From Patterns to Prediction Level 2, Sample 1

A

Exemplar Task

1. a) Use toothpicks to build the first four structures shown below.

Number of Triangles	Number of dots	Number of Segments
1	3	3
2	5	6
3	7	9
4	9	12
5	11	15
12	25	36
110	202	330

101 dots x2 = 202

___ Segments x2 =

B

b) What are some of the patterns you notice?

I noticed that the number of dots multiplied by 3 (the number of segments in 1 triangle) equals the total number of segments in the structures. I also noticed, as the number of triangles goes up in the structure, like 1, 2, 3, 4, the number of dots goes up by odds.

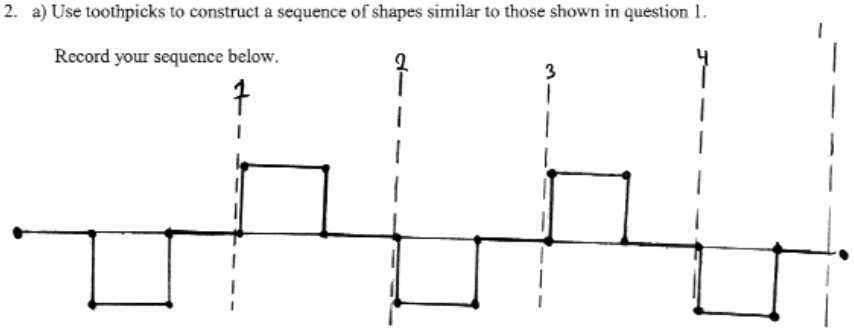
c) If there were *n* triangles, how many line segments would there be? Explain how you arrived at your answer. Show two different ways to arrive at your answer.

If there were n triangles there would be n x 3 = n line segments, another way is x x 3 = y.

C

2. a) Use toothpicks to construct a sequence of shapes similar to those shown in question 1.

Record your sequence below.



Squares	Number of dots	Number of line segments
1	6	6
2	10	10
3	14	14
4	18	18
5	22	22

D

b) Describe any patterns you observe in at least two different ways. You may use pictures, words, diagrams, or an algebraic expression.

My pattern has ~~one~~ six dots every 1 ~~Segment~~ Square, and 4 dots every Square.

c) Pose a question based on your pattern.

What if there happend to be 115 squares, how many line segments would be in the structure???

d) Now show how you would answer the question you have just posed.

$5\overline{)115}$ 23

22 × 23 = 506

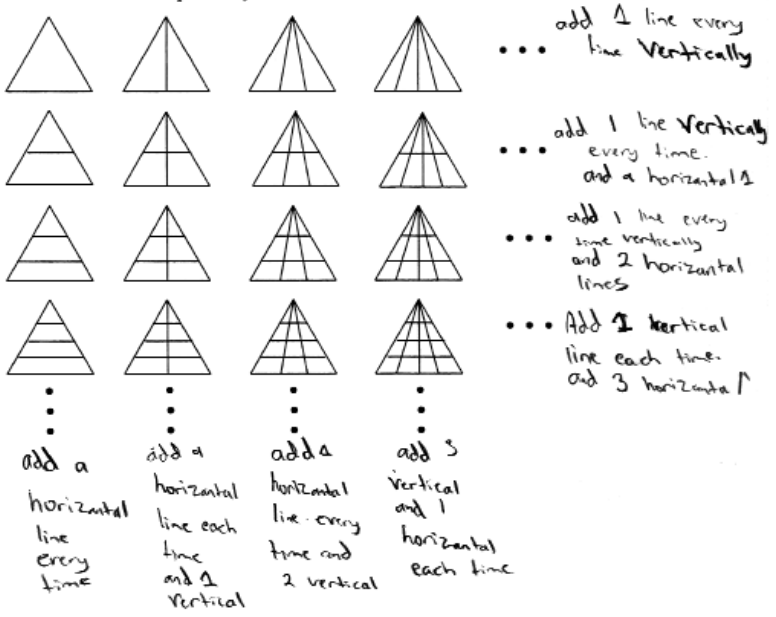
there would be 506 line segments

E

3. These triangles contain some vertical and horizontal lines.

You may want to use toothpicks to build them.

Identify and describe ***all*** of the patterns you observe.



