area models and diagrams (8m65);
4. apply the Pythagorean relationship to numerical problems involving area and right triangles (8m70);
5. explain the Pythagorean relationship (8m73).

Teacher Instructions

Prior Knowledge and Skills Required

To complete this task, students should have some knowledge or skills related to the following:
- the concepts of area, perimeter, square root, and perfect squares
- the properties of right-angled, obtuse, and acute triangles
- the difference between drawing a figure and constructing one

The Rubric*

The rubric provided with this exemplar task is to be used to assess students' work. The rubric is based on the achievement chart given on page 9 of *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*.

Before asking students to do the task outlined in this package, review with them the concept of a rubric.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

Materials and Resources Required

Before students attempt a particular task, provide them with the appropriate materials from among the following:
- a copy of the Student Package (see Appendix 1) for each student
- 100 tiles per pair of students (squares of construction paper could be substituted)
- numbered cubes – dice (a minimum 3 per student)
- rulers
- paper
- protractors
- calculators
- compasses
- geostrips (if available)
- optional: centimetre grid paper, toothpicks, geoboards and elastics, The Geometer's Sketchpad*

* The Geometer's Sketchpad software is licensed to the Ontario Ministry of Education (1999).

The rubric is reproduced on pages 13–14 of this document.

Task Instructions

Introductory Activities

The pre-tasks are designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

Pre-task 1: Constructing Squares

1. Provide each pair of students with 100 tiles.
2. Ask students to identify a square with an area of one square unit. Then ask them to use the tiles they have been given to construct different-sized squares.

   Ask students questions like the following:
   - How are the dimensions of the square you constructed related to the number of tiles you used?
   - How is the number of tiles per side related to the area of the square?
   - Do you notice anything of interest when you calculate the square root of the area of any square?
   - If you double the length of the sides of a square, what happens to the area?

Note that The Geometer's Sketchpad can be used effectively with this activity.

Pre-task 2: The Geometer's Sketchpad

This task can be done as a whole-class demonstration or students can work in pairs. Manipulatives (e.g., toothpicks, geoboards, geostrips) should be made available for students' use. Students may also draw by hand or investigate using The Geometer's Sketchpad.

Have students roll three numbered cubes (dice) at the same time. Then ask: "Is it always possible to create a triangle using the numbers on the dice to represent the lengths of the sides? For example, if the numbers 2, 2, and 3 were rolled, would you be able to form a triangle with sides of 2 units, 2 units, and 3 units? Experiment to find out."

Ask students to construct triangles from the numbers they rolled and investigate the types of triangles constructed.

Ask students questions like the following:
- Will it always be possible to make a triangle using the numbers rolled as the dimensions of the sides? Give reasons for your answer.
- Assuming that you roll the same number on all three dice, can you make more than one kind and size of triangle? Why or why not?
- What numbers is it possible to roll that cannot be used to create a triangle? Explain why.
- What is the probability of forming a triangle from three rolls of the dice?
- What is the probability of forming an equilateral triangle from one roll?

Students can continue to investigate the probability of creating a triangle given the numbers on regular dice. For variety, try using different types of dice (e.g, tetrahedron, octahedron, dodecahedron).

