Teacher Package

Mathematics Exemplar Task
Grade 7 – Patterning and Algebra

Teacher Package

Title: From Patterns to Prediction

Time requirement: 120 minutes (total)
- 40 minutes for the introductory activity
- two periods of 40 minutes each for the exemplar task

Description of the Task

This task requires students to:
- extend and describe a pattern;
- investigate sequences and patterns.

Following a given design, each student will build a series of patterned structures, describe the pattern, and use it to make a prediction. Each student will then construct his or her own pattern, record and describe it, and pose and answer a question about it. Next, students will identify and describe the patterns in a series of triangles. They will then determine the pattern of the number of tiles in a given series of rectangular shapes and in a series that they design; they will use the patterns to make predictions and/or solve problems.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from to the Ministry of Education's *Curriculum Unit Planner* (CD-ROM).

Students will:

1. identify the relationships between whole numbers and variables (7m66);
2. identify, extend, create, and discuss patterns using whole numbers and variables (7m67);
3. identify, create, and solve simple algebraic equations (7m68);
4. extend a pattern, complete a table, and write words to explain the pattern (7m71);
5. recognize patterns and use them to make predictions (7m72);

6. interpret a variable as a symbol that may be replaced by a given set of numbers (7m73);
7. present solutions to patterning problems and explain the thinking behind the solution process (7m75);
8. evaluate simple algebraic expressions by substituting natural numbers for the variables (7m76);
9. translate simple algebraic expressions by substituting natural numbers for the variables (7m77);
10. solve problems giving rise to first-degree equations with one variable by inspection or by systematic trial (7m79).

Teacher Instructions

Prior Knowledge and Skills Required

To complete this task, students should have some knowledge or skills related to the following:
- recognizing and discussing mathematical relationships between and among patterns
- identifying, extending, and creating patterns in a variety of contexts
- applying patterning strategies to problem-solving situations
- identifying, creating, and solving simple algebraic equations
- the following mathematical expressions: *term, variable, algebraic, expression, linear pattern, equation*

The Rubric*

The rubric provided with this exemplar task is to be used to assess students' work. The rubric is based on the achievement chart given on page 9 of *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*.

Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar tasks.

**The rubric is reproduced on pages 138–139 of this document.*

Materials and Resources Required

Before students attempt a particular task, provide them with the appropriate materials from among the following:

- a copy of the Student Package (see Appendix 2) for each student
- coloured tiles
- a computer and The Geometer's Sketchpad software (optional)
- grid paper
- geoboards
- toothpicks

Task Instructions

Introductory Activities

The pre-tasks are designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

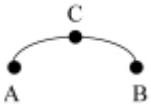
Pre-task: The Game of Sprouts

1. Many mathematicians recommend the use of a game to introduce, develop, or extend a topic. In this task, the game of Sprouts affords students the opportunity to spot patterns, to generalize, and to pose "what if" questions. "What if" questions are powerful in mathematics, as they enable students to see the big picture. Introduce students to the game of Sprouts, which was designed by the British mathematician John Conway. Begin by explaining as follows: "In the game of Sprouts, students start with a specific number of dots, which represent towns. Students join two towns (dots), or join one town (dot) to itself, with an arc to represent a road, which cannot cross any other road. They then place a new town on this new road. No town can have more than three roads attached to it."

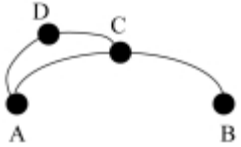
   Show students the board at the beginning of a game that starts with two towns (dots) and demonstrate how the game continues. Say:

   – "This is the start." (two dots)

   – "Connect A to B with a road (an arc) and add another town (C) on the road."



   – "Now connect A to C with another road and add another town (D) on the road."



   – "Continue joining towns with roads, making sure that no more than three roads converge on a town. Play until there are no more moves."

2. Have students play the game with different numbers of towns at the start, organize their findings, describe the pattern they discover, and use the pattern to make a prediction. (See Appendix 1: Game of Sprouts.)