• • • add 1 line every time Vertically

• • • add 1 line Vertically every time. and a horizontal 1

• • • add 1 line every time vertically and 2 horizontal lines

• • • Add 1 vertical line each time. and 3 horizontal

F

4. A rectangular pool is surrounded by a patio.

a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.

25
+25
+25
75
-25 Pool tiles
50+6 for the ends = 56

56 tiles are needed for the patio

b) If the pool has an area of ***n*** square units, how many tiles are needed for the patio? Show how you arrived at your answer.

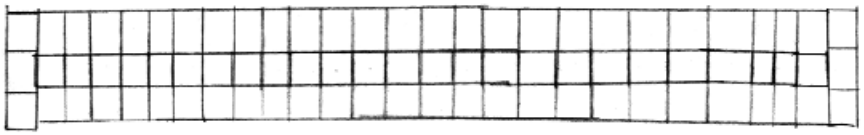
Pool □'s	Patio □'s
1	8
2	10
3	12
4	14
5	16
n	n+2

you would add 2 to n because you add 2 tiles every time.

n+2=

Or

x+2=y



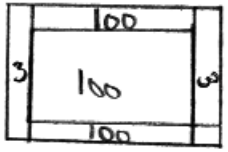
G

c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

6 = the ends tiles. 200 ÷ 2 because there is 100 patio tiles on each side.

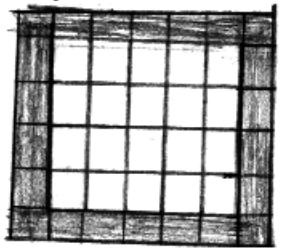
206 − 6 = 200 ÷ 2 = 100

There is 100 pool tiles you could build a pool that is 100 × 3.



H

5. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.



22 patio tiles

20 pool tiles

b) Describe how you would find out how many tiles would be in the thirteenth pool.

Pool #	# of tiles
1	8
2	10
3	12
4	14
5	16
6	18
7	20
8	22
9	24
10	26
11	28
12	30
13	32

There would be 32 tiles in the 13th pool.

Teacher's Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., in question 2, looks for patterns with a linear approach, but in question 2b, includes an explanation that does not support the pattern drawn; in question 4b, uses a chart to record information when looking for a pattern; in question 5b, incorrectly uses the same chart as that used in question 4b).
- The student selects and applies an appropriate problem-solving strategy to investigate number patterns, arriving at a partially complete and/or partially accurate solution (e.g., in question 3, uses a systematic approach to identify only simple adding patterns for each row and column).

Understanding of Concepts

- The student interprets a variable as a symbol that may be replaced by a given set of numbers with some success (e.g., in questions 4b and 1c, variables are shown, but the variables are not used to find solutions and are not defined or labelled).
- The student demonstrates some understanding of how to make a generalization from a pattern (e.g., in question 4b, states an incorrect formula, but solves subsequent questions correctly without the use of an algebraic expression).
- The student demonstrates some understanding of linear patterns (e.g., in question 3, identifies some patterns and, in question 4, explains how the original algorithm works, but, in question 5b, repeats the pattern from question 4 when a different pattern is required).

Application of Mathematical Procedures

- The student uses mathematical procedures that include some errors and/or omissions (e.g., in question 1b, attempts to develop a formula and apply it to question 1c; in question 4a, finds a solution without using a formula; in question 5b, does not apply the correct formula/pattern in the chart to find the tiles in the thirteenth pool).

Communication of Required Knowledge

- The student uses mathematical language and notation with some clarity to describe the various patterns (e.g., in question 4b, explains how the pattern grows rather than how to find the solution, while including a short explanation: "you would add 2 to *n* because you add 2 tiles every time").

