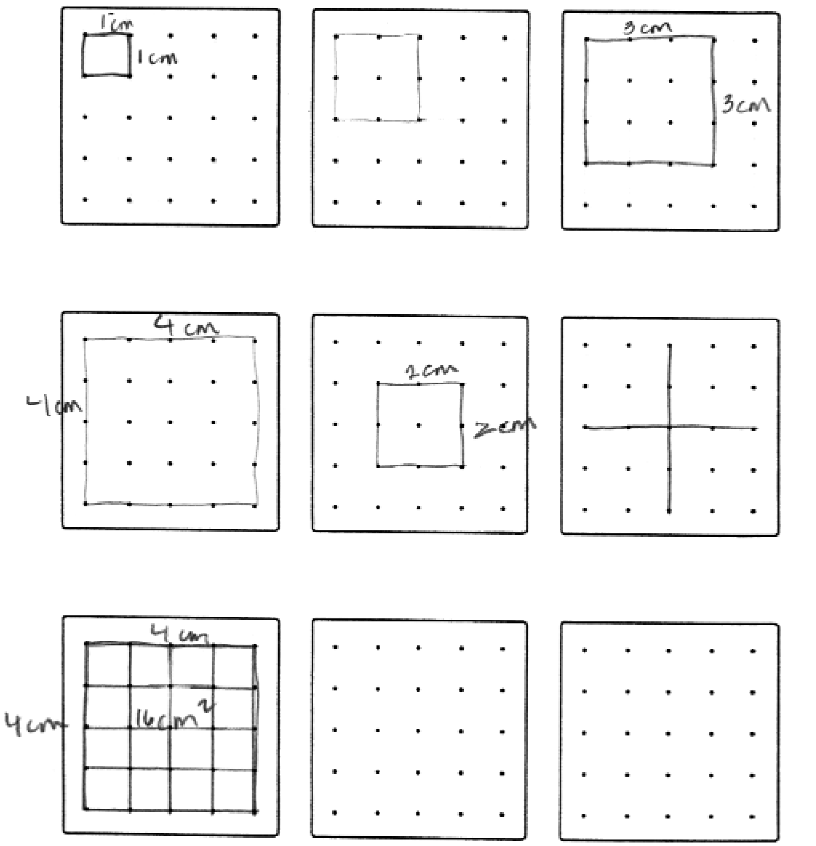
Exploring the Pythagorean Theorem Level 2, Sample 1

A

Exploring the Pythagorean Theorem

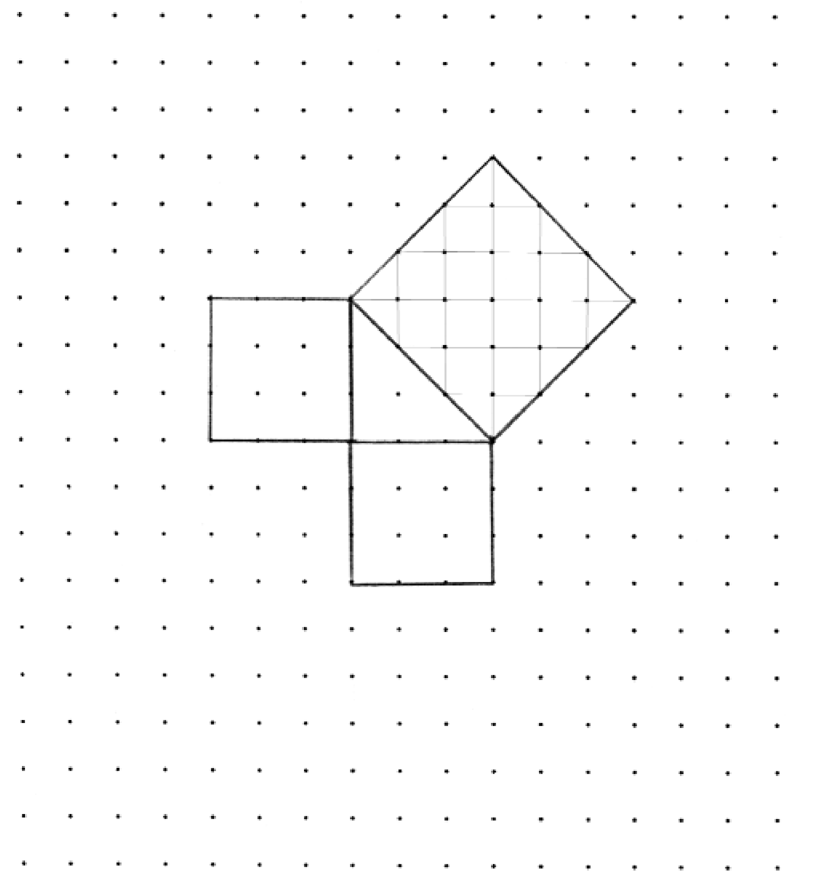
1. a) How many different sizes of squares can you draw on a 5 by 5 geopaper or make on a 5 by 5 geoboard? [There are more than 5].