3. You may also have students explore patterns using The Geometer's Sketchpad.

Exemplar Task

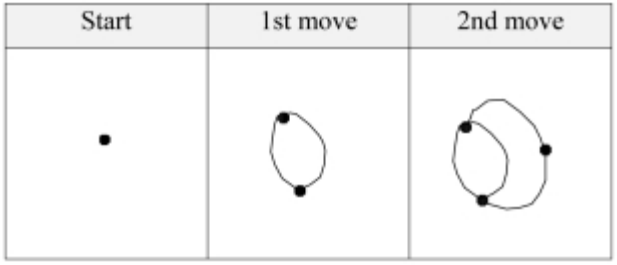
1. Distribute a copy of the Student Package to each student.
2. Tell students that they will be working individually and independently to complete the assigned task.
3. Remind students about the rubric and make sure that each student has a copy of it.
4. The following are notes about specific questions:
   - Question 1: Students investigate sequences and patterns and determine the rule. Students can use toothpicks to form the structures shown.
   - Question 3: Students investigate patterns with two variables.
   - Question 4: Students investigate patterns and make predictions.
5. The problem that the students will solve independently is provided in the worksheets in Appendix 2.

Appendix 1

Instructions for the Game of Sprouts

In the game of Sprouts, start with a certain number of towns (dots). Join two towns (dots), or join one town to itself, with a road (an arc), which cannot touch or cross any other road or town. Now place a new town on this new road. No town can have more than three roads attached to it.

If you start with one town, the game would look like this:



Play the game starting with 2, 3, or 4 towns. Organize your findings in a chart as follows:

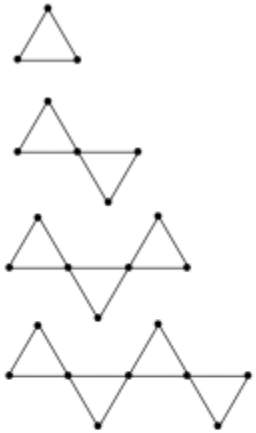
Towns at the start of the game	Towns at the end of the game
1	
2	
3	
4	
5	
etc.	

a) Describe your pattern so that someone looking at your description can see why you made the observations you did.

b) If there were *n* towns at the start, how many towns would there be at the end?

Appendix 2: Student Worksheets

1. a) Use toothpicks to build the first four structures shown below.



Number of triangles	Number of dots	Number of segments
1 →	3	3
2 →	5	6
3 →		
4 →		
5 →		
12 →		
110 →		

b) What are some of the patterns you notice?

c) If there were *n* triangles, how many line segments would there be? Explain how you arrived at your answer. Show two different ways to arrive at your answer.

2. a) Use toothpicks to construct a sequence of shapes similar to those shown in question 1.

Record your sequence below.

b) Describe any patterns you observe in at least two different ways. You may use pictures, words, diagrams, or an algebraic expression.

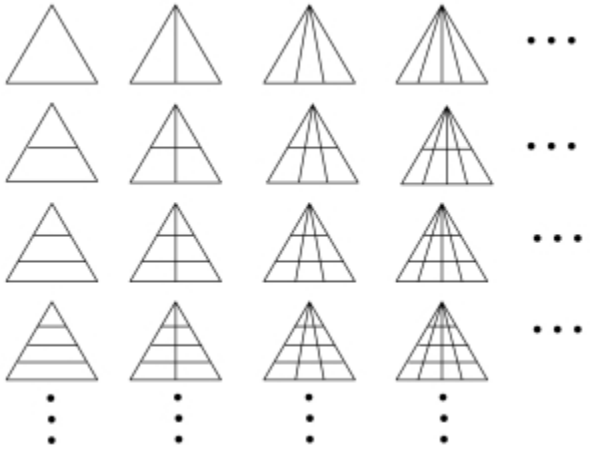
c) Pose a question based on your pattern.

d) Now show how you would answer the question you have just posed.

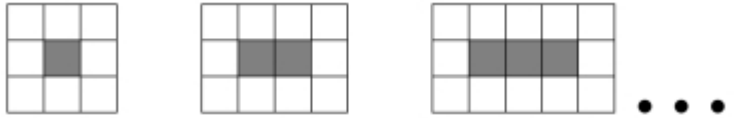
3. These triangles contain some vertical and horizontal lines.

   You may want to use toothpicks to build them.

   Identify and describe ***all*** of the patterns you observe.



4. A rectangular pool is surrounded by a patio.



   a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.

   b) If the pool has an area of *n* square units, how many tiles are needed for the patio? Show how you arrived at your answer.

c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

5. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.

b) Describe how you would find out how many tiles would be in the thirteenth pool.