Exemplar Task

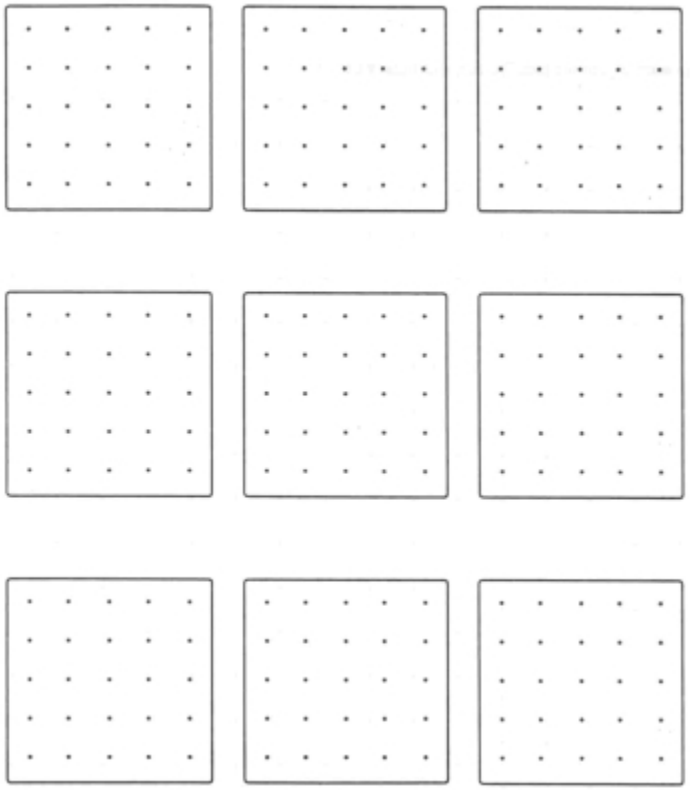
1. Distribute a copy of the Student Package to each student.
2. Students should have access to tiles and The Geometer's Sketchpad.
3. Tell students that they will be working individually and independently to complete the assigned task.
4. Remind students about the rubric and make sure that each student has a copy of it.
5. Students can use the space provided in the Student Package or additional sheets of paper for their responses. Computer printouts are encouraged.
6. The problem that the students will solve independently is provided in the worksheets in Appendix 1.

Appendix 1: Student Worksheets

Exploring the Pythagorean Theorem

1. a) How many different sizes of squares can you draw on a 5 by 5 geopaper or make on a 5 by 5 geoboard? [There are more than 5.]

   b) Show how you would determine the area of each of the squares.



2. a) On a geoboard or geopaper, make or draw a right-angled triangle.

   b) On each side of the triangle, make or draw a square.

c) How do the squares compare? Record any relationship you observe between or amongst the areas of the squares. Think of a way of recording your data so that someone looking at it will be able to tell what you are thinking.

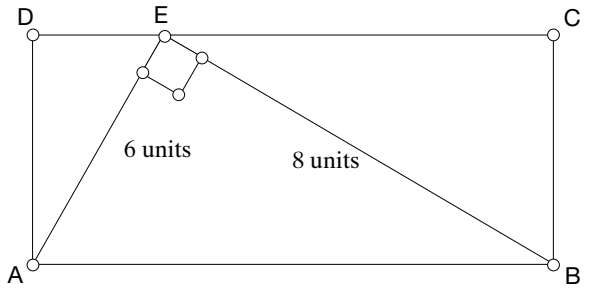
3. a) Using either geoboards, geopaper, Geometer's Sketchpad, or geostrips, construct two different right-angled triangles. Show your work below.

b) Construct, as you did in question 1, squares on each of the sides of the triangles you have just drawn.

c) Examine the relationships between and amongst the areas of these new squares you have just constructed. Summarize what you think is true about squares constructed on the sides of right-angled triangles.

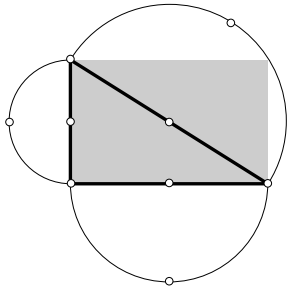
d) If you were to throw three numbered cubes – with each of the digits 1 to 6 on each cube – and use the three numbers facing up to construct a triangle, in how many of the cases would you be able to form a right-angled triangle?

4. ABCD is a rectangle. Find the area of ABCD using information from triangle AEB.



Explain how you found the area.

5. This is a diagram of a right-angled triangle with semi-circles constructed on each of the three sides.



Is the same relationship among the squares true for the semi-circles? Investigate.

Appendix 2

Geopaper