

From Patterns to Prediction **Level 3, Sample 1**

A

Exemplar Task

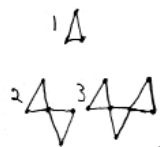
1. a) Use toothpicks to build the first four structures shown below.

	Number of Triangles	Number of dots	Number of Segments
	1	3	3
	2	5	6
	3	7	9
	4	9	12
	5	11	15
	12	25	36
	110	221	330

B

b) What are some of the patterns you notice?

The two patterns that I noticed were that the relationship between the number of triangles and the number of dots is multiplied by two and add one. The other pattern noticed is the relationship between the number of triangles and the number of line segments which is multiplied the number of triangles by three to get the number of line segments.



No. of Triangles	No. of Dots	No. of Triangles	No. of Line Segments
1	3	1	3
2	5	2	6
3	7	3	9
<hr/>		<hr/>	
320	661	330	990

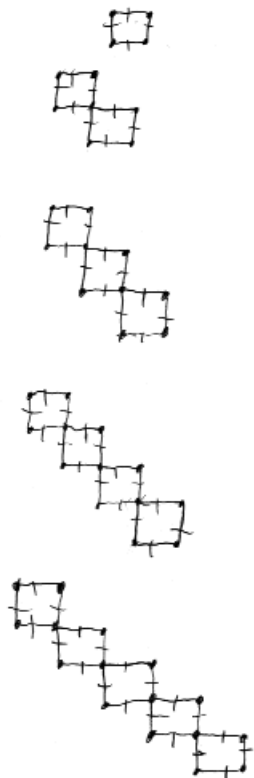
c) If there were n triangles, how many line segments would there be? Explain how you arrived at your answer. Show two different ways to arrive at your answer.

If there were n number of triangles as you wanted to know the number of line segments you would multiply n by 3 and then you would have your answer. You could also use the equation $l = nx + 3$ to get your answer. See equation $l = nx + 3$ means that the number of line segments is equal to the number of triangles (n) multiplied by 3.

C

2. a) Use toothpicks to construct a sequence of shapes similar to those shown in question 1.

Record your sequence below.



D

b) Describe any patterns you observe in at least two different ways. You may use pictures, words, diagrams, or an algebraic expression.

No. of Squares	No. of Dots
1	4
2	7
3	10
4	13
5	16

words
 To get the number of dots you must multiply the number of squares by 3 and then increase that number by one.

Algebraic Expression

$D = S \times 3 + 1$
 which means the number of dots is equal to the number of squares multiplied by 3 increased by one.

c) Pose a question based on your pattern.

eg. how many squares would you need to make out the number of dots.

d) Now show how you would answer the question you have just posed.

eg. you could try number of squares you would require to make number of dots by using the equation $D = nx + 1$ which means the number of dots is equal to the number of squares multiplied by 3 increased by 1

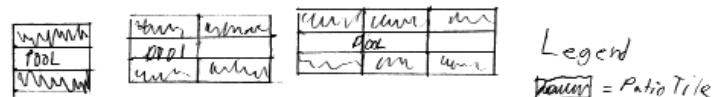
G

- c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

Because there were two hundred six patio tiles you could build a 100 square unit pool. You would do this because to figure out the number of patio stones you multiply the number of square units in a pool by two and six but now you want to do the opposite. You would divide the number of patio tiles 206 by two and decrease it by six to get 100.

H

5. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.



- b) Describe how you would find out how many tiles would be in the thirteenth pool.

You would build out how many tiles would be in the border of the pool. By dividing the number of tiles surrounding the pool by two.
 2. The equation is $P = T + 2$, which means that the number of tiles in the pool is equal to the number of tiles around it divided by two.

Number of tiles around pool	Number of tiles in pool
2	1
4	2
6	3
8	4
10	5
12	6
14	7
16	8
18	9
26	13

Transcription of Student's Answers

1. b) The two patterns that I noticed were that the relationship between the number of triangles and the number of dots is multiplied by two and add one. The other pattern I noticed is the relationship between the number of triangles and the number of line segments which is multiply the number of triangles by three to get the number of line segments.
- c) If there were n number of triangles and you wanted to know the number of line segments you would multiply n by 3 and then you would have your answer. You could also use the equation $l = n \times 3$ to get your answer. The equation $l = n \times 3$ means that the number of line segments is equal to the number of triangles (n) multiplied by 3.
2. b) Words
To get the number of dots you must multiply the number of squares by 3 and then increase that number by one.
- Algebraic Expression
-
- $$D = 5 \times 3 + 1$$
- Which means the number of dots is equal to the number of squares multiplied by 3 increased by one.
- c) If you had n number of squares how would you figure out the number of dots?
- d) If you had n number of squares you would figure out the number of dots by using the equation $D = n \times 3 + 1$ which means the number of dots is equal to the number of squares (n) multiplied by 3 increased by 1.
3. In one across the pattern would be divided by 1, 2, 3, 4 and so on. The pattern in one down would be the same only with horizontal lines. The pattern present in two across is divided by the multiples of two and the same pattern is in two down. The visible pattern in 3 across and down is divided by the multiples of three. In four across and down the pattern present is

divided by the multiples of four. If this overall pattern was to continue, the next set of triangles would have the pattern of divided by the multiples of five. The last pattern I see is that the pattern of division is the same for 1 across and 1 down, two across and two down, and so on.