   b) Show how you would determine the area of each of the squares.



B

2. a) On a geoboard or geopaper, make or draw a right-angled triangle.



b) On each side of the triangle, make or draw a square.

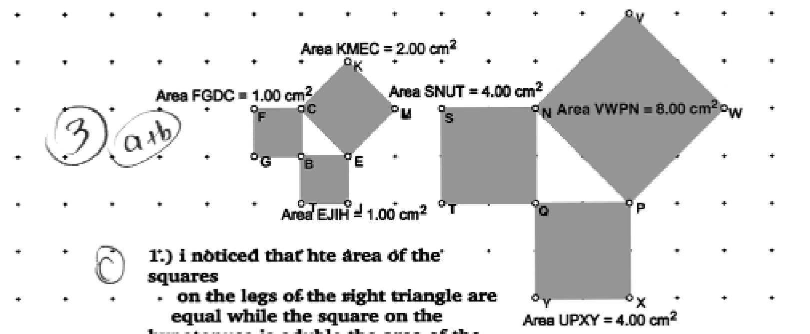
C

c) How do the squares compare? Record any relationship you observe between or amongst the areas of the squares. Think of a way of recording your data so that someone looking at it will be able to tell what you are thinking.

One of the relationships I noticed is that the square on the side of the hypotenuse is larger than the squares on the other two sides. I noticed that the area of the larger square is double the area of the other two squares.

D

3. a) Using either geoboards, geopaper, Geometer's Sketchpad, or geostrips, construct two different right-angled triangles. Show your work below.



c 1.) i noticed that hte area of the squares on the legs of the right triangle are equal while the square on the hypotenuse is oduble the area of the squares on the legs.

d 2.)6/216 times you would be able to form a right angle triangle. this is true because the first two numbers must add up to the other number.

b) Construct, as you did in question 1, squares on each of the sides of the triangles you have just drawn.

E

c) Examine the relationships between and amongst the areas of these new squares you have just constructed. Summarize what you think is true about squares constructed on the sides of right-angled triangles.

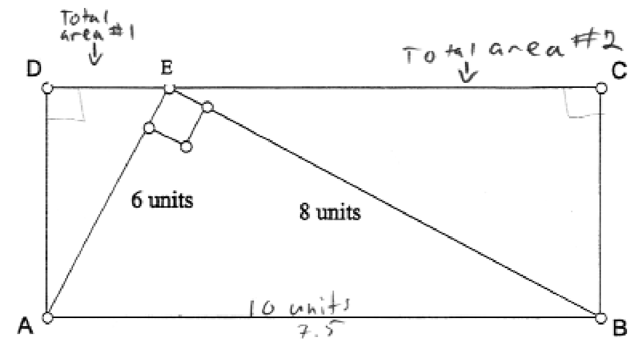
See page 5 *

d) If you were to throw three numbered cubes – with each of the digits 1 to 6 on each cube – and use the three numbers facing up to construct a triangle, in how many of the cases would you be able to form a right-angled triangle?

See page 5 *

F

4. ABCD is a rectangle. Find the area of ABCD using information from triangle AEB.



Total area # 1 = 29.75 u²

Explain how you found the area.