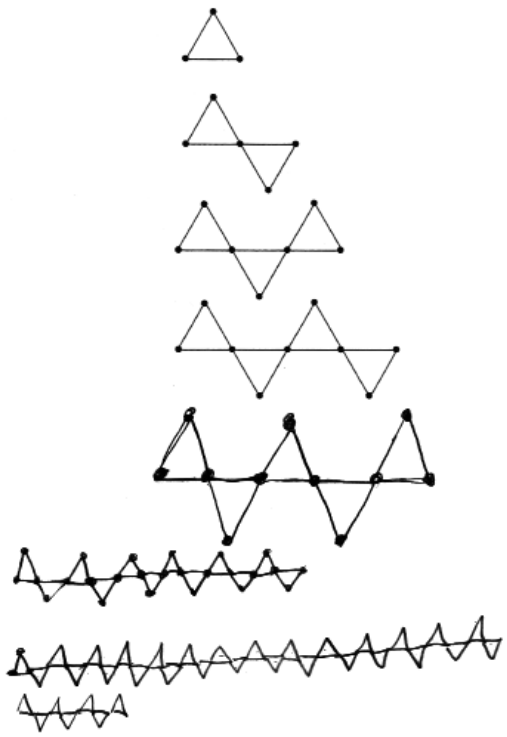
Comments/Next Steps

- The student should use the suggested manipulatives – toothpicks – to fully explore the patterns before attempting to answer the questions.
- The student should look more extensively for patterns, using a systematic approach, such as changing only one variable.
- The student could draw charts and diagrams as problem-solving strategies to assist in communicating solutions.
- The student should include more explanations and examples when communicating personal ideas or solutions.

A

Exemplar Task

1. a) Use toothpicks to build the first four structures shown below.



Number of Triangles	Number of dots	Number of Segments
1	3	3
2	5	6
3	7	9
4	9	12
5	11	15
12	25	36
110	220	330

B

b) What are some of the patterns you notice?

\# one is the # of dots go up by two 3, 5, 7, 9, 11, 13 and so on. Another one is the # of segments go up by 3.

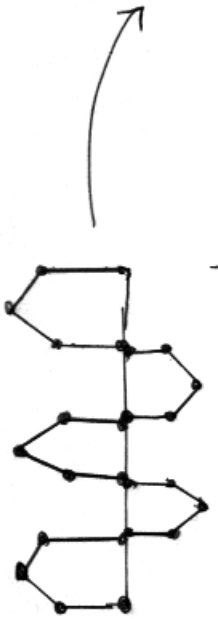
c) If there were *n* triangles, how many line segments would there be? Explain how you arrived at your answer. Show two different ways to arrive at your answer.

2N+1 I am not sure but

C

2. a) Use toothpicks to construct a sequence of shapes similar to those shown in question 1.

Record your sequence below.



# of shapes	# of dots	# of lines
1	5	5
2	9	10
3	13	15
4	17	20
5	21	25
8	33	40

D

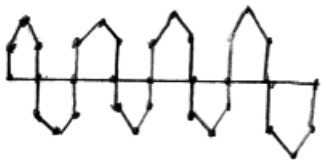
b) Describe any patterns you observe in at least two different ways. You may use pictures, words, diagrams, or an algebraic expression.

One thing is when you are adding more shapes you can see the differe in the # of dots and the lines 1st they are the same the second shape 1 difference 3rd 2 4th 3 and so on also the # of dots go up by four (four) and lines go up by 5

c) Pose a question based on your pattern.

On the 8th shape how many dots are there and how many lines are there?

d) Now show how you would answer the question you have just posed.



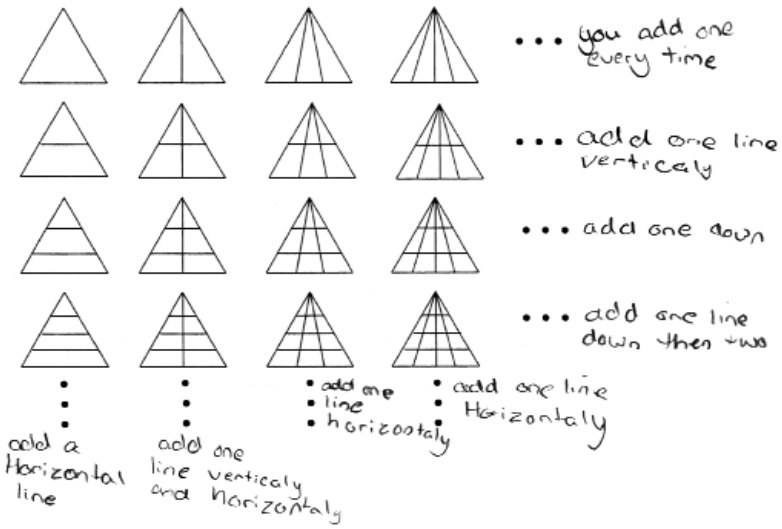
The would be 33 dots and there would be 40 lines.