4. a) If the pool has an area 25 units I think you will need 56 tiles for the patio because if you multiply the number of units in the pool by 2 and then add six it will give you the number of tiles you need for the patio (see diagrams above).
- b) If the pool has n number of square units and you wanted to figure out the number of patio stones that you would need to surround the pool you could use the equation $T = P \times 2 + 6$. This equation means that the number of patio tiles is equal to the number of square units of the pool multiplied by two increased by six.
- c) If there were two hundred six patio tiles you could build a 100 square unit pool. You could do this because to figure out the number of patio stones you multiply the number of square units in the pool by two and add six but here you want the opposite so you do the opposite. You would divide the number of patio tiles 206 by two and decrease it by six to get 100.
5. b) You would find out how many tiles would be in the thirteenth pool by dividing the number of tiles surrounding the thirteenth pool by 2. The equation is $P = T \div 2$, This means that the number of tiles in the pool is equal to the number of tiles around divided by two.

Teacher's Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., frequently and accurately uses charts to record patterns; in question 3, identifies multiplication patterns).
- The student selects and applies an appropriate problem-solving strategy to investigate number patterns, arriving at a generally complete and accurate solution (e.g., in question 4c, uses a formula from question 4b to work backwards; in question 5, draws a diagram and produces an accurate chart to record the patterns investigated).

Understanding of Concepts

- The student interprets a variable as a symbol that may be replaced by a given set of numbers with general success (e.g., uses appropriate substitutions throughout the task, including variables that are appropriate to the task: T = tiles and P = pool in question 4).
- The student demonstrates a general understanding of how to make a generalization from a pattern (e.g., in question 5b: “The equation is $P = T \div 2$, This means that the number of tiles in the pool is equal to the number of tiles around divided by two”; all formulas developed are appropriate).
- The student demonstrates a general understanding of linear patterns (e.g., in question 1c, writes an equation in two ways and identifies how the formulas are similar; in question 3, finds the multiplication patterns).

Application of Mathematical Procedures

- The student uses mathematical procedures that include few errors and/or omissions (e.g., in question 4c, applies the correct formula accurately to find a solution; and, in question 5b, demonstrates the answer with a chart).

Communication of Required Knowledge

- The student uses mathematical language and notation clearly to describe the various patterns (e.g., explanations generally include a written statement, a visual representation, and an algebraic expression, as in question 2b).

Comments/Next Steps

- The student should use a more systematic approach to finding patterns, such as only changing one variable when looking for patterns, to ensure that the pattern sought can be found efficiently.
- The student needs to use the suggested manipulatives – toothpicks – for specific questions to ensure that patterns are easily found and fully explored.
- Although not all the responses were correct, the overall level of the work is level 3.
- The student should use alternative methods (e.g., printing, word processing) to ensure that his or her work is legible.

From Patterns to Prediction

Level 3, Sample 2

A

Exemplar Task

1. a) Use toothpicks to build the first four structures shown below.



Number of Triangles	Number of dots	Number of Segments
1	→ 3	3



2	→ 5	6
---	-----	---



3	→ 7	9
---	-----	---



4	→ 9	12
---	-----	----

5	→ 11	15
---	------	----

12	→ 25	36
----	------	----

110	→ 221	330
-----	-------	-----

B

- b) What are some of the patterns you notice?

The patterns I notice is that for the # of triangles there are, the # of dots is twice it plus one and the segments are three times the # of triangles,

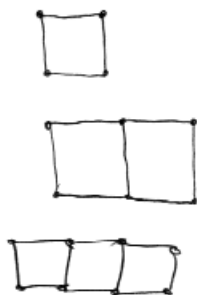
- c) If there were n triangles, how many line segments would there be? Explain how you arrived at your answer. Show two different ways to arrive at your answer.

If there are n triangles there would be $3n$ line segments. I figured this out because I saw that in the chart if there was four Δ 's, there would be 12 toothpicks. I saw the picture and the chart and then I built it myself,

C

2. a) Use toothpicks to construct a sequence of shapes similar to those shown in question 1.