$(6^2) + (8^2) = 100$

$\sqrt{100} = 10$ units

$10 \div 4 = 2.5$

$(2.5^2) - (6^2) =$

$36 - 6.25 = \sqrt{29.75} = 5.6^2 = 29.75$

$7.5^2 = 56.25$

$8^2 = 64 - 56.25 = \sqrt{7.75} = 2.7838822^2$

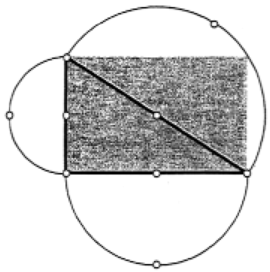
$= 7.75 =$ Total area #2 = 7.75

$29.75 + 7.75 = 37.5$ units²

* Page D in this sample

G

5. This is a diagram of a right-angled triangle with semi-circles constructed on each of the three sides.



$\pi \times r^2$
area of circle

Is the same relationship among the squares true for the semi-circles? Investigate.

The relationship is not the same because the radius^2 of the circle will not be the same as the the length of a square.
(ie 6 6 $6 \times 6 = 36 cm^2$, 6 radius$^2 = 9 \times 3.14 = 28.26$)

Teacher's Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy to determine the areas of some of the different-sized squares that can be drawn on a 5 x 5 geoboard or on 5 x 5 geopaper, arriving at a partially complete and/or partially accurate solution (e.g., in question 1, draws various squares, showing their dimensions and subdividing the last one drawn into 1 cm squares to show 16 cm^2).
- The student selects and applies an appropriate problem-solving strategy to solve a problem related to the Pythagorean theorem, arriving at a partially complete and/or partially accurate solution (e.g., in questions 2 and 3, draws or constructs only isosceles right-angled triangles and then concludes that the area of the square on the hypotenuse is "double the area of the other two squares" ["double the area of the squares on the legs"] rather than the sum of the areas of the other two squares).

Understanding of Concepts

- The student demonstrates some understanding of the Pythagorean theorem when analysing the data and looking for relationships (e.g., in question 2c, realizes that the square on the hypotenuse is larger than the other two squares, but identifies it as "double the area of the other two squares").

Application of Mathematical Procedures

- The student applies mathematical procedures with some errors and/or omissions when investigating the Pythagorean theorem (e.g., in question 4, applies the Pythagorean theorem correctly to find the length of the hypotenuse of the right-angled triangle, but the area of the rectangle is inaccurate; in question 5, identifies the formula for the area of a circle, but applies the formula to one circle rather than to all these semicircles).

Communication of Required Knowledge

- The student uses mathematical language and notation with some clarity to analyse and describe geometric relationships and concepts (e.g., in question 5, makes the following statement, but does not fully explain it: "The relationship is not the same because the radius^2 of the circle will not be the same as the length of a square").

Comments/Next Steps

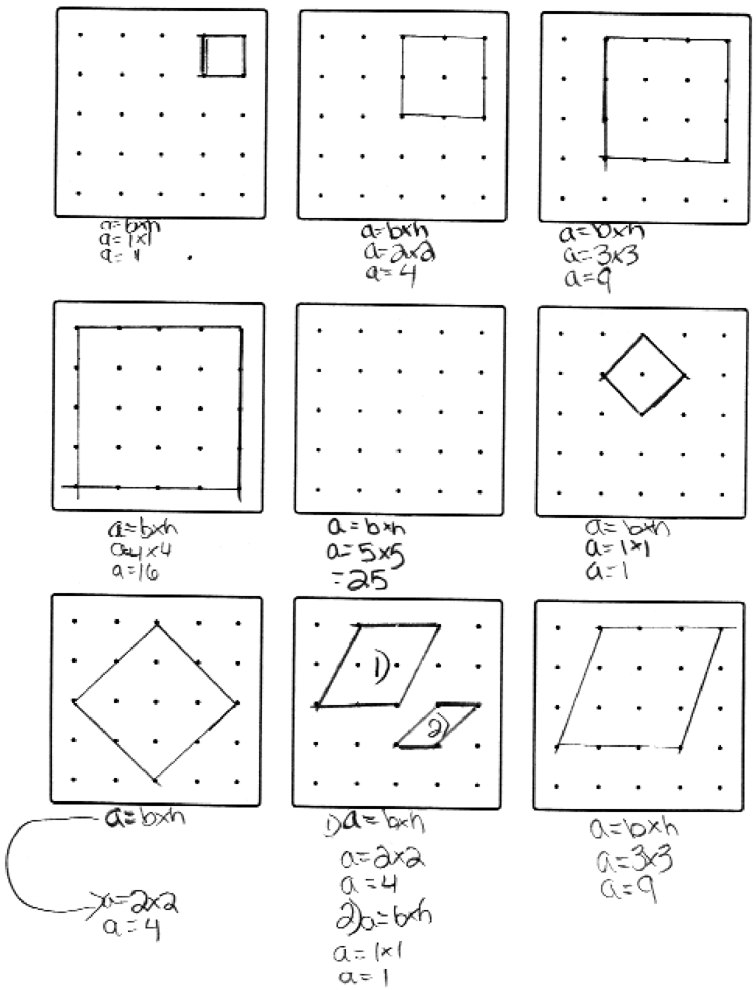
- The student should continue using mathematical software, such as The Geometer's Sketchpad, when solving problems and looking for relationships.
- The student needs to review his or her completed work, working backward to check solutions for accuracy.
- The student should record discovered relationships as equations in an effort to explain the relationships that are found.
- The student should make a conclusion based on a variety of scale drawings, not leap to a conclusion based on too special an example.

