Teacher Package

Mathematics Exemplar Task
Grade 7 – Measurement, and Number Sense and Numeration

Teacher Package

Title: Saving Space

Time requirement: 115 minutes (total)
- 40 minutes for the introductory activities
- two periods of 45 minutes for the exemplar task

Description of the Task

This task requires students to:
- construct open-top boxes with the greatest volume;
- investigate the relationship between surface area and volume.

For each of three sheets of square paper used to make open boxes, students will remove different sizes of squares from the sheet corners to find the open box with the greatest volume. They will use a guess-and-check or looking-for-a-pattern strategy and organize their findings in a chart to show how the volume changes as squares were removed. Students will then compare the volume of the rectangular prism having the greatest volume with the volume of four rectangular prisms made of sheets formed by dividing the original square sheet into four congruent squares. Next, students will determine how to wrap with the least amount of foil a chocolate bar of given dimensions. They will then determine the optimum size of packaging for soft drinks. Finally, they will support their reasons for agreeing or disagreeing with a statement about the relationship between surface area and volume.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from to the Ministry of Education's *Curriculum Unit Planner* (CD-ROM).

Number Sense and Numeration

Students will:

1. solve and explain multi-step problems involving simple fractions, decimals, and percents (7m6);
2. use a calculator to solve number questions that are beyond the proficiency expectations for operations using pencil and paper (7m8);
3. perform three-step problem solving that involves whole numbers and decimals related to real-life experiences, using calculators (7m14);
4. ask "what if" questions; pose problems involving simple fractions, decimals, and percents; and investigate solutions (7m23).

Measurement

Students will:

5. apply volume formulas to problem-solving situations involving rectangular prisms (7m31);
6. describe measurement concepts using appropriate measurement vocabulary (7m33);
7. make increasingly more informed and accurate measurement estimations based on an understanding of formulas and the results of investigations (7m35);
8. understand the relationship between the dimensions and the volume of a rectangular prism (7m44);
9. calculate the surface area and the volume of a rectangular prism in a problem-solving context (7m45).

Teacher Instructions

Prior Knowledge and Skills Required

To complete this task, students should have some knowledge or skills related to the following:
- thinking mathematically
- using concrete materials to develop a formula for finding the volume of a rectangular prism
- calculating surface area

The Rubric*

The rubric provided with this exemplar task is to be used to assess students' work. The rubric is based on the achievement chart given on page 9 of *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*.

Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

Materials and Resources Required

Before students attempt a particular task, provide them with the appropriate materials from among the following:

- a copy of the Student Package (see Appendix 1) for each student
- calculators
- interlocking cubes
- isometric dot paper
- scissors
- overhead projector (optional)
- computer and Geometer's Sketchpad software (optional)
- TI-83 calculator (optional)

Task Instructions

Introductory Activities

The pre-tasks are designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

Pre-task 1: Clayton Builds a Pen

Assign the following problem to students after explaining the meaning of the word *pen* as it is used in the problem:

> Clayton has just purchased a goat for his farm. He wants to build a pen so that his goat will not run away. Clayton has purchased 36 m of fencing to use to build a four-sided rectangular pen. How would he design a pen with the maximum grazing area?

You may use the following prompts during the task:

– "What length and width will give the maximum area?"

– "Is there only one answer?"

– "What, if anything, did you notice about the relationship between the length and the width?"

Students may find that the maximum area is achieved when the length and the width are the same magnitude. Also, the closer the numbers (length and width) are to each other, the larger the area will be.

Pre-task 2: Clayton's Ratio

1. Students will further explore the maximum area of the pen by trying to find the ideal ratio for creating the largest area. To find the maximum area, students must use decimals. Students should investigate how the use of integral numbers limits the possibilities.
2. Assign students the following problem:

   Clayton decides to start his own pen-making business. He wants to create some guidelines to help the business run smoothly.

   a) Find the length-to-width ratio for creating the pen with the largest area.
   b) Investigate with different lengths of fencing:
      i) 36 m of fencing
      ii) 25 m of fencing
      iii) 50 m of fencing
   c) Write guidelines that Clayton can apply for any length of fencing.

   You may model this activity using an overhead, the chalkboard, The Geometer's Sketchpad, and/or the TI-83 graphing calculator.

Pre-task 3: Building Rectangular Prisms

Assign the following task to students:

1. Have each student use interlocking cubes to build a rectangular prism.
2. Ask students to show the relationships among the length, width, height, and volume of the structures they built, using a chart like the following:

Length	Width	Height	Volume

3. Ask students: "What happens as you increase the height of your structure by 50 percent?" Encourage students to graph their results showing how volume is dependent on height.
4. Ask: "Where could you apply this concept in a real-life situation?"