E

3. These triangles contain some vertical and horizontal lines.

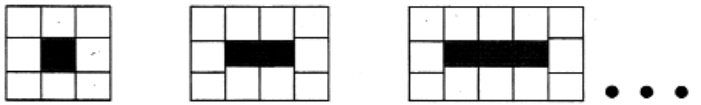
You may want to use toothpicks to build them.

Identify and describe ***all*** of the patterns you observe.

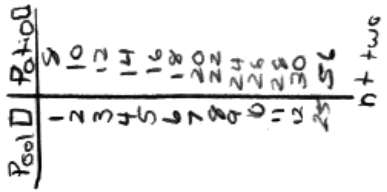


F

4. A rectangular pool is surrounded by a patio.



a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.

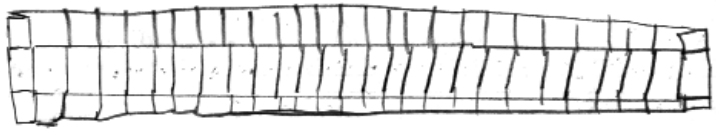


b) If the pool has an area of ***n*** square units, how many tiles are needed for the patio? Show how you arrived at your answer.

n+2 because you add two every time.

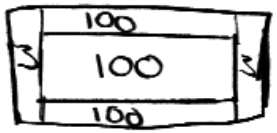
N+2

81
-25
56



G

c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

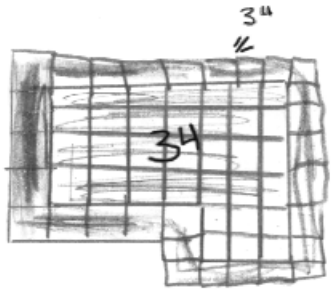


If you have 206 patio tiles you divide it by two the you 6 off that you have

100 on top
100 on the bottom
3 on each side
and 100 inside.

H

5. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.



there is 30 patio tiles and 34 ☐ for the pool

b) Describe how you would find out how many tiles would be in the thirteenth pool.

☐ Pool	☐ Patio
1	8
2	10
3	12
4	
5	14
6	16
7	18
8	20
9	22
10	24
11	26
12	28
13	30

there would be 30

Teacher's Notes

Problem Solving

- The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., uses charts to record patterns; in question 4c, uses a diagram; in question 5b, uses a chart, but the chart is not appropriate based on the diagram drawn in question 5a).
- The student selects and applies an appropriate problem-solving strategy to investigate number patterns, arriving at a partially complete and/or partially accurate solution (e.g., in question 3, identifies simple horizontal and vertical patterns, but does not identify all the patterns in each row or column).

Understanding of Concepts

- The student interprets a variable as a symbol that may be replaced by a given set of numbers with some success (e.g., in questions 1c and 4b, shows variables, but does not explain them or use them to find any solutions throughout the task).
- The student demonstrates some understanding of how to make a generalization from a pattern (e.g., in question 4, attempts to develop and use a formula, but relies on an alternative approach to find a solution).
- The student demonstrates some understanding of linear patterns (e.g., in question 1a, continues the pattern, but finds the solution for question 4a by showing a diagram in question 4b, and not using the number patterns that emerge from the chart in question 4a).

Application of Mathematical Procedures

- The student uses mathematical procedures that include some errors and/or omissions (e.g., in question 2b, identifies the pattern but not the formula; tends to find solutions without using a formula, as in questions 2d and 4c; in question 4b, the formula is based on the patio stones rather than on the relationship between the pool area and the corresponding number of patio stones).

Communication of Required Knowledge

- The student uses mathematical language and notation with some clarity to describe the various patterns (e.g., provides short explanations, as in question 1b; generally relies on diagrams as evidence of patterns and writes explanations that lack detail, as in question 3).

Comments/Next Steps

- The student should look more extensively for patterns using diagrams and should record results accurately on a corresponding chart .
- The student should use a systematic approach.
- The student needs to include more explanations and examples when communicating personal ideas or solutions. His or her written statements should be more detailed and presented in an organized, easy-to-read format.
- The student should use substitution in algebraic expressions to find accurate solutions.