A

Exploring the Pythagorean Theorem

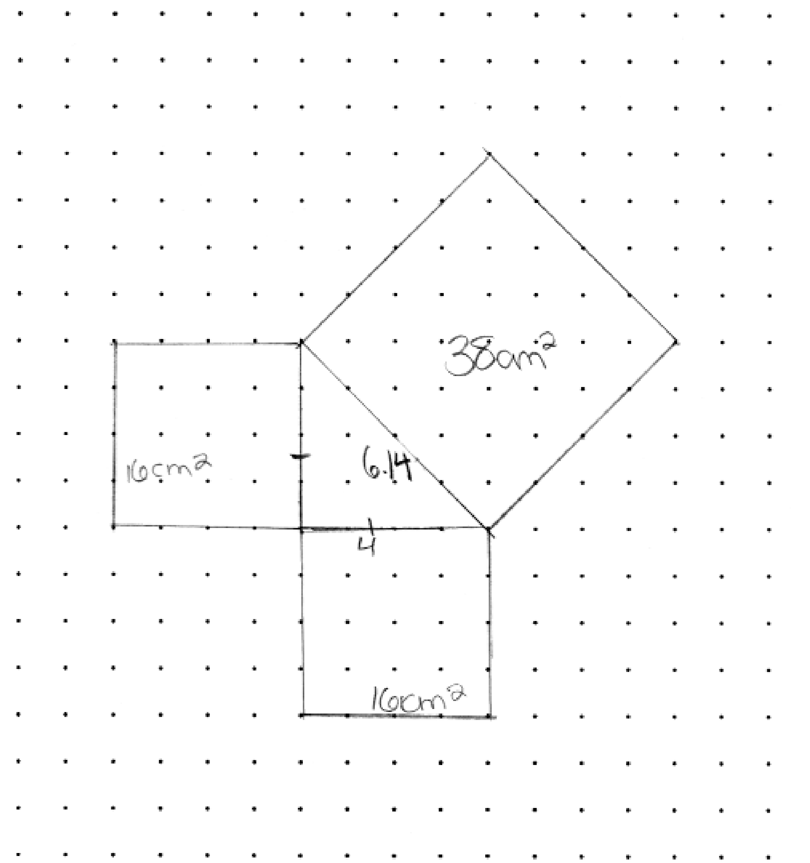
1. a) How many different sizes of squares can you draw on a 5 by 5 geopaper or make on a 5 by 5 geoboard? [There are more than 5].

   b) Show how you would determine the area of each of the squares.