**The rubric is reproduced on pages 14–15 of this document.*

Exemplar Task

1. Distribute a copy of the Student Package to each student.
2. Remind students about the rubric and make sure that each student has a copy of it.
3. Tell students that they will be working individually and independently to complete the assigned task.
4. Students will need different sizes of square grids and scissors. They will be required to cut out squares from the corners to form boxes. For question 3 involving chocolate bars, it would be a good idea to show students a real chocolate bar so that they can see how the foil is designed to wrap around the chocolate. Students may want to investigate whether the foil wrappings for different brands of chocolates are designed in the same way.
5. Have students follow the instructions and answer the questions in their Student Packages. Tell them that they are to put their answers in the spaces and use the grid paper provided. If necessary, they may use the back of the paper or use additional lined paper to show their calculations. Remind them that everything is to be handed in for assessment, including their calculations.
6. The problem that the students will solve independently is provided in the worksheets in Appendix 1.

Appendix 1: Student Worksheets

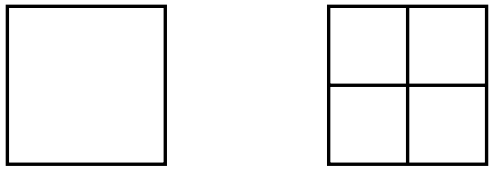
1. A cardboard box manufacturer has a flat sheet of square cardboard. She would like to construct open-top boxes with the greatest volume. This is done by cutting out corners as shown below.

a) Use different-sized sheets of square paper to find out how to obtain the greatest volume for open boxes from the following sheets of paper. Graph paper is provided at the end of the package.

   i) 16 x 16
   ii) 18 x 18
   iii) 20 x 20

b) Summarize your findings below. What did you discover? What are some of the problems you encountered along the way?

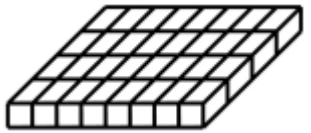
2. What if you were to cut a square piece of paper into four smaller congruent squares and from each construct a box with the maximum volume? How would the combined volume of these four rectangular prisms compare with the rectangular prism with the maximum volume constructed from the original square?



a) Form and record your hypothesis before investigating the problem.

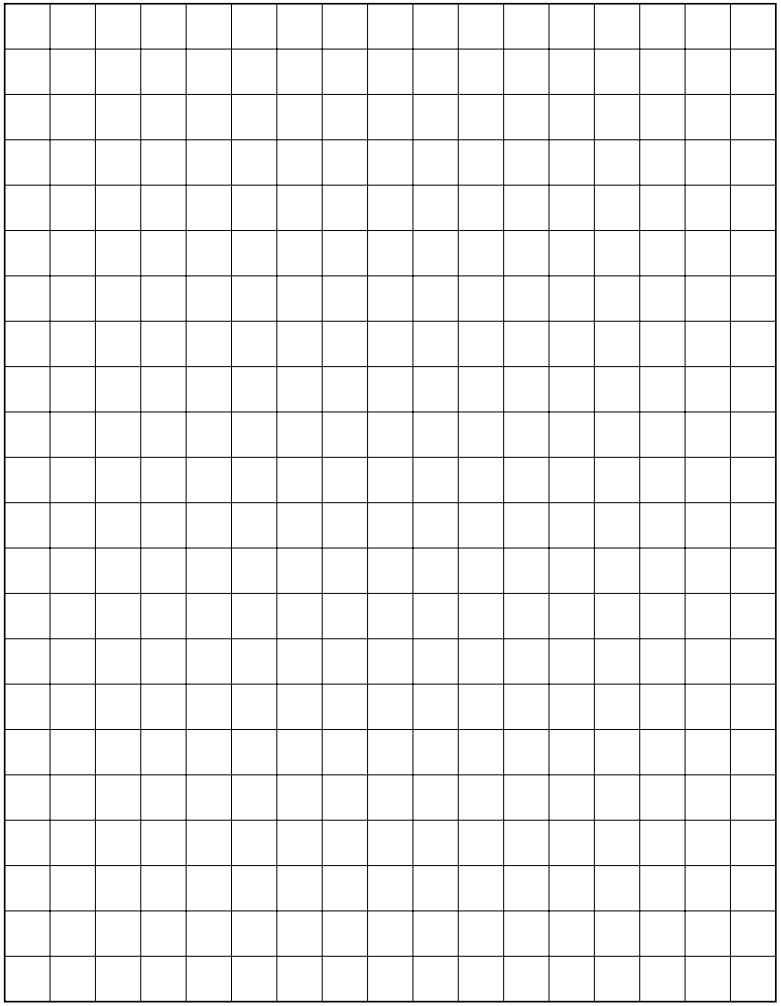
b) Now investigate the problem using whatever approach you believe would be appropriate. How will you modify your hypothesis in light of the results of your investigation?

3. An employee for a chocolate factory has just suggested to the manager that he has discovered a way of using foil to wrap a bar of chocolate that will use the least amount of foil. Prepare a report for management showing the least amount of foil that would be needed to wrap a bar 8 units long, 4 units wide, and 1 unit high.



4. Many brands of soft drink are packaged in rectangular boxes of 24 cans.
Which arrangement would you recommend that a soft drink company use? Why?

5. While investigating the types of boxes that can be made from a piece of paper, a student said that if you know the surface area of the box, you also know the volume of the box.
Show why you agree or disagree with his statement.



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