Record your sequence below.



# of □	# of /	# of •
1	4	4
2	7	6
3	10	8
4	13	10
25	51	52

D

b) Describe any patterns you observe in at least two different ways. You may use pictures, words, diagrams, or an algebraic expression.

I noticed that the # of segments was the # of $\square \times 3 + 1$, and I noticed that the # of •s was the # of $\square \times 2 + 2$

$$/ = n \times 3 + 1 \quad \bullet = n \times 2 + 2$$

c) Pose a question based on your pattern.

How many toothpicks would be needed if you had to make 100 squares?

d) Now show how you would answer the question you have just posed.

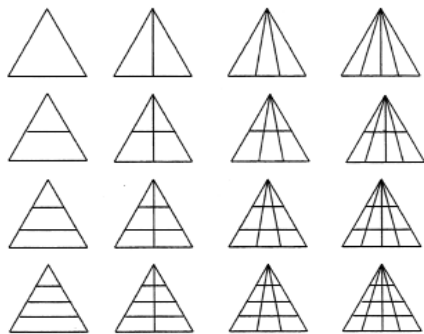
It can be answered like this:
 $n \times 3 + 1$
 $100 \times 3 + 1 = 301$

E

3. These triangles contain some vertical and horizontal lines.

You may want to use toothpicks to build them.

Identify and describe *all* of the patterns you observe.



... this goes up by one each time

... this goes up by two each time

... this goes up by three each time

... this goes up by four each time

this diagonal row goes 1, 6, 11

this goes up each time by 2

this goes up each time by 2

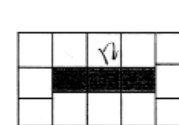
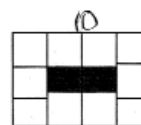
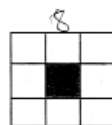
this goes up each time by three

this goes up each time by four

one line is added each time

F

4. A rectangular pool is surrounded by a patio.



the # of squares goes up by 2 each time

a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.

If you have a pool with the area $25u$, then the # of tiles is $25 \times 2 + 6$, which is 56

b) If the pool has an area of n square units, how many tiles are needed for the patio? Show how you arrived at your answer.

If a pool has an area of n , the # of tiles is $n \times 2 + 6$.

G

- c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

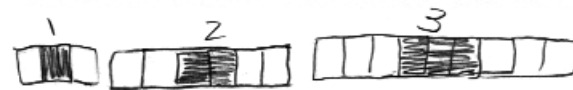
If there are 206 tiles, you subtract 6, and divide in half, getting 100. That's the area of the pool

$$n - 6 \div 2$$

$$\begin{array}{r} 206 \\ - 6 \\ \hline 200 \end{array} \qquad \begin{array}{r} 100 \\ 2 \overline{)200} \end{array}$$

H

5. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.



$$P = 2t$$

- b) Describe how you would find out how many tiles would be in the thirteenth pool.

For the 13th pool there would be 26 tiles because the # of ■'s is half as much as the # of □'s

Teacher's Notes**Problem Solving**

- The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., uses a formula developed in question 4b to solve the problem in question 4c, reversing the formula to work backwards).
- The student selects and applies an appropriate problem-solving strategy to investigate number patterns, arriving at a generally complete and accurate solution (e.g., in question 3, sees a diagonal pattern, as well as adding patterns; records patterns in charts and draws visual representations of the patterns).

Understanding of Concepts

- The student interprets a variable as a symbol that may be replaced by a given set of numbers with general success (e.g., in question 1c, defines n and uses it accurately in the formula proposed).
- The student demonstrates a general understanding of how to make a generalization from a pattern (e.g., in question 2b, uses observations from question 2a to develop appropriate formulas; in question 5a, develops his or her own formula, but finds many solutions without using formulas, as in questions 4c and 5b).
- The student demonstrates a general understanding of linear patterns (e.g., in question 3, sees the addition patterns, but does not identify them as multiplying patterns).

Application of Mathematical Procedures

- The student uses mathematical procedures that include few errors and/or omissions (e.g., in question 2a, makes a simple error in the chart; uses formulas to solve problems throughout).

Communication of Required Knowledge

- The student uses mathematical language and notation clearly to describe the various patterns (e.g., explanations include a written statement, a visual representation, and an algebraic expression, as in question 5; written statements are brief).

Comments/Next Steps

- The student must ensure that all components of questions are addressed, such as showing how solutions are reached in two different ways.
- The student should use a more systematic approach to finding patterns, such as creating a T-chart, to ensure that the pattern identified actually continues in further examples.
- The student must use the suggested manipulatives – toothpicks – for specific questions to ensure that patterns are found and fully explored.
- Although some of the solutions were not correct, the overall level of the work is level 3.