B

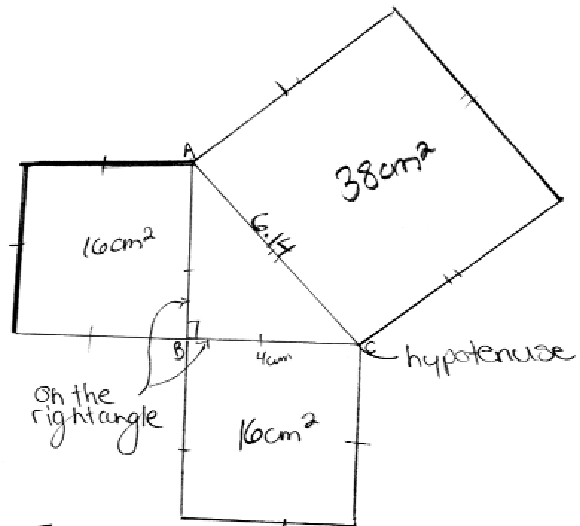
2. a) On a geoboard or geopaper, make or draw a right-angled triangle.



b) On each side of the triangle, make or draw a square.

C

c) How do the squares compare? Record any relationship you observe between or amongst the areas of the squares. Think of a way of recording your data so that someone looking at it will be able to tell what you are thinking.



$\overline{AB}$ and $\overline{BC}$ are on right angle. if you square the length of each line on the right angle and you add them together the sum is equal to the square of the length of the hypotenuse

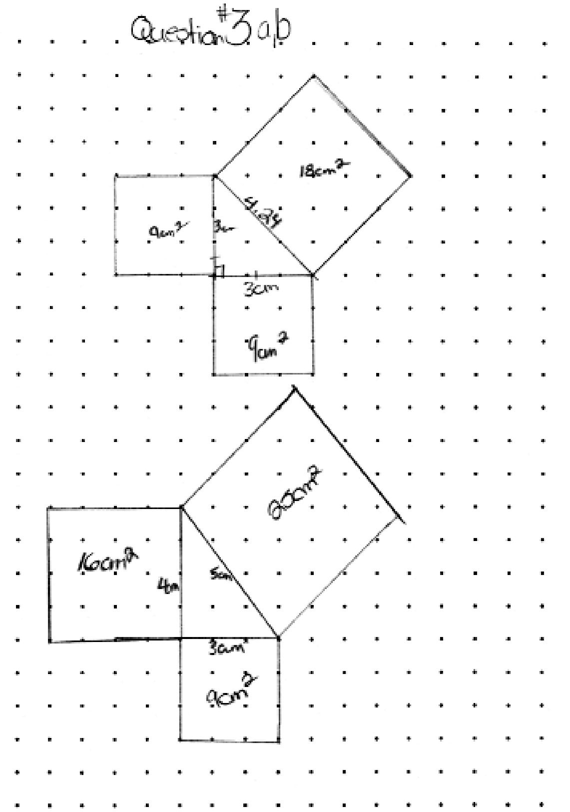
Proof: 4^2+4^2
$= 16+16$
$= 38$

$\sqrt{38} = 6.14$

$\overline{AC}^2 - \overline{BC}^2 = \overline{AB}^2$
$= 6.14^2 - 4^2 = \overline{AB}^2$
$= 38 - 16 = 16 (\overline{AB}^2)$

D

3. a) Using either geoboards, geopaper, Geometer's Sketchpad, or geostrips, construct two different right-angled triangles. Show your work below.



b) Construct, as you did in question 1, squares on each of the sides of the triangles you have just drawn.

E

c) Examine the relationships between and amongst the areas of these new squares you have just constructed. Summarize what you think is true about squares constructed on the sides of right-angled triangles.

the sum of the length squared on the right angle is equal to the length of the hypotenuse squared

d) If you were to throw three numbered cubes – with each of the digits 1 to 6 on each cube – and use the three numbers facing up to construct a triangle, in how many of the cases would you be able to form a right-angled triangle?

1,2,3
1,3,4
1,4,5
1,5,6
2,3,4
2,4,5
2,5,6
3,4,5
3,5,6
3,6,6

1,2,2
1,3,3
1,4,4
1,5,5
1,6,6
2,1,1
2,3,3
2,4,4
2,5,5
2,6,6

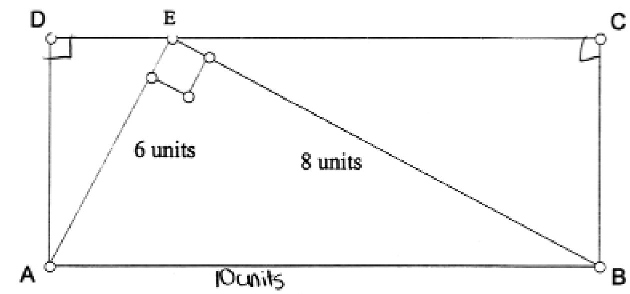
3,1,1
3,2,2
3,4,4
3,5,5
3,6,6
4,1,1
4,2,2
4,3,3
4,5,5
4,6,6
5,6,6
5,4,4
5,3,3
5,2,2
5,1,1

6,1,1
6,2,2
6,3,3
6,4,4
6,5,5
4,5,6

41 possibilities

F

4. ABCD is a rectangle. Find the area of ABCD using information from triangle AEB.



Explain how you found the area.

$aeb = 6^2 + 8^2 = \overline{AB}^2$
$= 36 + 64 = \overline{AB}^2$
$= \sqrt{100} = \sqrt{AB^2}$
$= 10 = \overline{AB}$

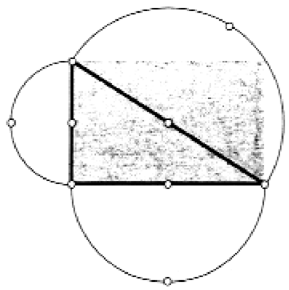
The triangle AEB is Half the retangle
so multiply it by 2

$10 \times 2 = 20$

$a = l \times w$
$a = 10 \times 2$
$a = 20$

G

5. This is a diagram of a right-angled triangle with semi-circles constructed on each of the three sides.



Is the same relationship among the squares true for the semi-circles? Investigate.

No if you do the work it's not the same

$a = \pi r^2$
$a = 3.14 \times 2.5^2$
$a = 3.14 \times 6.25$
$a = 19.629$

$\frac{19.629}{2} = 9.8145$

$a = \pi r^2$
$a = 3.14 \times 3.5^2$
$a = 3.14 \times 12.29$
$a = 38.469$

$\frac{38.469}{2} = 19.2329$

9.814
×2
19.629

The two on the right angle is a bid greater the the hypotoneuse

Teacher's Notes

Problem Solving

– The student selects and applies an appropriate problem-solving strategy to determine the areas of some of the different-sized squares that can be drawn on a 5 x 5 geoboard or on 5 x 5 geopaper, arriving at a partially complete and/or partially accurate solution (e.g., in question 1, draws different sizes of squares, uses a strategy to determine the areas of the squares, but experiences problems in determining the lengths of the diagonals).

– The student selects and applies an appropriate problem-solving strategy to solve a problem related to the Pythagorean theorem, arriving at a partially complete and/or partially accurate solution (e.g., in question 3d, lists many of the possible outcomes when rolling three dice; some of these will not form a triangle and only one – 3-4-5 – will form a right-angled triangle).

Understanding of Concepts

– The student demonstrates some understanding of the Pythagorean theorem when analysing the data and looking for relationships (e.g., in question 2c, identifies the Pythagorean theorem, but presents the area of the hypotenuse in the diagram of the right-angled triangle incorrectly as 38 cm^2 instead of 32 cm^2).

Application of Mathematical Procedures

– The student applies mathematical procedures with some errors and/or omissions when investigating the Pythagorean theorem (e.g., in question 4, uses the Pythagorean formula correctly to find the length of the rectangle, but does not use it correctly to find the rectangle's area).

Communication of Required Knowledge

– The student uses mathematical language and notation with some clarity to analyse and describe geometric relationships and concepts (e.g., in question 3c, uses some proper terminology in the explanation of the Pythagorean theorem: "the sum of the length squared on the right angle is equal to the length of the hypotenuse squared").

Comments/Next Steps

– The student should create charts to organize data and to identify patterns or relationships that become apparent.
– The student should use the correct conventions when mentioning polygons.
– The student needs to accurately apply the Pythagorean theorem when solving problems.
– The student should provide written explanations when asked to explain how an answer was derived.
– The student should consider using mathematical software, such as The Geometer's Sketchpad, when solving problems and looking for relationships.